Geometric Control and Learning for Aerial Robotics

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My research focuses on constructing *differential geometric* approaches for dynamics and control of complex systems evolving on a nonlinear *manifold*. In particular, these have been successfully applied to various autonomous unmanned aerial vehicles, explicitly demonstrating their efficacy and promises in control engineering and robotics through numerical simulations and flight experiments. Recently, geometric control and estimation have been further extended to be integrated with stochastic Bayesian learning and optimization to improve performances by recollecting prior experiences.

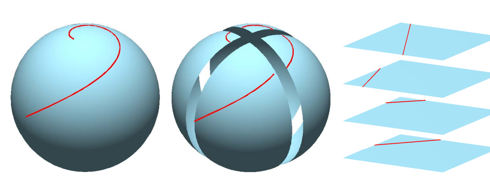
1 Why Does Geometry Matter?

Then, what is really geometric control, and how does geometry play a fundamental role in aerial robotics? While it can be interpreted in several ways, geometric controls defined here refers to controller design and stability analysis carried out *directly on a manifold*, which is a topological space that resembles Euclidean space at every point, but cannot be globally identified with a Euclidean space. If a manifold is accompanied with group structures, it is referred to as a *Lie group*. Simply speaking, geometric control is opposed to nonlinear controls formulated on a Euclidean space, or $\mathbb{R}^n$.

As the fundamental idea of geometric controls is formulating and resolving a control problem on a manifold, it is readily extended to other topics of dynamics and control system engineering, such as Lagrangian/Hamiltonian systems, optimization, uncertainty propagation, estimation, and learning, through which my research portfolio has been developed. In any case, geometric formulation on a manifold takes full advantages of the following properties.

Intrinsic Formulation Most of dynamic systems in aerial robotics, such as unmanned aerial vehicles with a robotic manipulator, evolve on a manifold. This is because any rotational maneuver cannot be formulated globally on any Euclidean space. For example, one-dimensional planar rotations, two-dimensional direction, and three-dimensional attitude of a rigid body are formulated in the embedded manifolds of the unit-circle, the unit-sphere, and the three-dimensional orthogonal group, respectively. As such, it is most natural to describe the dynamics of aerial vehicles directly on a manifold.

Global Formulation The most common approach to formulate dynamics and control on a manifold is utilizing local coordinates. This can be considered as partitioning the manifold into several parts such that each part can be identified as $\mathbb{R}^n$, as illustrated on the right, where the unit sphere is sliced into multiple planar patches. The issue is that they are



local, and as such, multiple sets of coordinates are required to cover the complete manifold. It causes *singularities* and *complexities* when putting together a complex trajectory covered by several sets of coordinates. Whereas, geometric controls completely avoid such issues, as they are formulated directly on the manifold. More importantly, the resulting stability properties hold *globally* for any aggressive maneuver visiting any part of the manifold.

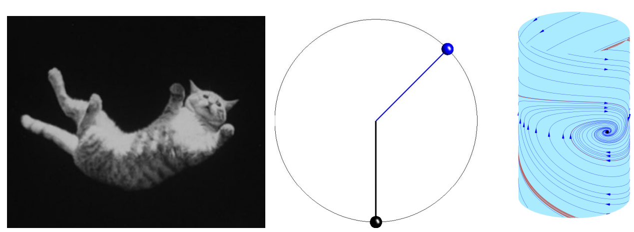
Structured Formulation Furthermore, geometric approaches yield a more structured, elegant form of control, especially when the dynamics of the controlled system is complicated. Consider an aerial manipulator or a micro flapping-wing aerial vehicle composed of articulated rigid bodies, which evolve on a high-dimensional manifold. The equations of motion written in local coordinates, such as Euler angles, would involve lengthy, complicated combinations of trigonometric functions, which prevent any reasonable analysis to grasp the underlying structures. Formulating the dynamics on the configuration manifold provides a more structured, simplified form that may provide valuable insight in control system design and streamline stability analysis.

