On the Wavelength Assignment Problem in Multifiber WDM Star and Ring Networks

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Abstract: *This paper studies the off-line wavelength assignment problem in star and ring networks that deploy multiple fibers between nodes and use Wavelength Division Multiplexing (WDM) for transmission. The results in this paper show that the ability to switch between fibers increases wavelength utility. In particular, sharper per-fiber bounds on the number of required wavelengths are derived for the multifiber version of the assignment problem in star and ring networks. Additionally, the complexity of the problem is studied and several constrained versions of the problem are also considered for star and ring networks. A summary of contributions is provided in the first section.*

Keywords: optical networks, wavelength division multiplexing, wavelength assignment.

1 Introduction

Optical networks hold out the promise of meeting the high transmission quality and large bandwidth desired by end-user applications. Among various transmission technologies for optical networks, Wavelength Division Multiplexing (WDM), because of its efficient use of bandwidth, has received considerable attention. In particular, because connections that share a link cannot use the same wavelength, WDM technology has focused attention on the combinatorial problem of deciding which route and which wavelength a connection uses – the so-called *wavelength routing and assignment problem.* In some cases, the routing component of this problem is either fixed (star networks, tree networks, rings with shortest-path routing) or given (when routing is decided prior to wavelength assignment) and thus, only wavelengths need to be assigned – this problem is referred to as the *wavelength assignment problem* (WAP). The off-line version of this latter problem is often formulated as follows: find the minimum number of wavelengths needed to accommodate a given set of connections (source-destination pairs) so that no two connections that share a link are assigned the same wavelength. (On-line versions consider connection arrival and departure times and assign wavelengths on a per-connection basis at the time of arrival, a problem formulation not considered here).

This paper studies the off-line wavelength assignment problem in star and ring networks that use a fixed number of multiple fibers between each pair of adjacent network nodes. While the off-line version of the problem is somewhat more limited in applicability than the on-line version (usually only when the connections are long-lived), bounds for the off-line problem also serve the useful purpose of providing a baseline for evaluating on-line algorithms. Such off-line results, in addition, often result in useful insight into the structure of the general problem.

We focus on those aspects of the assignment problem related to multiple fibers. While the single-fiber version of WAP has received considerable attention [7, 10, 18, 20], the common practice of bundling multiple fibers recommends a careful study of the multifiber version. Indeed, our results show that the additional choice brought by multiple fibers actually increases wavelength utility: it is possible to use fewer wavelengths per-fiber with multiple fibers than with a single fiber. For example, consider the following comparison between a single-fiber and a dual-fiber ring network. A well-known result states that a single-fiber ring requires $2L-1$ wavelengths in the worst case [18], where L is the "connection load" (the maximum number of connections carried by any link). Therefore, a dual-fiber ring could naively be decomposed into two parallel rings with approximately half the load ($L/2$) in each, suggesting that $2 \cdot L/2 = L$ wavelengths are required. However, we show that approximately $3L/4$ wavelengths are sufficient if one is allowed to switch between fibers at the network nodes. This result and others in this paper lend theoretical support to the notion of switching between fibers.

Consistent with much of the literature on the single-fiber version of the assignment problem, we present theoretical results in the form of bounds on the number of wavelengths as well as some complexity analysis (NP-completeness) of the problem. Thus, actual algorithms that meet the bounds are not explicitly presented but may be easily inferred from the proofs of the results. We consider both dual-fiber networks (two fibers in each link) as well as the general case of k fibers per link. The contributions in this paper can be summarized as follows:

- For dual-fiber ring networks, we give an upper bound of $3/2L-1$ for the total number of wavelengths required (counting wavelengths that are re-used in the second fiber), where L is the network load. When the number of network nodes is large enough, we show that this upper bound is tight. Note that this bound is a sharper per-fiber bound ($3L/4$) than the equivalent per-fiber bound (L) for two superposed single-fiber rings.

- The above upper bound is also extended to general k-fiber optical networks, resulting in at most $(k+1)/k \cdot L - 1$ required wavelengths. We show that this upper bound is also tight.

- In [12], we gave an upper bound for the well-known circular arc coloring problem, when no c or fewer arcs (connections) cover a ring. In this paper, we generalize the result to dual-fiber and k-fiber ring networks.

- For dual-fiber and k-fiber ring networks, we prove that the wavelength assignment problem remains NP-complete.

- For arbitrary dual-fiber undirected networks but where connection path-lengths are of length at most two, we show that the wavelength assignment problem is optimally solvable (in contrast to the single-fiber version, which is NP-complete).

Finally, we provide a few results for general topologies with k fibers that relate the number of wavelengths required to that of an equivalent single-fiber network.

Some of the results stated above involve a restriction on the connection set in an attempt to study simplifications of the problem that yield stronger bounds. Two types of restrictions are considered: a constraint on the length (number of hops) of a connection path (introduced in [10]) and a constraint on the number of paths that can together encircle a ring (introduced in [24]). From a theoretical point of view, the following example illustrates why these restrictions are important: the well-known $2L - 1$ bound for a single-fiber ring is obtained by depending on a connection set that has several long paths that use up much of the capacity. Thus, one is led to wonder whether fewer wavelengths can be used if the connection sets have paths that are more 'realistic' (shorter) in some sense. We show that this is indeed the case.

Path-length constraints sometimes also arise in algorithms, as the merging process in [7] shows and sometimes are the result of algorithms that generate routes [20]. Furthermore, the methods used to solve problems with path-length restrictions can be used as a first step in heuristics for the general problem: by first assigning the small paths near-optimally, the remaining connections, fewer in number, can be assigned heuristically. In the same vein, we take up the covering restriction introduced in [24] for ring networks where no c connections cover the ring. Note that shortest path routing, a common assumption, satisfies the restriction when $c = 2$.

The rest of this paper is organized as follows. Related work is discussed in Section 2. The assignment problem is formally described in Section 3 and an optimal algorithm for dual-fiber undirected stars and some extensions are given in Section 4. Results for dual-fiber ring networks are presented in Section 5 and results for general k-fiber ring networks in Section 6. Concluding remarks are given in Section 8.

2 Related Work

We assume familiarity with WDM optical networks [1, 15, 16, 21] and the wavelength assignment problem (WAP) [3, 16, 20] – see [10] for a recent survey. Related literature also includes the vertex coloring and arc coloring problems in graphs [4, 14, 24] as well as papers that study wavelength conversion [22] and scheduling [6] in optical networks.

Multifiber optical networks are a relatively recent topic. We are aware of only a few results [11, 23]. In [11], Jeong and Ayanoglu extend Barry and Humblet's blocking probability analysis model [2] to the multifiber case. With their stochastic approach, they find that multiple fibers can improve wavelength utility and network performance. A stochastic approach is similarly taken in [23]. Another related paper is [19], in which the authors consider fixed wavelength conversion capability within the nodes. However, the details are somewhat different; in [19], a wavelength can be converted into a different wavelength at each node. In contrast, we assume that a wavelength can be switched onto any outgoing fiber at a node.

