On Bounds for the Wavelength Assignment Problem on Optical Ring Networks.

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Abstract: *This research note presents two new bounds for the off-line wavelength assignment problem in optical ring networks that use wavelength division multiplexing. In this context, we consider a well-known bound [28, 35] that, to our knowledge, remains the only published bound for a ring topology and for which examples exist that show tightness. In this note, we show the bound can be improved in two ways: (1) by considering an additional parameter (the number of network nodes), we provide a sharper bound than the existing bound, and (2) we extend a classic result in [35] that was proved for the special case of $k = 3$ (no k connections cover the ring) to the general case of arbitrary k.*

Keywords: optical networks, wavelength division multiplexing, wavelength assignment problems.

1 Introduction

The wavelength assignment problem is usually framed as follows: given a set of connections, find the minimum number of wavelengths needed to transmit the connections simultaneously so that no two connections that share a link also share a wavelength. The requirement that connections on a link use unique wavelengths arises from wavelength division multiplexing, the technology used to multiplex several connections simultaneously on an optical link. Because current technology limits the number of available wavelengths and because the ring topology is a serious candidate for large scale commercial production, the assignment problem on rings has received substantial attention [3, 8, 10, 12, 27, 28, 31, 34]. (In graph theory, the equivalent problem of arc-coloring has also been the focus of many papers [16, 35, 32].) In the off-line version of the assignment problem, the entire set of connections is given in advance whereas in the on-line version the assignment must be done as the connections are revealed in sequence [10]. We only consider the off-line version in this paper.

Since it is NP-complete to compute the minimum number of wavelengths required to assign a given connection set [9], bounds on the number of required wavelengths are often the next source of detailed analysis. To our knowledge, the only known bound for single-fiber ring networks (Bounds for multifiber rings can be found in [19]) is the well-known bound of $2L - 1$ [28, 35], where L is the "connection load" (the maximum number of connections carried by any link). Examples can be constructed [10] to show that the bound is optimal (tight). However, these examples often use complex connection sets in which many connections are unrealistically long. Intuition suggests that sharper bounds may be obtained if the number of nodes in the ring is considered and if connection sets involve shorter connections. A similar observation was made in [8]. In this paper, we provide theoretical support for this intuition and show that sharper bounds are indeed possible:

- We improve the well-known $2L-1$ bound to $(2-1/\lfloor(n+1)/2\rfloor)L$, where n is the number of nodes.
- We show that for the well-known circular arc coloring problem, when no k or fewer arcs cover a ring, at most $\lceil kL/(k-1)\rceil$ colors are required where L is a lower bound on the required number of colors. This result generalizes the bound in [35] and is significant for two reasons. First, two important limiting cases closely match known results: $k=2$ (shortest-path routing, for which the best known bound is $2L$ [28]) and $k\to\infty$ (interval graph coloring, which can be colored with optimal L colors [32]). Second, the result can be used to derive bounds for constrained versions of the problem with path-length restrictions [12].

2 Related Work

The reader is referred to [24, 25, 30] for a broad survey of optical networks and to [3, 12, 15, 24, 29] for an overview of wavelength assignment problems. Since we are only concerned with the specific problem of providing sharper bounds for ring networks, no further survey of the assignment problem is given other than directly related papers.

For ring networks, NP-completeness is shown in [9] and a 2-approximation algorithm is given in [28] based on interval-graph coloring [32]. When path-lengths are restricted to length two, the problem is solvable (achieving L) with an even number of nodes and tight bounds exist for an odd number $(2k+1)$ of nodes: $L+\lfloor V_{min}/k\rfloor \leq \omega_{opt} \leq L+\lceil V_{min}/k\rceil$ [20], where V_{min} is the minimum number of connections passing through any node and ω_{opt} is the optimal number of wavelengths required. Scheduling problems on the ring are considered in [27, 34].

For the circular arc-coloring problem Tucker [35] shows that if no mutually overlapping three arcs cover the circle, at most $3L/2$ colors are needed. Extending the technique in [35], we show that if no k arcs cover the circle, at most $\lceil kL/(k-1)\rceil$ colors are needed. From this result, one infers that if path-lengths are bounded by m, at most $\lceil\lceil n/(m-1)\rceil L/(\lceil n/(m-1)\rceil-1)\rceil$ wavelengths are required, where n is the number of nodes in the ring network. Finally, bounds for multifiber rings are given in [19].