General Formulation Also, geometric approaches provides powerful machineries to construct control systems on an abstract manifold, or on an abstract Lie group. For example, the PI has shown that the Euler–Lagrange equation for a mechanical system on a Lie group G is formulated as

$$\frac{d}{dt}\mathbf{D}_\xi L(g,\xi) - \mathrm{ad}^*_\xi \cdot \mathbf{D}_\xi L(g,\xi) - \mathsf{T}^*_e\mathsf{L}_g \cdot \mathbf{D}_g L(g,\xi) = 0,$$

where $L(g,\xi)$ corresponds to the Lagrangian dependent of $(g,\xi) \in \mathsf{G} \times \mathfrak{g}$. While it relies on several operations that are uncommon in engineering [1], for dynamic systems considered in aerial robotics, they can be easily interpreted as familiar matrix operations. The desirable feature of the above intrinsic formulation is that it is applied to *any* mechanical system on a Lie group. For example, an optimal control theory developed for the above can be specialized to any aerial robotic system.

Characteristic Formulation Finally, dynamic systems evolving on a compact manifold may exhibit unique characteristics that cannot be described properly in Euclidean space. For example, a cat turn its body in the air by utilizing so-called *geometric phase* effects, which are caused by the curvature of the configuration manifold. Any smooth control system on a compact manifold cannot achieve global asymptotic stability due to the *topological restriction*. The right most figure illustrates the phase portrait of a control system on the unit circle, where the region of attraction to the desired equilibirum excludes a set of zero measure, denoted by a red curve. These phenomenon inherent to a nonlinear manifold can be characterized properly only if the controlled dynamics is formulated on the manifold.



2 Geometric Mechanics and Control for Aerial Robotics

While the above motivation for geometric may sound abstract and distant, geometric approaches are particularly useful for non-trivial maneuvers of complex dynamic systems. The section summarizes a selected set of examples for geometric approaches applied to aerial robotics.

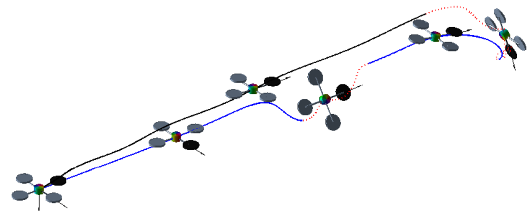
2.1 Geometric Control for Quadrotor

The PI has developed geometric control systems for multi-rotor unmanned aerial vehicles on the special Euclidean group SE(3), which is the semi-direct product of the special orthogonal group SO(3), and the

[1] See for example, J. Marsden and T. Ratiu, *Introduction to Mechanics and Symmetry*, Springer 1999.

three-dimensional Euclidean space $\mathbb{R}^3$ that represent the attitude and the position of the vehicle, respectively.

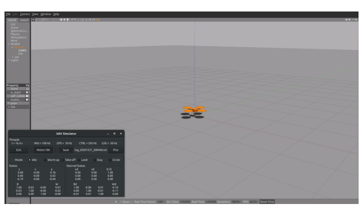
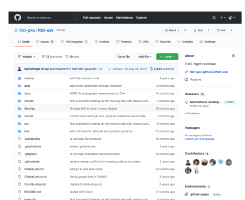
The multi-rotor aerial vehicles are under-actuated in the sense that the direction of the resultant thrust is always fixed to the body, i.e., to change the direction of the thrust, the body should be rotated. As such its attitude dynamics is tightly coupled to the translational dynamics. The PI has proposed a backstepping approach, where the desired attitude is formulated to follow a given desired position trajectory, and the attitude is controlled on SO(3) [1]. It is illustrated that this geometric approach can be utilized for acrobatic maneuvers, for example multiple flipping as shown above. This approach has been one of *de facto* approaches, which has been adopted to various quadrotor control systems, and the corresponding IEEE CDC paper [1] has been well cited.