Several papers consider the wavelength assignment (and routing) problem in single-fiber networks. We only provide a brief review of results related to star and ring networks since summaries of prior work can be found elsewhere [10, 12]. In [18], Raghavan and Upfal prove that $3L/2$ wavelengths and $2L - 1$ wavelengths are sufficient for undirected trees and rings respectively. For trees, Erlebach and Jansen [7] improve the tree bound in [18] to $\lfloor 1.1\omega_{opt} + 0.8 \rfloor$ using the edge coloring algorithm in [17], where ω_{opt} is the minimum number of wavelengths needed. They also present an optimal algorithm for undirected binary trees. An alternative optimal algorithm for binary trees that also works for ternary trees is presented in [13]. Scheduling problems on the ring are considered in [5].

In prior work [12], we generalized the following result on the circular arc-coloring problem [24]: if no mutually overlapping three arcs cover the circle, at most $3L/2$ colors are needed. In [12], we showed that if no c arcs cover the circle, at most $\lceil cL/(c-1) \rceil$ colors are needed. In this paper, the above result is extended to general k-fiber ring networks, we prove that $\lceil ck/(ck-1) \cdot L \rceil$ colors are sufficient, a result that is consistent with the single-fiber case ($k = 1$).

3 Problem Formulation

The following notation and definitions will be used in the paper:

- $G(V, E, k)$: a connected network graph, where each edge has k parallel fibers. Here the term *fiber* refers to an individual fiber between two nodes. We assume that each fiber can carry the same number of wavelengths and that network nodes can switch an incoming wavelength onto any outgoing fiber, provided that the fiber is not already using that wavelength for some other connection.
- Let $n = |V|$, the number of network nodes.
- *Routing.* We do not consider the additional problem of routing in this paper; that is, we assume routes are given. Note that the WAP on a unidirectional ring is equivalent to that on an undirected ring with the same routes. Therefore, since a directed ring with full-duplex links between each pair of nodes is equivalent to two unidirectional rings, we need only consider the assignment problem (with fixed routes) on the undirected ring.
- $I = \{(s_1, d_1), ..., (s_k, d_k) : s_i, d_i \in V\}$: a collection of source-destination connection requests. Note that several connections in I may have the same source-destination pair.
- $L_e(I)$: the number of connections in a connection set I that use edge e. Let $L(I) = \max_e L_e(I)$, the *connection load*, be the largest number of connections using any link for a given connection set I. We will drop I where the connection set implied is obvious.
- *Uniform connection set.* A path set I is said to be *uniform* if the number of routes traversing any link is the same.

The wavelength assignment problem we consider is: given G and I, find the minimum number of wavelengths needed to schedule all the connections in I so that no two connections that share a *fiber* use the same wavelength.

4 Dual-fiber Star Networks

It is known that WAP is NP-complete for single-fiber undirected star networks [18]. Interestingly, as we show next, the dual-fiber version admits an optimal polynomial-time algorithm. This result is used later in the section to solve the assignment problem when path lengths are restricted.

To proceed, we first (for the proof) assign directions to all the request paths. If the paths along each link are directed such that exactly half traverse the link in one direction and the other half traverse the link in the opposite direction, then we call the directed path set *a balanced set of directions* [19]. Then the following result was shown in [19].

Lemma 1 *[19] Consider an undirected tree network T with L even. Every uniform path set I with load L may have its paths directed such that it has a balanced set of directions.*

Proof: For reader convenience, we outline the main steps in the proof. From the tree $T(V, E)$ and path set I, we can construct a undirected multi-edge graph $G(V, E')$, such that $\forall v_1, v_2 \in V, (v_1, v_2) \in E'$ if and only if $(v_1, v_2) \in I$. Consider $v \in V$ in tree T and suppose v has x edges. Then the x edges have total load xL. Let n_1 be the number of paths that terminate at v and n_2 be the number of paths that pass through v. Then $n_1 + 2n_2 = xL$. Since L is even, n_1 is even. This means that each node in $G(V, E')$ has an even number of incident edges. Therefore an Euler tour can be constructed in $G(V, E')$. Along the (directed) Euler tour, we direct all paths. The directed path set is a balanced set of directions. ∎

Theorem 1 *In a dual-fiber star network, any path set I with load L can be assigned with optimal $\lceil L/2 \rceil$ wavelengths in each fiber.*

Proof: Without loss of generality, we assume that I is a uniform path set (otherwise, we can add one-hop paths such that I satisfies this condition). First consider the case $L = 2x$, x an integer. According to the above lemma, the paths (connections) in I can be assigned directions to create balanced set of directions. In a directed star network with load x, x wavelengths are sufficient to assign these directed paths. Returning to the dual-fiber star and the balanced set of directions, we see that each fiber requires only x wavelengths in each fiber. In general, $\lceil L/2 \rceil$ wavelengths per fiber are sufficient. ∎

The above result, while appearing counter-intuitive, actually has a plausible explanation. We know that WAP is optimally solvable for directed star networks in which each edge represents two separate fibers (one for each direction). Thus, a dual-fiber undirected star network is really a "sort-of" directed star network and therefore it is reasonable to expect an optimal solution for the dual-fiber case.

Our next result for star networks concerns the special case when connection lengths are restricted. In [10], the wavelength assignment problem for arbitrary networks with paths of length at most two was shown to be NP-complete assuming that each link only has a single fiber. However, for dual-fiber optical networks, the same problem becomes polynomially solvable. This is because any arbitrary network with paths of length at most 2 can be transformed into a star network with the same load. This transformation is described next.

Given any arbitrary network $G = (V, E)$, we construct a star network $G_s = (V_s \cup \{s\}, E_s)$ with s as the center and each $v \in V_s$ as a leaf. Next, each link e in G is represented by a distinct edge in G_s that is also labeled e (Thus, $|V_s| = |E|$). For any request set I in G with path-lengths at most 2, we can construct a request set I' for G_s, such that $\forall e_1 e_2 \in I$ if and only if $e_1 e_2 \in I'$. Then the load in G is equal to the load in G_s, i.e., $L(G) = L(G')$. The wavelength assignment for I' on G_s can be translated to a wavelength assignment for I on G in a straightforward way because wavelengths are assigned to the same path in the two instances. In dual-fiber star networks, we have seen that the wavelength assignment problem can be solved optimally and thus, we have following corollary:

COROLLARY **1** *For any path set I with load L on an arbitrary dual-fiber network, if the path length at most 2, then the connections can be assigned with optimal $\lceil L/2 \rceil$ wavelengths per fiber.*

It is straightforward to extend these results to k-fiber optical networks. If k is even, we have following corollary:

COROLLARY **2** *For any k-fiber star network and k even, any path set I with load L can be assigned with optimal $\lceil L/k \rceil$ wavelengths in each fiber.*

COROLLARY **3** *For any path set I with load L on a k-fiber network of arbitrary topology and k even, if the path length is at most 2, the connections can be assigned with optimal $\lceil L/k \rceil$ wavelengths per fiber.*

5 Dual-fiber Ring Networks

5.1 Complexity

The wavelength assignment problem in ring networks is closely related to the problem of coloring circular-arc graphs, which has been shown to be NP-complete [8]. However, for dual-fiber ring networks, it is not immediately clear whether the complexity is the same since there is added flexibility in selecting fibers. Nevertheless, we now show that the dual-fiber version of the assignment problem is also NP-complete.

