Analyze a Load Bearing Structure using MATLAB

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1 Brief Discussion of Theory

We begin with a quick and simple review of some of physics of loads applied to solids. It is important to note that this simple introduction is essential in starting to think about the how a solid structure begins to deform under external forces without going into complex mathematical arguements and defining the most rudimentary physical parameters. The theory that is described is contextual to the problem which the student is expected to solve using MATLAB.

1.1 Stress

Stress is the internal distribution of forces within a body that balance and react to the loads applied to it. It is defined as force per unit area. It has the same units as pressure, and in fact pressure is one special variety of stress. However, stress is a much more complex quantity than pressure because it varies both with direction and with the surface it acts on. [1]
Stress, therefore has units of pressure (*psi, pascals etc.*).
There are different kinds of stresses a material, solid or fluid can experience. [1]

1. Compression: Stress that acts to shorten an object. [1]
2. Tension: Stress that acts to lengthen an object. [1]
3. Normal Stress: Stress that acts perpendicular to a surface. Can be either compressional or tensional. [1]
4. Shear: Stress that acts parallel to a surface. It can cause one object to slide over another. It also tends to deform originally rectangular objects into parallelograms. The most general definition is that shear acts to change the angles in an object. [1]
5. Hydrostatic: Stress (usually compressional) that is uniform in all directions. A scuba diver experiences hydrostatic stress. Stress in the earth is nearly hydrostatic. The term for uniform stress in the earth is lithostatic. [1]

6. Directed Stress: Stress that varies with direction. Stress under a stone slab is directed; there is a force in one direction but no counteracting forces perpendicular to it. This is why a person under a thick slab gets squashed but a scuba diver under the same pressure doesn't. The scuba diver feels the same force in all directions. [1]

In reality we never see stress. We only see the results of stress as it deforms materials. Even if we were to use a strain gauge to measure in-situ stress in solid materials, we would not measure the stress itself. We would measure the deformation of the strain gauge (that's why it's called a "strain gauge") and use that to infer the stress. [1]

1.2 Strain

Strain is quite simply, the relative distortion of a solid or ratio of the change in length to original length. This is a dimensionless parameter or, in other words has no units.
It is also defined as the amount of deformation an object experiences compared to its original size and shape. For example, if a block 10 cm on a side is deformed so that it becomes 9 cm long, the strain is (10-9)/10 or 0.1 (sometimes expressed in percent, in this case 10 percent.). [1] There are different kinds of strains a material, solid or fluid can experience. [1]

1. Longitudinal or Linear Strain: Strain that changes the length of a line without changing its direction. Can be either compressional or tensional. [1]
2. Compression: Longitudinal strain that shortens an object. [1]
3. Tension: Longitudinal strain that lengthens an object. [1]
4. Shear: Strain that changes the angles of an object. Shear causes lines to rotate. [1]
5. Infinitesimal Strain: Strain that is tiny, a few percent or less. Allows a number of useful mathematical simplifications and approximations. [1]
6. Finite Strain: Strain larger than a few percent. Requires a more complicated mathematical treatment than infinitesimal strain. [1]
7. Homogeneous Strain or Uniform strain: Straight lines in the original object remain straight. Parallel lines remain parallel. Circles deform to ellipses. Note that this definition rules out folding, since an originally straight layer has to remain straight. [1]
8. Inhomogeneous Strain: Deformation varies from place to place. Lines may bend and do not necessarily remain parallel. [1]

1.3 Young's Modulus

Stress and Strain related to each other by a proportionality constant viz., Young's Modulus (E). Young's modulus therefore has units of pressure (*psi, pascals etc.*).

It's numerical value is indicative of the "stiffness" of the material: smaller values indicate that less stress is required for more strain. [2]

The Eq. (1) is known as *Hooke's Law*, which states that for a perfectly elastic body, Stress (τ) is directly proportional to Strain (ϵ).

$$\tau \propto \epsilon \tag{1}$$

$$\tau = E\epsilon \tag{2}$$

Figure 1 shows two concentric cylinders made of steel and brass. When a load is placed on them or, subjected to a uniform external force, these cylinders would experience a slight change in length [figure 2].