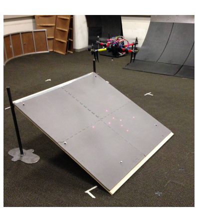
Later, this idea has been extended to various directions: geometric PID control [2], geometric adaptive control [3, 4], nonlinear robust control [5, 6], tethered quadrotor control [7], decoupled yaw control [8, 9], neural-network based adaptive learning control [10, 11]. The geometric formulation has been further utilized in extended Kalman filter [12, 13], and system identification [14] for quadrotor as well.

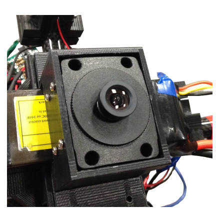
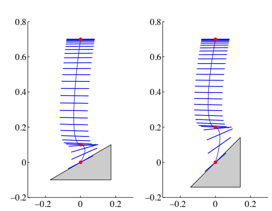
2.2 Autonomous Flight Experiments

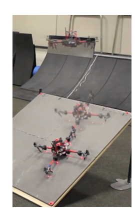
The desirable properties of geometric control have been illustrated by various autonomous flight experiments as well. The PI's lab has designed and developed both of flight software and hardware in-house. The flight hardware is composed of a compact computing module with GPU (NVidia Jetson TX2) running a linux operating system, which can be connected to various sensors (IMU, camera, event camera, Lidar, and GPS) and wireless communication links (WIFI. Bluetooth). The flight software is developed in C++ with multi-threads that allows multiple tasks of control, estimation, communication, and data logging simultaneously. It is connected to ROS/Gazebo for standardized packaging and simulation, and it is maintained in Github.

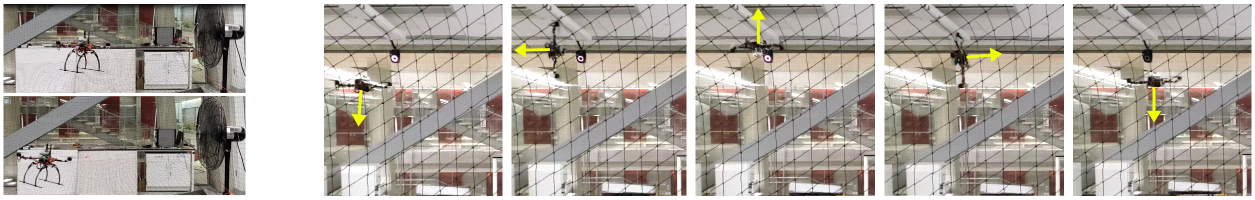

Indoor Flight Experiments For indoor flight experiments, the position of the quadrotor is measured by an infrared motion capture system. Notable experiments include backflipping [4], and landing on an inclined surface [15, 16], as shown below: the inclination and height of the surface are estimated by capturing a pattern projected by laser points, and the landing trajectory is planned based on the estimation results, before the landing maneuver is executed.



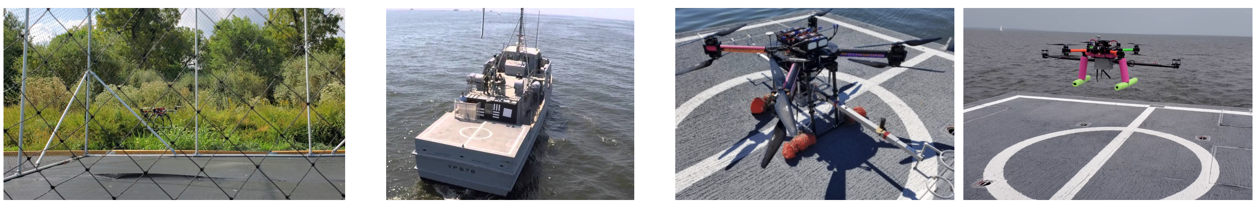





Another interesting experiment is rejecting the wind disturbance caused by an industrial fan [10, 11], where the unknown effects of turbulent wind is compensated by neural networks. First, the advantages over conventional adaptive control were demonstrated (left), followed by performing an aggressive maneuver under the wind gusts (right).