As with the single-fiber version, the arc-coloring problem is used in the reduction although the details are different. An instance of arc-coloring is given by a family $P = \{A_1, A_2, ..., A_n\}$ of circular arcs and a positive integer K. Each arc $A_i \in P$ is described by a pair (a_i, b_i) with $a_i, b_i \in \{1, ..., m\}$. We call a_i and b_i the endpoints of arc A_i. Points located consecutively in anti-clockwise order around a circle are labeled $\{1, ..., m\}$.

The span $sp(A_i)$ of arc A_i is the set $\{a_i + 1, a_i + 2, ..., b_i\}$ if $a_i < b_i$ and $\{a_i + 1, ..., m, 1, ..., b_i\}$ if $a_i > b_i$. Two arcs A_i and A_j intersect if $sp(A_i) \cap sp(A_j) \neq \emptyset$. It is NP-complete to decide whether the arcs in P can be colored with K colors such that arcs with the same color do not intersect [8].

Note that we can assume without loss of generality that $m \leq 2n$, $K \leq n$, and each number $i \in \{1, ..., m\}$ is contained in the span of exactly K arcs, otherwise, new arcs of the form $(i, i + 1)$ or $(m, 1)$ can be introduced without changing the K-colorability.

Now consider the dual-fiber ring network. The idea is to view an instance I of the arc-coloring problem as a request set in single fiber networks and then transform it into a request set I' in dual-fiber ring networks, such that I is K-colorable if and only if I' can be assigned wavelengths in each fiber with the K available wavelengths.

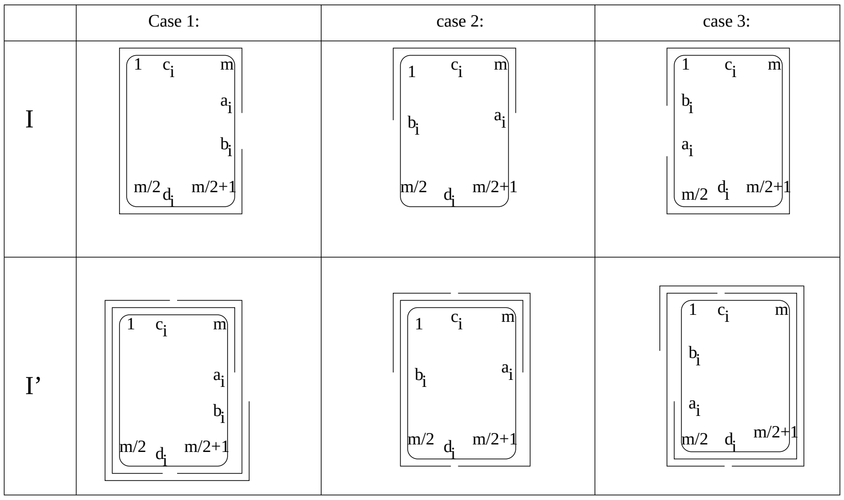

Figure 1: The transformation of I to I'.

In order to transform I into I', we introduce $2K$ new points on the circle, K points between $\lfloor m/2 \rfloor$ and $\lfloor m/2 \rfloor + 1$ and K points between m and 1. We label the new points between m and 1 by $c_1, c_2, ..., c_K$ and the new points between $\lfloor m/2 \rfloor$ and $\lfloor m/2 \rfloor + 1$ by $d_1, d_2, ..., d_K$. By our assumption, there are exactly K arcs $B_1, ..., B_K$ spanning point 1. For each $B_i = (a_i, b_i)$, we identify three possible cases and we replace the arc with three new arcs depending on the different cases (see Figure 1):

- Case $a_i > b_i > \lfloor m/2 \rfloor$: replace B_i with $B_i^1 = (a_i, d_i), B_i^2 = (d_i, c_i), B_i^3 = (c_i, b_i)$.
- Case: $a_i > \lfloor m/2 \rfloor \geq b_i$: replace B_i with $B_i^1 = (a_i, d_i), B_i^2 = (d_i, c_i), B_i^3 = (c_i, b_i)$.
- Case $\lfloor m/2 \rfloor \geq a_i > b_i$: replace B_i with $B_i^1 = (a_i, c_i), B_i^2 = (c_i, d_i), B_i^3 = (d_i, b_i)$.

Because B_i spans point 1, there are no other cases to consider. Other arcs that do not span point 1 are copied to instance I' without modification.

From the construction, each of the new points is an endpoint of exactly two arcs. In the instance I', each point is contained in the span of exactly $2K$ arcs, because each B_i adds one more arc in the span of each point and there are K such arcs. Each pair of arcs that share a new point as an end point must necessarily be assigned the same color in any proper K-wavelength assignment for the new family of arcs because they intersect the same $2K - 1$ other arcs. For example, consider point c_i. Suppose that c_i is a common end point of both B_i^2 and B_i^3. Then c_i can't be any other arc's end point according to our construction. However, $c_i \notin sp(B_i^3)$. Then there exist other $2K - 1$ arcs that pass through point c_i, which means B_i^2 and B_i^3 intersect with these $2K - 1$ arcs. So, the arcs B_i^2, B_i^3 must have the same wavelength. By the same reason, B_i^1, B_i^2 must have the same wavelength. So, the new three arcs B_i^1, B_i^2, B_i^3 for each i must share the same wavelength.

Now, if I is K-colorable, I' can be assigned using K wavelengths in each fiber in a dual-fiber ring network. On the other hand, if I' can be assigned by K wavelengths per fiber in a dual-fiber ring network, we can replace back each group of three new arcs by the original arc that was removed and, using the same wavelength, get a K-color assignment for I. Finally, noting that the problem of K-color assignment for I is in the class NP, we get the following result.

Theorem 2 *The wavelength assignment problem in dual-fiber ring networks is NP-complete.*

5.2 An upper bound

For single-fiber ring networks, we are aware of only one upper bound on number of wavelengths required: $2L-1$ [18]. As mentioned earlier, this upper bound loosely translates to L wavelengths per fiber if two fibers are used in parallel. However, we now show that the ability to switch between fibers allows the bound to be decreased to $\lceil 3/4 \cdot L - 1/2 \rceil$ wavelengths per fiber.

