Complexity:

Highly Optimized Tolerance

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Papers

J.M.Carlson and J.Doyle.
Highly optimized tolerance: a mechanism for power laws in designed systems.
Phys. Rev. E 60, 1412-1427. (HOT:PL)

J.M.Carlson and J.Doyle.
Complexity and robustness.
Proc. Nat. Acad. Sci. 99 , 2538-2545. (CaR)

Review:

Last week focused on Cellular Automata

CAs are rules to turn collections of 1s and 0s into a 1 or 0 in a specific location

CA can be made to accomplish interesting computational tasks--discussion focused on $p>0.5$ question

Goal: Explain Power Law Behavior

Recall "Power Law Behavior" - observation that small/short measure events have a high frequency, large/long events have low frequency - or $p(L) = k*L^{(-a)}$

Carson & Doyle argue that this goal is a central object of "Complexity" science

Self-Organized Criticality (SOC)

SOC is an explanation of systems that have equilibria at critical transition points.

SOC proposed as one explanation of power law behavior.

Highly Optimized Tolerance (HOT)

HOT turns on the notion of design, either deliberate (e.g., engineered) or by preferential selection (e.g., evolution), creating systems which are internally complex but produce "reliable" or "robust" external behavior for frequent events. Infrequent events produce catastrophic results.

HOT proposed as explanation of power law behavior in the subject papers.

HOT vs SOC

Table from CaR 1

Seems somewhat semantic?

Internal Complexity in HOT
Qualitatively related to number of components and "heterogenity" of components

Robustness in HOT
Qualitatively related to the range of variables over which the system can operate satisfactorily

Catastrophic Failure in HOT
Drastic changes in performance relative to small changes that the system was NOT designed to accommodate

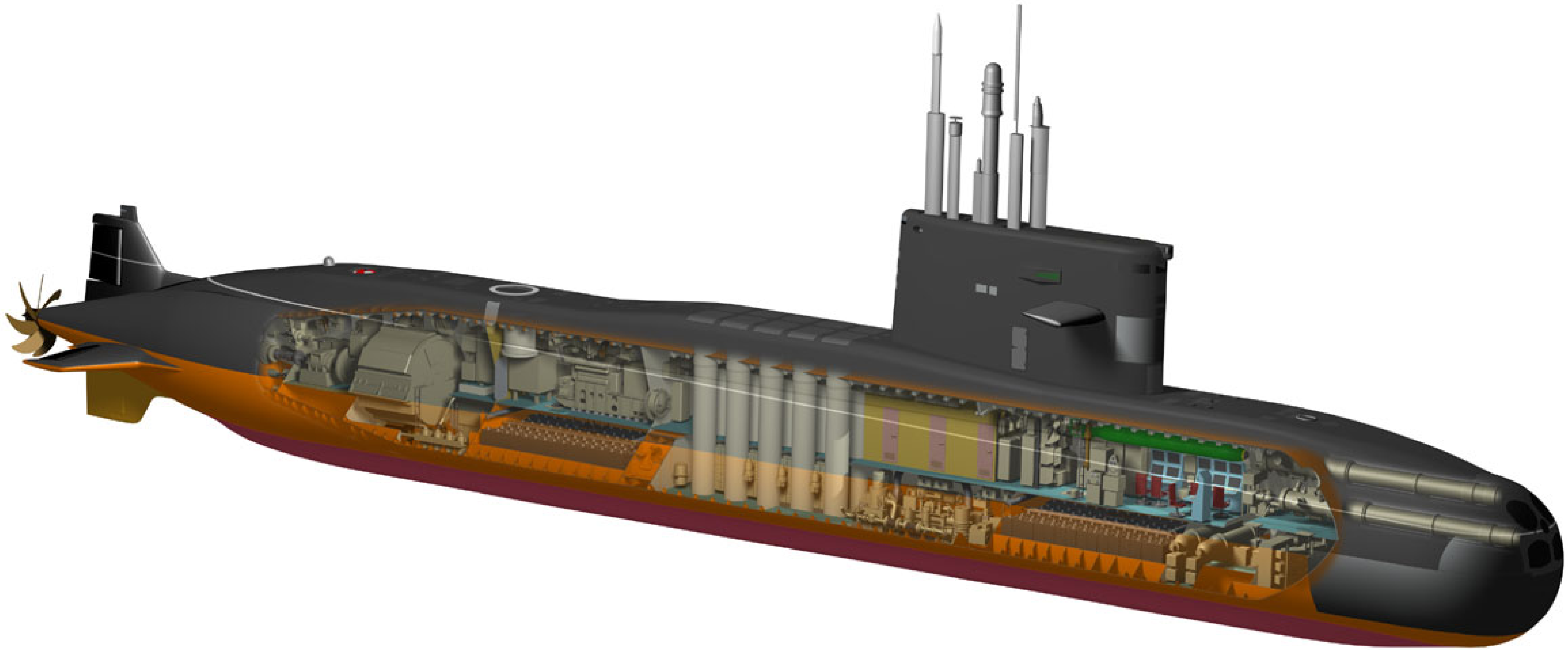
HOT Intuitive Examples

Cells, Boeing 777, Computers, Internet

Also note notion of increasing internal complexity vs "robustness" and potential for catastrophic failure

Simple cells much less capable to survive in broad range of environments; complex cells can completely fail based on small changes regulatory networks