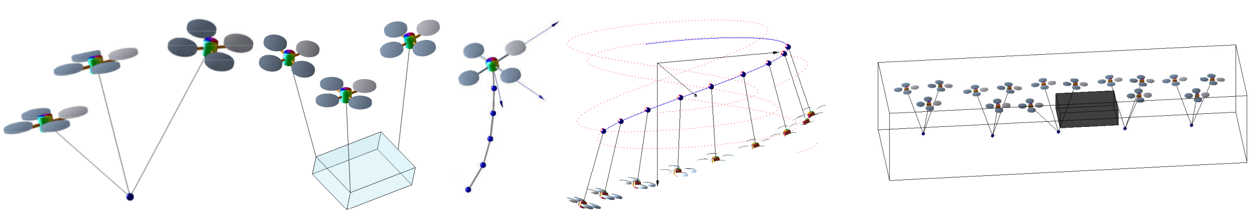
Outdoor Flight Experiments Utilizing a low-cost, real-time kinematic (RTK) GPS, outdoor flight tests have been performed. An extended Kalman filter on SE(3) is extended to handle the delayed measurements of GPS [17]. After verifying stability of controlled flights with RTK GPS in grounds, we are performing flight experiments in Chesapeake Bay, MD for autonomous landing on a USNA research vessel.



2.3 Geometric Control for Aerial Transportation

The next application of geometric control is aerial transportation of a cable-suspended payload. Compared with attaching a payload rigidly, connecting it with a cable is desirable when transporting a large object with collaborating, multiple aerial vehicles, or when transporting in rough terrains where it is challenging to identify a safe landing/take-off site. It also addresses safety issues in urban areas as it can avoid flights in close-proximity to recipients. However, considering the dynamic coupling between payload, cable, and multiple UAV in control system design is challenging due to the inherent complexities.

The PI has formulated the complicated dynamics in a higher-dimensional manifold using geometric mechanics, and design geometric control systems such that the trajectory of the payload asymptotically follow given desired trajectories, under various assumptions including a single point mass payload with one or multiple quadrotors [18–20], a rigid body payload [21, 22], transportation with flexible cables [23–26]. It is further extended to control of a flying, inverted spherical pendulum, and concurrent formation control to avoid collision [27].



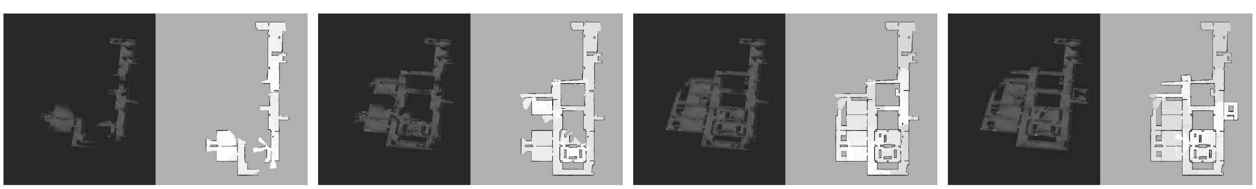
The proposed control systems have been verified indoor flight experiments for stabilization of a point mass [24], stabilization of a rod with two quadrotors [26], and tracking a point mass with a single quadrotor [28]

2.4 Autonomous Aerial Exploration

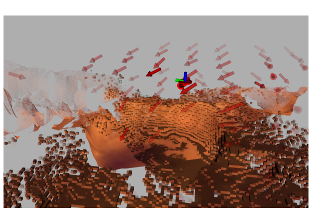
Autonomous aerial exploration is for multiple UAVs actively planning their paths to build a map of an unknown area of interests. Compared with simultaneous localization and mapping (SLAM), where the vehicles passively follow a trajectory determined by a user, in exploration, the aerial vehicles should decide where to go to complete a map. The map is often constructed with a depth sensor, such as stereo vision or Lidar, that can provides a set of points for closest objects or walls, referred to as point cloud.