3 Problem Formulation

The following notation and definitions will be used in the paper:

- $G(V,E)$: a connected network graph for both directed and undirected versions. Let $n=|V|$.
- *Routing.* We do not consider the additional problem of routing in this paper. We assume unidirectional clockwise routing.
- $I=\{(s_1,d_1),...,(s_k,d_k): s_i,d_i\in V\}$: a collection of source-destination connection requests. Note that several connections in I may have the same source-destination pair.

- $L_e(I)$: the number of connections in a connection set I that use edge e. Let $L(I) = \max_e L_e(I)$, the largest number of connections using any link for a given connection set I. We will drop I where the connection set implied is obvious.

The wavelength assignment problem we consider is: given G and I, find the minimum number of wavelengths needed to schedule all the connections in I so that no two connections that share an edge use the same wavelength.

4 A Sharper Bound

In this section, we show that $(2 - 1/\lfloor(n+1)/2\rfloor)L$ wavelengths are sufficient for ring networks. This represents an improvement over the current, well-known bound of $2L-1$. (The improvement, however, is modest for large n and clearly decreases with increasing n). A request set is called *a uniform request set* if the number paths on each edge is the same. For any request set with load L, we can construct an equivalent uniform request set with load L by adding one-hop requests. A similar construction is described in [31].

LEMMA **1** *Given a uniform request set* $I = \{p_0, p_1, ..., p_{|I|-1}\}$ *on a ring network* G*, the number of requests that terminate at each node is even.*

Proof: Consider an arbitrary node u in the ring G, and let $e_l(u), e_r(u)$ be the two edges adjacent to u: suppose e_l is the left edge and e_r the right edge. Note that since I is uniform, each edge has exactly L paths passing it. Thus, $L_{e_l} + L_{e_r} = 2 \cdot L$. Now, let n_1 denote the number of paths that terminate at u, and let n_2 denote the number of paths that have u as an intermediate node. Therefore, $n_1 + 2n_2 = L_{e_l} + L_{e_r} = 2 \cdot L$. So n_1 must be even. ∎

Next, for the proof we will construct a directed path-edge multigraph $M(V,E)$ from G and I where $V(M) = V(G)$ and in which all paths have clockwise direction. Here, each $e = (u,v) \in M(E)$ if and only if $(u,v) \in I$. Note that M is a directed multigraph where each of its nodes has an even number of incident edges because the number of paths of I that terminate at any node in G is even. Moreover, in each node, the number of incoming edges is equal to the number of outgoing edges. Then M can be partitioned into several circuits by Euler's theorem [11]. See Figure 1.

LEMMA **2** *Given a uniform request set* $I = \{p_0, p_1, ..., p_{|I|-1}\}$ *on a ring network* G*,* I *can be partitioned into subsets* $I_1, ..., I_J$*, such that each subset is a uniform request set.*

If a uniform request set cannot be partitioned into more than one uniform request subset, then we call it a *nondecomposable uniform request set.*

LEMMA **3** *Given a uniform request set* $I = \{p_0, p_1, ..., p_{|I|-1}\}$ *on a ring network* G*,* I *can be partitioned into subsets* $I_1, ..., I_J$*, such that each subset is a nondecomposable uniform request set.*

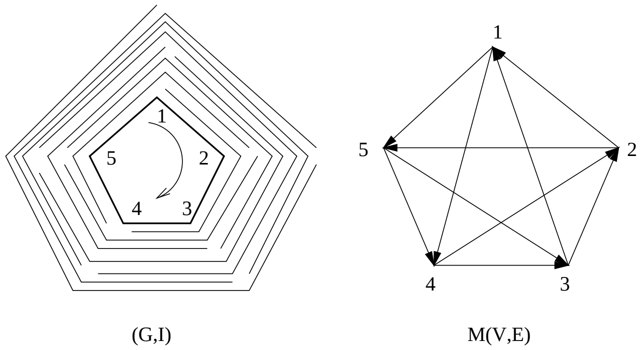

Figure 1: Multigraph $M(V, E)$ construction example.