Lemma 2 *For any request set I with load L in a ring network, I can be partitioned into three parts: I_1, I_2, I_3, such that the loads of I_1, I_2, I_3 are at most $L/2, 1, L/2$ respectively.*

Proof: Without of loss generality, we assume I is a uniform request set. Pick an arc A_0 and let p be the counterclockwise endpoint of A_0. Moving clockwise around the circle, let A_1 be the the arc that passes through p for the $L/2$-th time. Then, I_1 is assigned all the arcs before A_1, I_2 includes arc A_1 only, and I_3 is assigned all the remaining arcs. Obviously, I_1, I_2, I_3 form a partition of I, and the loads of I_1, I_2, I_3 are at most $L/2, 1, L/2$ respectively. ∎

Next, observe that in dual-fiber networks, any request set with load at most 2 needs just one wavelength. We will use this observation as the base case in a decomposition argument below.

Theorem 3 *In a dual-fiber ring network, any request set with load L can be assigned with $\lceil 3/4 \cdot L - 1/2 \rceil$ wavelengths per fiber.*

Proof: Let $T(L)$ be the total number of wavelengths (across both fibers) required in dual-fiber ring networks for a request set with load L. According to the above lemma, we have

$$T(L) \leq 2T(L/2) + 1$$

because the request set can be partitioned as shown. Here, the initial condition is $T(2) = 2$. Solving, we get $T(L) \leq 3/2 \cdot L - 1$. Because the number of wavelengths must be an integer, $T(L) \leq \lceil 3/2 \cdot L - 1 \rceil$. This bound applies to the total number of wavelengths. Dividing by two provides the per-fiber bound. ∎

For the wavelength assignment problem in single-fiber ring networks, a static scenario is given in [9] that proves that the known upper bound of $2L-1$ is optimal when the number of nodes is large enough. (Note that an example for the arc-coloring problem is given in [24]). In this paper, we present a similar scenario and show that our upper bound for dual-fiber ring is also tight when the number of nodes is large enough.

THEOREM 4 *Given a dual-fiber ring with $N > 3/2L$ nodes, there exist connection sets that require $W = \lceil 3/2L - 1 \rceil$ wavelengths overall.*

Proof: For convenience, we will devise an instance in which L is divisible by 2 and N is divisible by 3. The connection requests are divided into four groups: $\mathcal{A} = \{a_0, ..., a_{L/2-2}\}, \mathcal{B} = \{b_0, ..., b_{L/2-2}\}, \mathcal{C} = \{c_0, ..., c_{L/2-2}\}, \mathcal{D} = \{d_0, d_1\}$. The nodes of the ring network are numbered starting at an arbitrary node 0, proceeding clockwise up to node $N-1$. We define:

$$a_i = (i, 2/3N + i), b_i = (2/3N + i, 1/3N + i), c_i = (1/3N + i, i + 1), i = 0, ..., L/2 - 2,$$

$$d_0 = (1/3N - 1, 1), d_1 = (1, 0).$$

All these paths are clockwise (the direction from the first endpoint to the second is clockwise). It is easy to verify that the maximal load is L. All paths in group $\mathcal{A}$ overlap on both links $[1/3N - 1, 1/3N]$ and $[2/3N - 1, 2/3N]$. Similarly, all paths in group $\mathcal{B}$ overlap on both links $[0, 1]$ and $[1/3N - 1, 1/3N]$; all paths in group $\mathcal{C}$ overlap on both links $[2/3N - 1, 2/3N]$ and $[0, 1]$; both d_0 and d_1 overlap all these three links. In addition, $\forall a \in \mathcal{A}, b \in \mathcal{B}, c \in \mathcal{C}$, the sum of path-lengths of a, b, c is greater than $2N$, which means a, b, c need three wavelengths overall (counting re-use of a wavelength across the two fibers); if a, b, c come from at most two of the four groups, they must share at least one common link, which means a, b, c need three wavelengths also. Totally, we have $3(L/2 - 1) + 2 = 3/2L - 1$ paths and each needs a different wavelength, resulting in our tight bound. ∎

5.3 Covering restriction

When the path sets are such that only shortest paths are used, the bound for the single-fiber version remains the same, $2L - 1$ wavelengths. However, in dual-fiber ring networks, we now provide a sharper per-fiber bound: $2L/3$ wavelengths.

We say a set of paths $p_1, ..., p_n$ cover the ring R if and only if $\forall x \in R, \exists \epsilon > 0, i \in \{1, ..., n\}$, such that path $(x - \epsilon, x + \epsilon) \subset p_i$.

LEMMA 3 *In a dual-fiber ring network $\lceil 2L/3 \rceil$ wavelengths per fiber are sufficient to assign a connection request set with load L in which no two paths cover the whole ring.*

Proof: As in Section 4, we assume, without loss of generality, that the connection set is uniform. We prove our result by induction on L. First, consider $L = 2$. Here, wavelength 0 in each fiber can be used to assign the paths. Next, for the induction hypothesis, we assume that when $L < r$, $\lceil 2L/3 \rceil$ wavelengths per fiber are sufficient. We wish to prove that when $L = r$, our result is still true.

We will use X to denote one fiber and Y to denote the other. Colors (wavelengths) will be denoted by integers starting with 0. Pick any connection path, call it A_0 and color it $X(0)$ (i.e., color 0 on fiber X). Let p be the counterclockwise end of A_0. Now, moving in the clockwise direction, identify successive non-overlapping arcs after the clockwise end of A_0. Go around until an arc crosses p or ends at p. Call this arc A_1. If A_1 ends at p color it $X(0)$ otherwise use $X(0)$ for A_1 until p and switch to $Y(0)$ (no change in wavelength) at p. This completes one round. Continue for one more round in the clockwise direction, coloring arcs with $Y(0)$ until we reach the last arc to start from or overlap p in this round. After this we use $X(1)$ starting from the last arc onwards. Call this arc A_2 and let its counterclockwise end be q. At this time we will have completed two rounds of the circle. Continue identifying arcs in the clockwise direction using color $X(1)$ until q is reached and switch to $Y(1)$ at point q. If p has been passed, stop; else continue with $Y(1)$ until p has been passed or a path ends at p. At this time, we will have circumscribed the circle three times.

Note that the last "round" is possible because of the assumption that no two paths cover the whole ring. To see why, label the last arc in the third round (the one that passes or ends at p) by the label A_4. Now, the last round would not necessarily be possible if A_4 overlapped the point q (since A_4 is assigned $Y(1)$). But if it did, then we have the following situation: A_2 spans the range (q,p) and A_4 spans (p,q) and therefore together they would cover the ring – a contradiction.

Let I_1 denote the all paths colored starting with A_0 and stopping at A_2 above. Let $I_2 = I - I_1$. Then $I = I_1 \cup I_2, I_1 \cap I_2 = \emptyset$. It is easy to see that $L(I_2) \leq L(I) - 3 = r - 3$ since we have covered the ring three times. By the induction hypothesis, the path set I_2 can be assigned by using $\lceil 2L(I_2)/3 \rceil \leq \lceil 2L(I)/3 \rceil - 2$ wavelengths on each fiber. But we have used only two wavelengths for the the paths in I_1. Hence, the path set I can be assigned by using $\lceil 2L/3 \rceil - 2 + 2 = \lceil 2L/3 \rceil$ wavelengths in each fiber. ∎

COROLLARY 4 *In a dual-fiber ring network with shortest-path routing,* $\lceil 2L/3 \rceil$ *wavelengths per fiber are sufficient to assign a connection request set with load* L.