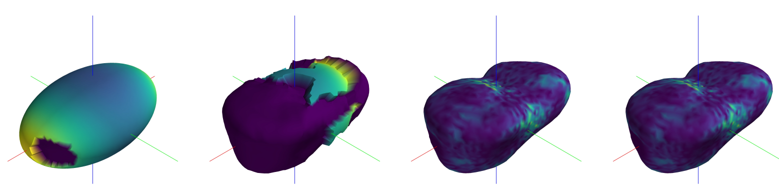
The PI has developed an exact Bayesian inverse sensor model, where the probability of occupancy in each cell of a 3D grid is determined by point clouds [29]. The proposed probabilistic formulation of 3D mapping naturally leads to autonomous exploration, where the motion of aerial vehicles are planned to minimize the uncertainties of mapping measured by Shannon's entropy. The following figures show a flight experiment performed at US Naval Research Lab, where a large area is explored by a quadrotor with depth sensors to build a 3D map [30–32].

This has further developed into autonomous aerial patrol [33]. Assuming that the probabilistic map diffuses over time, minimizing map uncertainties causes the vehicles to revisit particular areas that were mapped a while ago. The following figures illustrate collaborative 3D patrol for a large building.



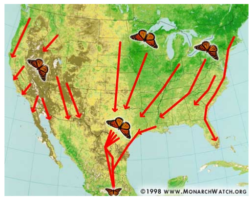
This idea has been applied to space systems, for Mars surface exploration [34] (left) and for asteroid shape mapping [35–37] (right).

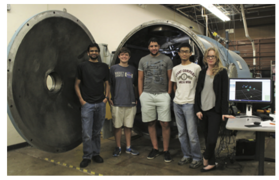




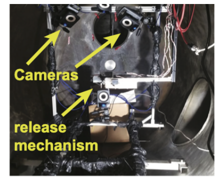
2.5 Supervised Learning Control for Micro Flapping-Wing UAV

Monarch is one of the most common butterfly species in North America. They exhibit remarkable flight characteristics migrating over 3000 miles, which is the longest range among the similar-sized insects. Compared with other insects, Monarch has relative-large wings flapping at a lower frequency. To develop micro flapping-wing UAV inspired by Monarch, the PI is collaborating with Dr. Kang at the University of Alabama, Huntsville, who has a facility to capture the flapping motion of a live Monarch butterfly and perform CFD analysis with fluid-structure interaction.

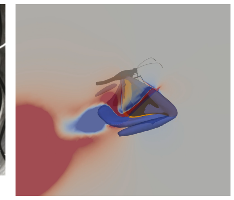




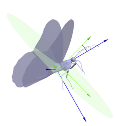
a) Large vacuum chamber and students

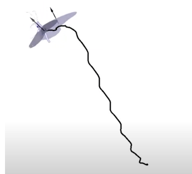


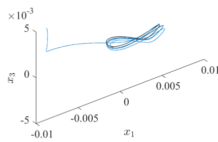
b) Setup inside the chamber



The PI has developed a dynamic model on a manifold, for an articulated rigid body model that characterizes wing flapping and abdomen undulation [38, 39]. Due to the complexity of the aerodynamics and the higher dimension of control inputs, it is challenging to design a control system via conventional techniques.







We have proposed supervised learning control, where a neural network learns optimal control trajectories that minimize the tracking error. Due to their ability of consistent generalization, the learned neural network successfully stabilize the complicated flapping motion. More importantly, the training process is carefully designed such that the Floquet stability of the periodic motion of flapping is guaranteed along the controlled dynamics. This is being further extended to on-line learning.

