

**A REMINISCENCE ABOUT SHORTEST SPANNING SUBTREES**

JOSEPH B. KRUSKAL

*I am delighted to provide this in honor of Professor Otakar Borůvka*

It happened at Princeton, in old Fine Hall, just outside the tea-room. I don't remember when, but it was probably a few months after June 1954, the date of my Ph.D. graduation. Someone—his name vanished from my mind within days, if it was ever there—handed me two pages of very flimsy paper stapled together. He told me it was “floating around the math department”. What little else he may have said came from text itself.

The pages were typewritten, carbon copy, and in German. They plunged right in to mathematics, and described a result about graphs, a subject which appealed to me. I didn't understand it very well at first reading, just got the general idea. I never found out who did the typing or why.

At the end, the document described itself as the German-language abstract of a 1926 paper by Otakar Borůvka. The name of the Journal, in an unfamiliar language, was given also in German. Such details didn't interest me much, however, and I simply thought of it, using my inexpert translation of the German, as “the annals of the Moravian academy of science”. After a few more readings, and writing down the Moravian name of the Journal, I passed the document on to someone else, telling him as little as I had been told.

The abstract described a method for constructing the shortest spanning subtree of a graph whose edges have known lengths, and from this method trivially derived the corollary that the shortest spanning tree is unique if no two of the lengths are equal. For me, and it appears for almost everyone else, the interest of the paper was the method of construction, not the corollary.

In one way, the method of construction was very elegant. In another way, however, it was unnecessarily complicated. A goal which has always been important to me is to find simpler ways to describe complicated ideas, and that is all I tried to do here. I simplified the construction down to its essence, but it seems to me that the idea of Professor Borůvka's method is still present in my version.

After reaching this simplification, I started wondering whether it was worth publication. I had published only one paper at the time, in 1953 while a graduate student at Princeton. While my Ph.D. thesis was already on the shelves at Princeton University, publication of the Ph.D. result was still 6 years in the future.

I remember seeking advice from someone—who could it have been?—about whether this work was worth submitting for publication: the reasoning it uses is so very simple. If memory is correct, this important discussion took place while we were walking on a minor road between Route 1 and Princeton. Fortunately he advised me to go ahead, and many years passed before another of my publications became as well-known as this very simple one.

There was an additional bonus from this paper a few years later, in 1959, when I wanted to leave university life because of my emotional problems with teaching. I knew some people at the Bell Labs mathematics group, and thought to look for a job there.

The head of the mathematics group at the time, and also the person I called when seeking a job, was Bob Prim, whose 1957 paper on the same topic is also widely known. Until my visit to Bell Labs for the employment interview, I was unaware of Prim's paper, though his paper cited mine. There is little doubt that this connection was helpful in starting my 34-year position at Bell Labs.

Finally, I regret that the phrase “minimum spanning tree” has taken hold. For one thing “minimum” often degenerates to the badly incorrect “minimal”, and for another, “minimum” is so vague; minimum in what way? I always think of the concept as “shortest spanning subtree”, and hope someday to see SST replace MST.

JOSEPH B. KRUSKAL  
BELL LABS  
LUCENT TECHNOLOGIES  
ROOM 2C-281  
MURRAY HILL, NJ 07974