Proof: In a ring network with shortest path routing, no two paths cover the whole ring. ∎

In this paper, we generalize the above upper bound in two ways: by considering the number of paths that form the restriction (no c paths cover the ring) and by considering the number of fibers. Both results end up generalizing Tucker's result [24] for three arcs. The dual-fiber generalization is presented next whereas the bound for general k fibers is presented later.

THEOREM 5 *In a dual-fiber ring network, if no* c *paths cover the whole cycle, then* $\lceil c/(2c-1) \cdot L \rceil$ *wavelengths per fiber are sufficient, where* L *is the connection load.*

Proof: We prove this theorem by induction on c. We know that when $c = 2$, the result is true from the preceding lemma. Suppose it is true for any $c < m, m > 2$, we will prove that this theorem is true for $c = m$.

When $L < 2m - 1$, we consider two cases:

- Case 1: $L = 2k$. Here, $2k < 2m - 1$, and thus $k < m$. By assumption, we only need $\lceil k/(2k-1) \cdot 2k \rceil = k + 1$ wavelengths each fiber, which happens to be $\lceil m/(2m-1) \cdot 2k \rceil = k + 1$.

- Case 2: $L = 2k+1$. Here, $2k+1 < 2m-1$, and thus $k+1 < m$. By assumption, we only need $\lceil (k+1)/(2k+1) \cdot L \rceil = k+1$ wavelengths each fiber, which happens to be $\lceil m/(2m-1) \cdot (2k+1) \rceil = k+1$.

This means that if $L < 2m-1$, the result holds.

Next, we assume that $L < r$ is true, and we show that the result holds for $L = r$, where $r \geq 2m-1$.

We use the idea in the previous lemma (A similar idea is used in Tucker's proof [24]). Pick a point p, such that S_p, the set of arcs containing p, has size r. Let A_1 be the arc in S_p that extends the shortest length on the counterclockwise side of p. Starting with A_1, we move clockwise round and round the circle indexing the arcs $A_1, A_2, \ldots$ by the rule: if A_i is the arc previously indexed, then A_{i+1} is the first unindexed arc to begin after the end of A_i. If there are multiple such arcs, pick one randomly. As before, let A_{b_1} be the second arc to overlap the counterclockwise end of A_1; let $A_{b_{i+1}}$ be the second arc to overlap the counterclockwise end of $A_{b_i}, i = 1, \ldots, m-2$. Finally, if $A_{b_{m-1}}$ does not pass p, let A_{b_m} be the second round arc to pass point p. If during the second round, there is a gap in passing point p, let A_{b_m} be the last arc ending before point p. If $A_{b_{m-1}}$ does pass point p, let A_{b_m} be the arc passing point p next time. If there is a gap in passing point p, let A_{b_m} be the last arc ending before point p.

Let $I_{b_m} = I - \{A_1, A_2, \ldots, A_{b_m}\}$. Then $L(I_{b_m}) \leq L(I) - (2m-1)$. By the induction hypothesis, I_{b_m} can be colored by $\lceil m/(2m-1) \cdot L(I_{b_m}) \rceil$ wavelengths. We need to prove that $A_1, \ldots, A_{b_m}$ can be colored by m colors. We claim that $A_{b_{m-1}}$ and A_{b_m} can't overlap, otherwise $A_{b_1}, A_{b_2}, \ldots, A_{b_m}$ will cover the whole cycle. Then, we can assign color 0 to paths before A_{b_1}; we can assign color i to paths from A_{b_i} to $A_{b_{i+1}-1}$, here $i = 1, \ldots, m-2$. Because $A_{b_{m-1}}$ and A_{b_m} do not overlap, we can assign color $m-1$ to the paths between these two arcs, including these two arcs. Thus $A_1, \ldots, A_{b_m}$ can be colored by m colors. Hence, I can be colored by $m + \lceil m/(2m-1) \cdot L(I_{b_k}) \rceil \leq m + \lceil m/(2m-1) \cdot (L - (2m-1)) \rceil = \lceil m/(2m-1) \cdot L \rceil$ colors. ∎

Using this theorem, we generalize the earlier result on path-length restricted connection sets to the case when path-lengths are at most m.

Corollary 5 *In a dual-fiber ring network, if all connection paths have path-length at most m, $\omega_{opt} \leq \lceil n/(m-1) \rceil L / (2\lceil n/(m-1) \rceil - 1) \rceil$ for each fiber.*

Proof: The result follows from the simple observation that no $\lceil n/(m-1) \rceil$ paths cover the whole ring.

6 Multifiber Ring Networks

In this section, we consider ring networks in which each link has k, $(k > 1)$ fibers. We show that all our results for dual-fiber ring networks can be extended to k-fiber ring networks although the proofs are more involved.

6.1 Complexity

In Section 5, we proved that the wavelength assignment problem in dual-fiber ring networks is NP-complete by reducing from the problem of coloring circular-arc graphs. In this section, we use the same technique to prove that for any k-fiber ring networks (k arbitrary), the wavelength assignment problem is still NP-complete.

We use the same instance I, K of an arc-coloring problem and the same assumptions as in Section 5. The difference here is that we view the instance I of the arc-coloring problem as a request set in single fiber networks and then transform it into a request set I'' for a k-fiber ring network, such that I is K-colorable if and only if I'' can be assigned with K wavelengths in each fiber.

Generalizing the idea used earlier, we transform I into I'', by introducing $2(k-1)K$ new points on the circle, $(k-1)K$ points between $\lfloor m/2 \rfloor$ and $\lfloor m/2 \rfloor + 1$ and $(k-1)K$ points between m and 1. We label the new points between m and 1 by $c_1^1, c_2^1, ..., c_K^1, ..., c_1^{k-1}, c_2^{k-1}, ..., c_K^{k-1}$ and the new points between $\lfloor m/2 \rfloor$ and $\lfloor m/2 \rfloor + 1$ by $d_1^1, d_2^1, ..., d_K^1, ..., d_1^{k-1}, d_2^{k-1}, ..., d_K^{k-1}$. By our assumption, there are exactly K arcs $B_1, ..., B_K$ spanning point 1. For each $B_i = (a_i, b_i)$, there are three possible cases to consider (as we did earlier). In each case, we replace arc B_i with $2(k-1)+1$ new arcs depending on the case:

- Case $a_i > b_i > \lfloor m/2 \rfloor$: replace B_i with $B_i^1 = (a_i, d_i^1), B_i^2 = (d_i^1, c_i^1), B_i^3 = (c_i^1, d_i^2), B_i^4 = (d_i^2, c_i^2), ..., B_i^{2(k-1)} = (d_i^{k-1}, c_i^{k-1}), B_i^{2(k-1)+1} = (c_i^{k-1}, b_i)$.
- Case $a_i > \lfloor m/2 \rfloor > b_i$: replace B_i with $B_i^1 = (a_i, d_i^1), B_i^2 = (d_i^1, c_i^1), B_i^3 = (c_i^1, d_i^2) B_i^4 = (d_i^2, c_i^2), ..., B_i^{2(k-1)} = (d_i^{k-1}, c_i^{k-1}), B_i^{2(k-1)+1} = (c_i^{k-1}, b_i)$.
- Case $\lfloor m/2 \rfloor > a_i > b_i$: replace B_i with $B_i^1 = (a_i, c_i^1), B_i^2 = (c_i^1, d_i^1), B_i^3 = (d_i^1, c_i^2) B_i^4 = (c_i^2, d_i^2), ..., B_i^{2(k-1)} = (c_i^{k-1}, d_i^{k-1}), B_i^{2(k-1)+1} = (d_i^{k-1}, b_i)$.