3 Future Directions

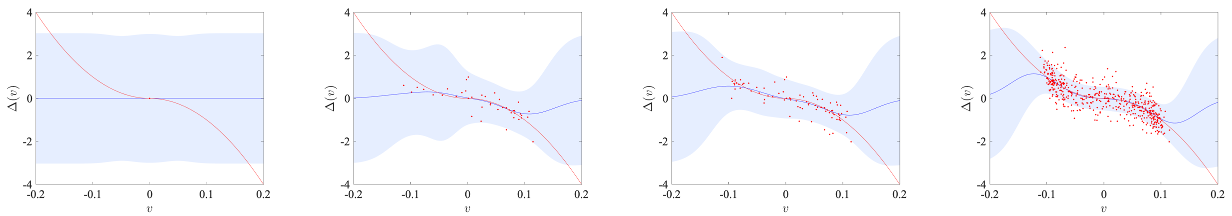
Based on the preceding success with geometric controls, the following directions, especially in stochastic learning, are being currently investigated and planned for futures.

3.1 Concurrent Bayesian Learning and Bayesian Estimation

Despite numerous fascinating applications of model-free reinforcement learning, completely discarding the underlying dynamics might not be desirable in various control problems in aerial robotics, as by doing so, valuable information encoded in the dynamics will be lost. A more reasonable approach would be identifying the discrepancy between the pre-determined mathematical model and the actual response, or constructing a dynamic model completely from experiences.

In such cases, it is critical to formulate a measure of confidence in the learned model such that the corresponding controller becomes aware of risky actions caused by large uncertainties in the learned model, or to avoid the model being constantly drifted by noisy measurements. Bayesian learning, often described as a neural network which is aware of what it does not know, will address such issue of learning with uncertainties. However, in learning a dynamic model, we often don't have a direct access to the sample values of unknown terms. Instead, a subset of the state or a lower-dimensional function of state is measured by a sensor. For example, unmanned aerial vehicles are rarely equipped with a velocity sensor, and the velocity is estimated by measurements of position and acceleration.

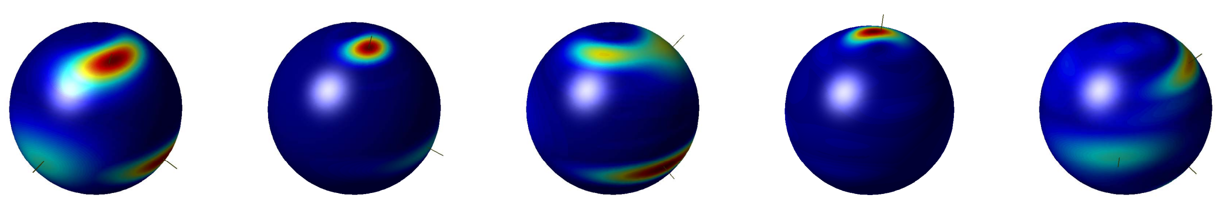
As such, Bayesian learning should be integrated with Bayesian estimation for stochastic learning of dynamics. But most of learning-based control systems often assume that the state is available such that the uncertain term can be directly inferred. Concurrent stochastic learning and estimation is one of the main directions that I plan to pursue in the next five years. Figures below illustrate one preliminary result, where the uncertain part of the dynamics (red) is learned as a function of state with a Gaussian process (blue) that is extended to handles uncertainties in the input [40].



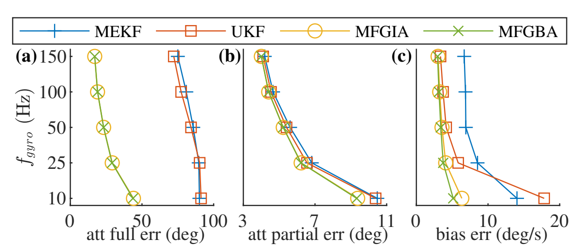
3.2 Geometric Stochastic Analysis on a Manifold

To perform Bayesian learning and estimation on a manifold, we should be able to formulate probability densities intrinsic to the manifold. The common approach is adopting Gaussian distributions for local coordinates of a manifold. Due to the several reasons discussed in Section 1, this causes singularities and complexities especially for large uncertainties. More formally, they don't even satisfy the fundamental property of a density function that should be normalized to one.