Proof: Partition the path-edge multigraph M into several circuits such that each circuit only has degree 2. From each circuit, construct a nondecomposable uniform request set. ∎

Lemma 4 *In each nondecomposable uniform request set, there are at most n paths.*

Proof: Because each circuit only has degree 2, each circuit at most has n edges. So, each single uniform request set at most has n paths. ∎

Consider Figure 1. In this five-node-ring network, we have a 10-connection uniform request set $I = \{(1,4),(4,2),(2,5),(5,3),(3,1),(1,5),(5,4),(4,3),(3,2),(2,1)\}$. From the directed multigraph M, it is easy to see that I can be partitioned into two nondecomposable uniform request set I_1, I_2, where $I_1 = \{(1,4),(4,2),(2,5),(5,3),(3,1)\}$ and $I_2 = \{(1,5),(5,4),(4,3),(3,2),(2,1)\}$. Furthermore, we have $L(I_1) = 3$ and $L(I_2) = 4$. For I_1, we need at most $2L(I_1) - 1 = 5$ colors, but for I_2, we need at most 5 instead of $2L(I_2) - 1 = 7$ colors because there are only 5 connections. This is the basic idea to reduce the upper bound of total number of wavelengths. Mathematically, we observe that at most $\min(2L - 1, n)$ wavelengths are needed to assign each nondecomposable uniform request set with load L. This is because there are at most n paths and because the previous bound of $2L - 1$ applies to all instances. This is the intuition for our main result.

Theorem 1 *Given a ring network with n nodes, for any request set I with load L, we need at most $(2 - 1/\lfloor (n+1)/2 \rfloor) \cdot L$ wavelengths.*

Proof: Without loss of generality, we assume that I is a uniform request set. Then I can be partitioned into J nondecomposable uniform request sets, $I_1, ..., I_J$. For each I_i with $L(I_i) \leq \lfloor (n+1)/2 \rfloor$, we need at most $2L(I_i) - 1$ wavelengths to assign I_i. On the other hand, for any I_j with $L(I_j) \geq \lceil (n+1)/2 \rceil$, we need at most n wavelengths to assign it. Suppose now that there are J_1 nondecomposable uniform request sets with load equal or greater than $\lceil (n+1)/2 \rceil$. Let the sum of the loads of these J_1 nondecomposable uniform request sets be L_1. Then at least we have $(L - L_1)/\lfloor (n+1)/2 \rfloor$ nondecomposable uniform request sets with load not greater than $\lfloor (n+1)/2 \rfloor$. So, totally, we need at most $(2(L - L_1) - (L - L_1)/\lfloor(n+1)/2\rfloor + (L_1/\lceil(n+1)/2\rceil \cdot n) = (2 - 1/\lfloor(n+1)/2\rfloor)L - (2 - 1/\lfloor(n+1)/2\rfloor - n/\lceil(n+1)/2\rceil) \cdot L_1 \leq (2 - 1/\lfloor(n+1)/2\rfloor) \cdot L$. The last inequality is true because $(2-1/\lfloor(n+1)/2\rfloor - n/\lceil(n+1)/2\rceil)L_1 \geq 0$ when $n > 0$. Actually, let $f = 2-1/\lfloor(n+1)/2\rfloor - n/\lceil(n+1)/2\rceil$. If $n = 2k+1, f = 2 - 1/(k+1) - (2k+1)/(k+1) = 0$; if $n = 2k, f = 2 - 1/k - 2k/(k+1) = (k-1)/(k(k+1)) \geq 0$. ∎

From the bound, the following result is apparent.

Corollary **1** *For any fixed ring network, there is an approximation algorithm with an approximation ratio less than 2.*

Theorem **2** *If the routing is restricted to shortest paths, then* $(2-4/(n+2)) \cdot L$ *wavelengths are sufficient.*

Proof: If we restrict the routing to shortest paths, then no two paths can cover the whole ring. This means each wavelength can cover at least 2 paths. Each nondecomposable request set therefore needs at most $\min(2L(I_i) - 1, n/2)$. Suppose we have J_1 nondecomposable uniform request sets with load greater than $(n+2)/4$ and the total load of the J_1 nondecomposable request sets is L_1. Then, there are at least $(L - L_1)/((n+2)/4)$ nondecomposable request sets with load not greater than $(n+2)/4$. So, totally, we need at most $2(L - L_1) - (L - L_1)/((n+2)/4) + L_1/((n+2)/4) \cdot n/2 = (2 - 4/(n+2))L - (2 - 4/(n+2) - 2n/(n+2))L_1 = (2 - 4/(n+2)L$ wavelengths. ∎

5 A Covering Restriction

In this section, we take up a generalization of shortest paths first considered in [35]. With shortest paths, no two overlapping paths together cover the ring; we consider the case where no k mutually overlapping paths cover the whole ring. Since the case $k = 3$ was proved in the context of the equivalent arc-coloring problem in graph theory, we state our results in those terms in this section. In this context, a collection of arcs I on a circle are to be colored such that no overlapping arcs are assigned the same color.