Because B_i spans point 1, there are no other cases to consider. Other arcs that do not span point 1 are copied to instance I'' without modification. Then each of the new points is an endpoint of exactly two arcs. In the instance I'', each point is contained in the span of exactly kK arcs, because each B_i adds $k-1$ more arcs in the span of each point and there are K such arcs. By the same reasoning as in Section 5, I is K-colorable if and only if I'' can be assigned by K wavelengths in each fiber for the k-fiber ring network. In this way, we have shown that the wavelength assignment problem for general k-fiber ring networks is still NP-complete. ∎

6.2 An upper bound

We now generalize the upper bound for dual-fiber rings (and, for that matter, single-fiber rings) to the general case of k-fiber rings. We show that $\lceil (k+1)/k \cdot L - 1 \rceil$ wavelengths overall, or approximately $(k+1)L/k^2$ wavelengths per fiber are sufficient. Furthermore, generalizing the instance used to show tightness of the bound for dual-fiber rings, we show that the bound below is tight. Following these results, we consider path-length and covering restrictions described earlier.

THEOREM **6** *If a ring network has k-fibers in each link, then for any request set with load L, $\lceil (k+1)/k \cdot L-1\rceil$ wavelengths totally over all k fibers are sufficient to assign the request set.*

Proof: We use the same idea as in dual-fiber ring networks. Let $T_k(L)$ be the number of wavelengths required in k-fiber ring networks for a load-L request set. Then, we have

$$T_k(L) \leq 2T_k(L/2) + 1$$

with initial condition $T_k(k) = k$. Solving, we get $T_k(L) \leq (k+1)/k \cdot L - 1$. Because the number of wavelengths must be an integer, $T_k(L) = \lceil (k+1)/k \cdot L - 1\rceil$. ∎

Note that this bound generalizes several previous bounds for rings; for example, taking $k = 1$ gives the single-fiber bound and $k = 2$ gives the dual-fiber bound; if $k = L$, the bound is L, which means only one wavelength per fiber is required, as we would expect.

Next, we construct a connection request set to show that the upper bound is tight.

THEOREM **7** *Given a general k-fiber ring with $N > (k+1)/k \cdot L$ nodes, there exist connection requirement instances that require $W = \lceil (k+1)/k \cdot L - 1\rceil$ wavelengths.*

Proof: In the instance we create, L is divisible by k and N is divisible by $k+1$. The connection requests are divided into $k+2$ groups: $\mathcal{A}^0 = \{a_0^0, ..., a_{L/k-2}^0\}, \mathcal{A}^1 = \{a_0^1, ..., a_{L/k-2}^1\}, ..., \mathcal{A}^k = \{a_0^k, ..., a_{L/k-2}^k\}, \mathcal{B} = \{b_0, b_1, ..., b_{k-1}\}$. The nodes of the ring network are numbered starting at an arbitrary node 0, proceeding clockwise up to node $N-1$. We define:

$$a_i^0 = [i, k/(k+1)N + i], a_i^1 = [1/(k+1)N + i, 1 + i], i = 0, ..., L/k - 2,$$

$$a_i^j = [j/(k+1)N + i, (j-1)/(k+1)N + i], i = 0, ..., L/k - 2, j = 2, ..., k,$$

$$b_i = [1/(k+1)N, 0], i = 0, ..., k-1.$$

All these paths route along clockwise direction. Let A_0 be the link $[N-1, 0]$ and A_i be the link $[i/(k+1)N - 1, i/(k+1)N]$ where $i = 1, ..., k$. We define $A = \{A_i : i = 0, ..., k\}$. It is easy to verify that the maximal load is L. All paths in group $\mathcal{A}_i$ overlap on all links of $A - A_i$. Paths in group $\mathcal{B}$ overlap on all links of A. So, any $k+1$ paths from k or fewer groups at least overlap one link of A and should use $k+1$ wavelengths. If $k+1$ paths from $\mathcal{A}_0$,...,$\mathcal{A}_k$ $k+1$ groups, the total path-length of the $k+1$ paths is greater than kN and should use $k+1$ wavelengths. So, in this connection request set, each path should use a different wavelength. Totally, we have $(k+1)(L/k-1) + k = (k+1)/k \cdot L - 1$ paths. This shows our upper bound in k-fiber ring networks is tight when the number of nodes is large enough. ∎

Next, we consider path-length restrictions on connection sets for k-fiber ring networks. Note that, trivially, for any path set I with load $L \leq k$, one wavelength is sufficient.

Theorem 8 *Consider a ring network of each link with $k, k > 1$, fibers. For any path set I with load L, if no 2 paths cover the whole ring, then, $\lceil 2/(2k-1) \cdot L \rceil$ wavelengths per fiber are sufficient.*

Proof: When $k < L \leq 2k-1$, we can separate I into two parts I_1, I_2, such that $I_1 \cap I_2 = \emptyset, I_1 \cup I_2 = I$, and $L(I_1) \leq k, L(I_2) \leq k$. Therefore, we can assign I_1 with wavelength 1, I_2 with wavelength 2. Thus, in this case, we only need $\lceil 2/(2k-1) \cdot L \rceil = 2$ wavelengths.

Now, we assume the result holds for $L < r$, where $r > 2k-1$. We will prove that it holds for $L = r$.

Without loss of generality, we assume that I is a uniform path set (otherwise, we can add one-hop paths to make it so). Pick any arc and label it A_0, and its counter clockwise endpoint p. Along the clockwise direction, label all paths $A_1, A_2, ...$ by the rule: A_{i+1} immediately follows A_i. Because the path set is uniform, A_{i+1} immediately follows A_i. We label the arc passing through p for the k-th time or starting from point p as A_{b_1}, and label the arc passing through p for the $(2k-1)$-th time or ending at point p as A_{b_2}. We define $I_{b_2} = I - \{A_0, ..., A_{b_2}\}$, then $L(I_{b_2}) \leq L - (2k-1)$. By assumption, I_{b_2} can be assigned with $\lceil 2/(2k-1) \cdot L(I_{b_2}) \rceil \leq \lceil 2/(2k-1) \cdot (L-(2k-1)) \rceil = \lceil 2/(2k-1) \cdot L \rceil - 2$ wavelengths. Now, we want to prove that $A_0, ..., A_{b_2}$ can be assigned with 2 wavelengths in each fiber. $A_0, ..., A_{b_1-1}$ can be assigned by wavelength 1, and $I_{b_1,b_2} = \{A_{b_1}, ..., A_{b_2}\}$ can be assigned wavelength 2, because $L(I_{b_1,b_2}) \leq L$ as long as A_{b_1} and A_{b_2} do not cover the whole cycle. Thus, putting these facts together, we need at most $\lceil 2/(2m-1) \cdot L \rceil$ wavelengths per fiber. ∎

The above theorem can be extended to the general case in which no c paths cover the whole ring.