In one of my prior attempts, a global form of probability densities on $\mathsf{SO}(3) \times \mathbb{R}^n$ is defined by non-commutative harmonic analysis, which is essentially Fourier analysis on a Lie group. According to Peter-Weyl theorem, irreducible unitary representations on a Lie group serve as an orthogonal basis for square-integrable functions on the group, through which an arbitrary density function on the group is defined naturally. The following figures show one particular application to uncertainty propagation for attitude dynamics [41, 42].



While the above approach addresses the problem of defining a density function on a manifold, it is not suitable for real-time application in aerial robotics, due to computational complexities associated with non-commutative harmonic analysis. Recently, the PI has proposed a more regularized form of density on $\mathsf{SO}(3) \times \mathbb{R}^n$, referred to as *Matrix Fisher–Gaussian Distribution*, and developed a Bayesian estimator for attitude and gyro bias [43–45]. This approach is benchmarked against multiplicative extended Kalman filter and unscented Kalman filter to illustrate substantial improvements in estimation accuracy.

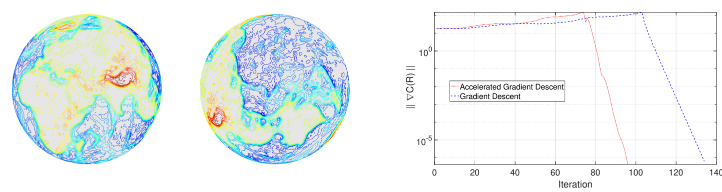


Formulating a density function is a fundamental question of statistics. Stochastic analysis on a manifold will be investigated continually, for both of theoretical perspectives and application to Bayesian learning.

3.3 Accelerated Symplectic Learning Optimization

In machine learning, the task of learning is often formulated as numerical optimization. As such, there has been increased efforts to improve gradient-based optimization techniques, among which momentum-based approaches have become popular. The idea is that optimization is considered as finding the lowest point of terrain defined by the objective function, and descending along the gradient can be accelerated as if a ball rolling down a hill is accelerated. In other words, an accelerated optimization algorithm can be defined by discretizing a mechanical system analogous to gradient descent schemes.

The PI has developed geometric numerical integration schemes, referred to as *Lie group variational integrator* for Hamiltonian systems on a Lie group [46, 47]. They exhibit long-term structural stability in numerical simulation, as both of sympletic structures of Hamiltonian systems and Lie group structures are preserved concurrently. Recently, a Lie group variational integrator has been utilize to construct accelerated gradient-descent optimization scheme on SO(3) [48], and it is applied to spherical shape matching. It is shown that the desirable properties of variational integrators improves computational efficiency of optimization. This idea of integrating numerical optimization for learning and structure-preserving integration on a manifold will be investigated continually.



3.4 Large-Scale Aerial Transportation

Besides the above theoretical research, two particular applications of geometric control will be pursued. The first is large-scale aerial transportation. While geometric controls for autonomous transportation have been illustrated under various assumptions, flight experiments are limited by one or two quadrotors transporting a fictitious payload in a lab environment. Upon availability of more resources, such as a larger outdoor space for flight experiments, I envision a larger scale aerial transportation, where several aerial vehicles transporting a sizable payload through cluttered environments.

3.5 Drone Racing: Tight-Coupling Between Perception and Control

The next application is autonomous drone racing. The PI is well posed to drone racing, particularly for designing geometric time-optimal controls involving complex maneuvers and developing flight hardware/software. Beyond control and optimization, another important aspect of drone racing is vision-based perception. After recognizing my lack of expertise in computer vision and perception, I decided to organize a new graduate course in *Robotics Vision and Perception* in Spring 21, to train myself and my students in depth. The materials in this course will be utilized in drone racing for short-term future, and more interestingly, tight-coupling between perception and control, bypassing the traditional pipeline of estimation, planning, and control, in long-term future.



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