HOT Aside

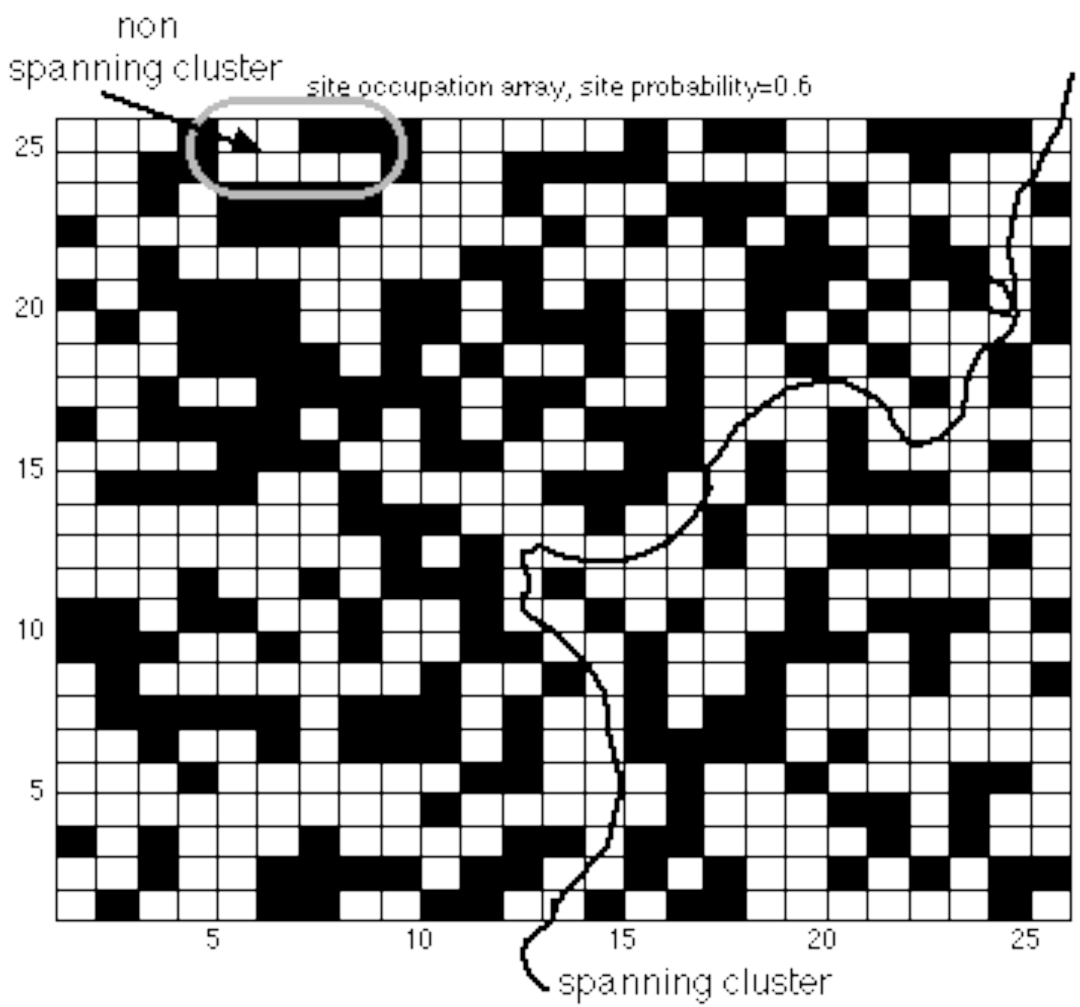


HOT Quantitative Discussion

Forest-Fire Models
Website Optimization
Data Compression
Sand Piles

Yield Graph from CaR3

Forest-Fire Model
Starts with 2D percolation picture



some sites are occupied--i.e., trees present

some sites are unoccupied--i.e., no trees

"spark" randomly applied, burns affected cluster

Image taken from Technion,
The Israeli Institute of Technology, Physics department

Forest-Fire Model

Several pertubations on basic model, papers cover single spark burning connected cluster

Question to be answered: how to maximize yield of trees after burn

Forest-Fire Model

Trying to avoid obviating B2 presentations, but:

For creating the percolated configuration: if the "forest" is a homogenous object, SOC results are obtained--i.e., ideal yield obtained by percolation with critical probability.

How does this correspond to assignment 1 results?

Forest-Fire Model

If the percolated configuration is "designed" instead, higher yields are obtained

However, design becomes weak against small defects (e.g., a tree appearing in a firebreak) and different environmental conditions (e.g., change in spark distribution)

Sample lattices: CaR5

Sandpile Model

Recall the discussion of sandpile/avalanche models from the first class, then extend it to 2D

If yield is defined as un-perturbed sand, then the problem become conceptually very similar to the forest fire problem

Image HOT:PL6

Sandpile Model + Time

Any starting distribution for the aforementioned sandpile rules will progress to the critical height density--i.e., dynamically, all initial states decay to SOC configuration

Generalizing Concept

Authors interested in generalizing problem framework. Went to Probability-Loss-Resource model, which is a generaliztion of Shannon coding theory (!)

Some Equations - CaR6

Other Problems in those Terms

Source words become 0D objects w/ 0D breaks

Websites become 1D objects w/ 0D breaks

Forest Fire/Sandpile becomes 2D object, 1D breaks

Philosophical Aside

Tenor of CaR paper implies XOR for SOC vs HOT explaining complex phenomena. My opinion: this implication is incorrect.

HOT:PL implies complementary roles--i.e., systems with design produce HOT-like results, systems absent design tend towards SOC-like results.

HOT vs SOC, round II

SOC produces power laws only as the system achieves critical state; HOT provides power law results over a variety of states

Small changes to state do not affect SOC results; they do affect HOT results

"Yield" and "Performance" measures on SOC systems are small than those in HOT systems, despite similar power law results for losses

Potential HOT Investigations

Internet - traffic "burstiness," website design

Ecosystem - explanation of evolutionary trends (i.e., punctuated equilibria), ecosystem stability/consequences of invasive species