Theorem 9 *Consider a ring network of each link with $k, k > 1$ fibers. For any path set I with load L, if no c paths cover the whole ring, then $\lceil c/(kc-1) \cdot L \rceil$ wavelengths per fiber are sufficient.*

Proof: We use proof by induction on parameter c. First, we show that when $L \leq kc-1$, the theorem is true.

- If $L \leq k-1$, we can use one wavelength in each fiber to assign the path set I, which means the result holds for this case.
- If $(i-1)k \leq L \leq ik-1, i = 2, ..., c$, then no i paths cover the whole ring by assumption, and thus we only need $\lceil i/(ik-1) \cdot L \rceil = i$ wavelengths for each fiber to color the path set I, which is the value of $\lceil c/(ck-1) \cdot L \rceil = i$. So, the theorem is true in this case.

Now, we assume that the result is true for $L < r$, where $r \geq kc-1$; we want to prove that it holds for $L = r$.

Without loss of generality, we assume that I is a uniform path set. Pick any arc and label it as A_0, and its counter clockwise endpoint as p. As in the dual-fiber version, along the clockwise direction, we label all paths as A_i by the rule: A_{i+1} immediately follows A_i. We label the arc passing through or starting from point p for the k-th time as A_{b_1}, the arc passing through or starting from the start point of A_{b_1} k-th time as A_{b_2}. In general, we label the arc passing through or starting from the starting point of

A_{b_i} for the k-th time as $A_{b_{i+1}}$, where $i = 1, ..., c-2$. We label the arc passing through or ending at point p for the $kc-1$-th time as A_{b_c}. Define $I_{b_c} = I - \{A_0, ..., A_{b_c}\}$, then $L(I_{b_c}) \le L-(ck-1)$. By assumption, I_{b_c} can be assigned with $\lceil c/(ck-1) \cdot L(I_{b_c}) \rceil \le \lceil c/(ck-1) \cdot (L-(ck-1)) \rceil = \lceil c/(ck-1) \cdot L \rceil - c$ wavelengths. Now, we want to prove that $A_0, ..., A_{b_c}$ can be assigned by c wavelengths in each fiber. $A_0, ..., A_{b_1 - 1}$ can be assigned by wavelength 1, and $A_{b_i}, ...A_{b_{i+1}-1}$ can be assigned by wavelength $i+1$, where $i = 1, ..., c-2$. And $I_{b_{c-1}, b_c} = \{A_{b_{c-1}}, ..., A_{b_c}\}$ can be assigned wavelength c, because $L(I_{b_{c-1}, b_c}) \le k$ as long as $A_{b_1}, A_{b_2}, ..., A_{b_c}$ do not cover the whole ring. Thus $\lceil c/(ck-1) \cdot L \rceil$ wavelengths per fiber are sufficient. ∎

Finally, using this theorem, we obtain an upper bound when the paths are length-restricted in the ring.

COROLLARY **6** *If all connection paths have path-length at most m in a k-fiber ring network, $\omega_{opt} \le \lceil \lceil n/(m-1) \rceil L/(k \lceil n/(m-1) \rceil - 1 \rceil$, where n is the number of nodes in the ring.*

Proof: Note that no $\lceil n/(m-1) \rceil$ paths cover the whole ring.

7 General Networks

In this section, we present a few results for arbitrary topologies. Because no assumptions are made about the topology, the results are not as strong as results for specific topologies. Let $\omega(k)$ be the minimum number of wavelengths required to support a given connection set on a network in which each link consists of k fibers. (Thus, $\omega(1)$ is the number of wavelengths required in a single-fiber network). We now derive some results relating $\omega(k)$ and $\omega(1)$, starting with the simple observation that $\omega(k) \le \lceil \omega(1)/k \rceil$.

LEMMA **4** *For any graph $G = (V, E)$ and any connection set I, we have $\omega(k) \le \lceil \omega(1)/k \rceil$.*

Proof: For single-fiber network, the optimal solution is $\omega(1)$. Distributing $\omega(1)$ among k fibers, each fiber needs $\lceil \omega(1)/k \rceil$ wavelengths, which is an upper bound for $\omega(k)$. ∎

It is easy to construct a network and a connection set such that above upper bound is tight. For example, take G to be a a single link with L connections. Obviously, $\omega(1) = L$, and $\omega(k) \ge \lceil L/k \rceil = \lceil \omega(1)/k \rceil$. So $\omega(k) = \lceil \omega(1)/k \rceil$ in this case.

A natural question arises at this point: is there a limit to the benefit incurred with additional fibers? One way to address this question is to examine whether $\omega(k)/\omega(k+1)$ is bounded by a constant. We now show that $\omega(k)/\omega(k+1)$ can be arbitrarily large, providing some theoretical support to the notion that an additional fiber in general can greatly decrease the number of required wavelengths. First, we consider the case $k = 2$.

LEMMA **5** *For any $n > 0$, there exist a network and a connection set, such that $w(2) \le w(1)/n$.*

Proof: Consider a $n \times n$ mesh network $M = \{(i,j) : i,j = 1,...,n\}$ and n fixed path connections $p_i, i = 1,...,n$, where

$$\begin{aligned}
p_1 &: \quad (1,1) - (1,2) - (2,2) - (2,3) - (3,3) - ... - (n-1,n) - (n,n), \\
p_2 &: \quad (2,1) - (2,2) - (1,2) - (1,3) - (2,3) - ... - (n-2,n) - (n-1,n), \\
&\dots \\
p_i &: \quad (i,1) - (i,2) - (i-1,2) - ... - (1,i) - (1,i+1) - ... - (n-i,n) - (n-i+1,n), \\
&\dots \\
p_n &: \quad (n,1) - (n,2) - (n-1,2) - (n-1,3) - ... - (2,n) - (1,n).
\end{aligned}$$

From the construction, it is easy to verify that each pair of connections share a common link. Thus, $\omega(1) = n$. However, at most 2 connections share one link, and therefore $L = 2$. Hence, $\omega(2) = 1$. ∎

This result can be extended to any value of k.