The arcs $A_1, A_2, ..., A_n$ are said to *cover* the circle if they are mutually overlapping and collectively they contain every point on the circle. Let $L(I)$ be the maximum number of arcs passing any point in arc set I. Without loss of generality, we assume that I is a uniform request set with load $L(I)$. Tucker [35] has proved the following result by induction:

Lemma **5** *If no set of three or fewer arcs in the arc set* I *cover the circle,* $3L(I)/2$ *colors are sufficient.*

The proof technique in [35] can be used to generalize the result in two stages. First, we prove the following result.

Lemma **6** *If no set of four or fewer arcs in the arc set* I *cover the circle,* $\lceil 4L/3 \rceil$ *colors are sufficient.*

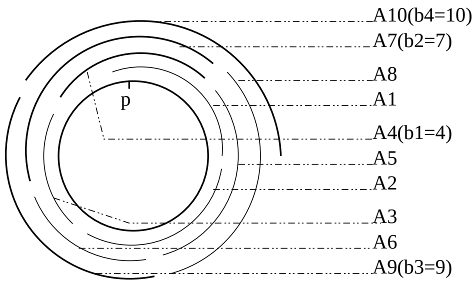

Figure 2: Lemma 6 proof example.

Proof: If $L(I) = 1$, the result is obviously true. We will use induction on the value of L: assume that the result is valid when $L < r$ for some $r > 1$; we will show it remains valid for any arc set I with $L(I) = r$.

First, when $r = 2$ Tucker's lemma shows that $3r/2$ colors are sufficient. But $3r/2 = \lceil 4r/3 \rceil$ and hence we only need to consider the case $r > 2$.

Pick a point p such that S_p, the set of arcs that contain point p, has size r. We start with Tucker's construction: let A_1 be the arc in S_p that extends the shortest length on the counter-clockwise side of p. Starting with A_1, we consider successive arcs in the clockwise direction and index a subset of the arcs by the rule: if A_i is the arc previously indexed, then A_{i+1} is the first unindexed arc to begin from the *clockwise* end of A_i. Note that A_{i+1} always exists because of the uniform request set assumption.

Among the A_i's, let b_1 be the index of the arc that passes point p a second time (A_1 passes p first in the indexing). If on this second pass, p is in the space between the previously indexed arc and the next arc to be indexed, let b_1 be the index of the next arc indexed. Now, let b_2 be the index of the arc being indexed when we first pass the *counterclockwise* end of A_{b_1}; as above, if A_{b_1} ends in a space, then A_{b_2} is the next arc. Let A_{b_3} be the arc being indexed when we first repass the counterclockwise end of A_{b_2}; as before, if A_{b_2} ends in a space, then A_{b_3} is the next arc. Finally, if A_{b_3} has not passed around p, let A_{b_4} be the next arc in the sequence after A_{b_3} to pass p, in this case; if no such arc exists, let A_{b_4} be the last arc ending before p; if A_{b_3} does pass p, we will let $b_4 = b_3$ for convenience. See Figure 2 for an example of the labeling.

It is easy to see that $L(I_{b_1}) \leq L(I) - 1$, where we let I_{b_i} denote $I - \{A_1, ..., A_{b_i}\}$. Similarly, $L(I_{b_4}) \leq L(I) - 3$ because the arcs up to and including b_4 cover the ring three times and removing them reduces L by three. If we can show that $A_1, ..., A_{b_4}$ can be 4-colored, then by the induction hypothesis, I_{b_4} can be colored with $\lceil 4L(I_{b_4})/3 \rceil$ colors, thereby completing the proof.

Observe that $A_1, ..., A_{b_3-1}$ can be 3-colored: $A_1, ..., A_{b_1-1}$ get color 1, $A_{b_1}, ..., A_{b_2-1}$ get color 2, $A_{b_2}, ..., A_{b_3-1}$ get color 3. Now consider arcs A_{b_3} and A_{b_4}. There are three cases to consider:

- Case 1: $b_4 = b_3$. Here, A_{b_3} covers p and thus four colors are sufficient, since arcs A_1, A_{b_1}, A_{b_3} cover p and A_{b_2} may or may not cover p.