Lemma 6 *For any $n > 0, k > 0$, there exists a network and a connection set, such that $\omega(k)/\omega(k+1) > n$.*

Proof: Construct a $(k+1)n \times C_{k+1}^{(k+1)n}$ mesh with $(k+1)n$ connections such that any $k+1$ connections share a link and at most $k+1$ connections share a single link. (Here, $C_{k+1}^{(k+1)n}$ denotes the combinatorial number of ways of choosing $k+1$ items from $(k+1)n$ items). If each link has $k+1$ fibers, we only need 1 wavelength per fiber because at most $k+1$ connections use a link. If each link only has k fibers, then each wavelength at most can be assigned to k connections. There are totally $(k+1)n$ connections, so we need at least $(k+1)n/k > n$ wavelengths. In this case, we have $\omega(k)/\omega(k+1) > n$. ∎

Next, we consider some bounds for $\omega(k)$ in terms of L, the connection load. From the first lemma, we have following corollaries:

Corollary 7 *If a network has m edges, then $\omega(k) \leq \lceil \sqrt{m}L/k \rceil$.*

Proof: For any connection set with load L, construct the path-conflict graph. Each link with load L produces $L(L-1)/2$ edges in the path-conflict graph. Since there are m edges in the network, the path-conflict graph has at most $mL(L-1)/2$ edges. If the vertices with the same color are placed in a group, there are $\omega(1)$ groups and each group pair must share at least one edge, otherwise, we can put these two group together, which means $\omega(1)$ is not minimal, contradiction. So, we get $\omega(1)(\omega(1)-1)/2 \leq mL(L-1)/2$. Then $\omega(1) < \sqrt{m}L, m > 1$. By the first lemma, $\omega(k) \leq \lceil \sqrt{m}L/k \rceil$. ∎

Corollary 8 *If all connections have path-length no more than l, then $\omega(k) \leq \lceil (l(L-1)+1)/k \rceil$.*

Proof: For any connection set with load L and all path-lengths no greater than l, we construct the path-conflict graph. Each connection shares a link with at most $l(L-1)$ others. So the degree of path-conflict graph is at most $l(L-1)$. Then we need at most $l(L-1)+1$ colors for vertex-coloring the path-conflict graph. So $\omega(1) \leq l(L-1)$. By the first lemma, $\omega(k) \leq \lceil (l(L-1)+1)/k \rceil$. ∎

8 Conclusion

In this paper, we considered the wavelength assignment problem in multifiber networks, in which each link has exactly k parallel fibers. Our results for star and ring networks show that fewer wavelengths per fiber are needed to assign all requests than in a corresponding collection of parallel single-fiber rings. For the ring, our upper bounds are summarized on following table.

	shortest-path routing	no c paths cover the ring	general
single-fiber	$2L-1$	$\frac{c}{c-1}L$	$2L-1$
dual-fiber	$\frac{4}{3}L-1$	$\frac{2c}{2c-1}L$	$\frac{3}{2}L-1$
k-fiber	$\frac{2k}{2k-1}L-1$	$\frac{kc}{kc-1}L$	$\frac{k+1}{k}L-1$

Some results were also given for general topologies. Future work is likely to involve specific network topologies for which the routing component must also be considered.

References

[1] A. Acampora. The scalable lightwave network. *IEEE Comm. Mag.*, pages 36–42, Dec. 1994.

[2] R.A. Barry and P.A. Humblet. Models of blocking probability in all optical networks with and without wavelength changers. *IEEE JSAC*, 14(5):858–867, June 1996.

[3] B. Beauquier, J.C. Bermond, L. Gargano, P. Hell, S. Perennes, and U. Vaccaro. Graph problems arising from wavelength routing in all optical networks. *Proc. 2nd Workshop in Optics and Computer Science*, April 1997.

[4] A. Blum. Algorithms for approximate graph coloring. *J. ACM*, 41(4):630–647, July 1994.

[5] X. Zhang C. Qiao and L. Zhao. Scheduling all-to-all connections in wdm rings. *SPIE Proceedings, All Optical Communication Systems: Architecture, Control and Network Issues, vol. 2919*, pages 218–229, 1996.

[6] H. Choi, H-A. Choi, and M. Azizoglu. Efficient scheduling of transmissions in optical broadcast networks. *IEEE/ACM Trans. Networking*, 4(6):913–920, December 1996.

[7] T. Erlebach and K. Jansen. The complexity of call-scheduling in trees, rings, and meshes. *30th Hawaii Int. Conf. System Sciences*, March 1997.

[8] M. Garey, D. Johnson, G. Miller, and C. Papadimitriou. The complexity of coloring circular arcs and chords. *SIAM J. Disc. Math.*, 1(2):216–227, June 1980.

[9] O. Gerstel, R. Ramaswami, and G. Sasaki. Dynamic channel assignment for wdm optical networks with little or no wavelength conversion. *Proc. 1996 Allerton conference*, 1996.

[10] E.J. Harder. *Routing and wavelength assignment in all-optical WDM wavelength-routed networks.* PhD thesis, George Washington University, May 1988.

[11] G. Jeong and E. Ayanoglu. Comparison of wavelength-interchanging and wavelength-selective cross-connects in multiwavelength all-optical networks. *INFOCOM'96*, March 1996.

[12] G. Li and R. Simha. On bounds for the wavelength assignment problem in optical ring networks. *Submitted*, 1998.

[13] G. Li and R. Simha. New results for the wavelength assignment problem in tree and ring networks. *ACM-SE'99*, 1999.

[14] D. Matula, G. Marble, and J. Isaacson. *Graph Coloring Algorithms.* 1972.

[15] B. Mukherjee. Wdm-based local lightwave networks part ii: multihop systems. *IEEE Network Mag.*, pages 20–31, July 1992.

[16] B. Mukherjee. *Optical Communication Networks.* Mc-Graw-Hill, 1997.

[17] T. Nishizeki and K. Kashiwagi. On the 1.1 edge-coloring of multigraphs. *SIAM J. Disc. Math.*, 3(3):391–410, August 1990.

[18] P. Raghavan and E. Upfal. Efficient routing in all-optical networks. *Proc 26th ACM Symp. on Theory of Computing, NewYork*, pages 134–143, 1994.

[19] R. Ramaswami and G. Sasaki. Multiwavelength optical networks with limited wavelength conversion. *INFOCOM 97*, pages 489–498, 1997.

[20] R. Ramaswami and K.N. Sivarajan. Routing and wavelength assignment in all-optical networks. *IEEE/ACM Trans. Networking*, 3(5):489–500, October 1995.

[21] R. Ramaswami and K.N. Sivarajan. *Optical networks: a practical perspective.* Morgan-Kaufman, 1998.

[22] S. Subramaniam, M. Azizoglu, and A.K. Somani. All optical networks with sparse wavelength conversion. *IEEE/ACM Trans. Networking*, 4(4):544–557, August 1996.

[23] S. Subramaniam and R. Barry. Wavelength assignment in fixed routing wdm networks. *Proceedings of the IEEE International Conference on Communications*, pages 406–410, May 1997.

[24] A. Tucker. Coloring a family of circular arcs. *SIAM J. Appl. Math.*, 29(3):493–502, November 1975.