- Case 2: A_{b_3} is the last arc in sequence and does not cover p. In this case, four colors are sufficient.

- Case 3: $b_4 \neq b_3$. Here, if A_{b_4} did not wrap around and overlap A_{b_3}, four colors would be sufficient because A_{b_3} and A_{b_4} would use the same color. Thus, we only need to worry about possible overlap between A_{b_4}'s clockwise end and A_{b_3}'s counterclockwise end. We show that this is impossible. If it were the case, $A_{b_4}, A_{b_3}, A_{b_2}$ and A_{b_1} would cover the circle and violate the assumption. Note that A_{b_1} covers point p because any arc that overlaps the counterclockwise end of A_1 must cover p. ∎

The above argument can be generalized to the case when no k or fewer arcs cover the circle.

Theorem 3 *If no k or fewer arcs in the arc set I cover the circle, $\lceil kL/(k-1) \rceil$ colors are sufficient.*

Proof: From the above lemmas, the result is true for $k = 3$ and $k = 4$. Suppose, the result is true for any integer $k > 3$; we will show it holds for integer $k + 1$.

By assumption, no $k+1$ or fewer arcs in the arc set I cover the circle. According to our assumption, when $L < k$, the circular arcs can be colored by $\lceil kL/(k-1) \rceil = \lceil L + L/(k-1) \rceil = L + 1$ colors, which is $\lceil (k+1)L/k \rceil = L + 1$. Thus, we need only consider the case when $L \geq k$. We will prove the theorem by induction on L. Assume it is true for any $L < r$ is true, where $r \geq k$. We want to prove that it holds for $L = r$.

We use the idea in the previous lemma. Pick a point p, such that S_p, the set of arcs containing p, has size r. Let A_1 be the arc in S_p that extends the shortest length on the counterclockwise side of p. Starting with A_1, we move clockwise round and round the circle indexing the arcs $A_1, A_2, A_3, ...$ by the rule: if A_i is the arc previously indexed, then A_{i+1} is the first unindexed arc to begin after the end of A_i. If there are multiple such arcs, pick one randomly. As before, let A_{b_1} be the first arc to overlap the counterclockwise end of A_1; let $A_{b_{i+1}}$ be the first arc to overlap the counterclockwise end of A_{b_i}, $i = 1, ..., k-1$. Finally, if A_{b_k} does not pass p, let $A_{b_{k+1}}$ be the next arc to do so.

Note that $L(I_{b_{k+1}}) \leq L(I) - k$. By the induction hypothesis, $I_{b_{k+1}}$ can be colored by $\lceil (k+1)L(I_{b_k})/k \rceil$ colors. We need to prove that $A_1, ..., A_{b_{k+1}}$ can be colored with $(k+1)$ colors. By the argument used earlier, $A_1, ..., A_{b_k - 1}$ can be colored by k colors. It is easy to check that A_{b_k} and $A_{b_{k+1}}$ either represent the same arc or cannot overlap; otherwise, $A_{b_1}, ..., A_{b_{k+1}}$ will cover the whole circle. In either case, only one color is needed for both. Thus, $A_1, ..., A_{b_{k+1}}$ can be colored by $k+1$ colors. Hence I can be colored by $k + 1 + \lceil (k+1)L(I_{b_k})/k \rceil \leq k + 1 + \lceil (k+1)(L-k)/k \rceil = \lceil (k+1)L/k \rceil$ colors. ∎

Using this theorem, we can estimate an upper bound for some wavelength assignment problems with path-length restrictions.

Corollary 2 *If all connection paths have path-length at most m in a ring network, $\omega_{opt} \leq \lceil \lceil n/(m-1) \rceil L/(\lceil n/(m-1) \rceil - 1) \rceil$.*

Proof: Note that no $\lceil n/(m-1) \rceil$ paths cover the whole ring and apply Theorem 3. ∎

6 Summary

We presented two sharper bounds for the wavelength assignment problem on ring networks. The bounds serve as useful baseline comparisons for more practical on-line algorithms, as well as provide new theoretical insight into the assignment problem itself. Future work will focus on other network topologies.

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