If the orginial length of the cylinders is denoted by L and the change in length is denoted by δ, Strain induced due to the load is defined as

$$\epsilon = \frac{\delta}{L} \tag{3}$$

Substituting Eq.3 in Eq.2 we have

$$\frac{P}{A} = E\frac{\delta}{L} \tag{4}$$

Re-arranging Eq.4 we arrive at this very important equation.

$$\delta = \frac{P}{A}\frac{L}{E} \tag{5}$$

Due to the load acting on cylinders uniformly, there would be an equal change in length, in both the outer brass cylinder and inner steel cylinder [figure 2]. Hence,

$$\delta_s = \delta_b \tag{6}$$

$$i.e., \frac{P_b L_b}{A_b E_b} = \frac{P_s L_s}{A_s E_s} \tag{7}$$

Here, P_b and P_s are the forces the brass and steel cylinders experience respectively, due to some load placed on the structure [figure 2]. A_b and A_s are the respective areas of crossection of the hollow brass and solid steel cylinders.

2 Problem Description

A solid steel cylinder of radius r and a hollow brass cylinder of outer radius $3r/2$ support the load. Both cylinders are of the same length ($L_b = L_s$) [figure 1]. Assume the Young's Modulus of Steel (E_s) is *twice* that of Brass (E_b). When a load is placed on this arrangement, the distance (δ_s, δ_b) through which the cylinders are compressed the same.

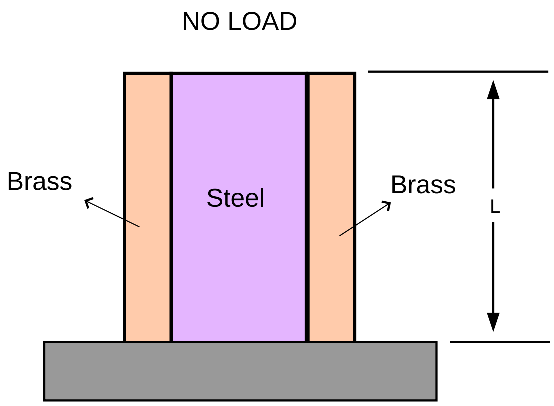


Figure 1: Inner Steel cylinder and outer hollow Brass cylinder without load

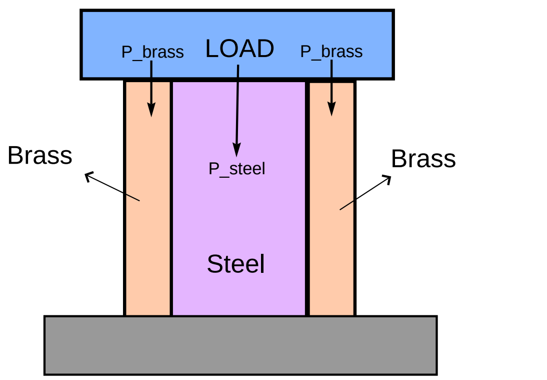


Figure 2: Inner Steel cylinder of radius r and outer Brass cylinder of radius $3r/2$

3 Question

Vary P_b between 10 Newtons and 100 Newtons by increments of 10. Calculate P_s for each value of P_b. Use MATLAB to plot P_b vs. P_s.
The commands you are most likely to use are `plot, title, xlabel` and `ylabel`.

3.1 Solution

We begin solving Eq.(7)

$$\frac{P_b L}{A_b E_b} = \frac{P_s L}{A_s E_s} \tag{8}$$

Since we know that $L_b = L_s = L$ (say).

$$\frac{P_b L}{A_b E_b} = \frac{P_s L}{A_s (2E_b)} \tag{9}$$

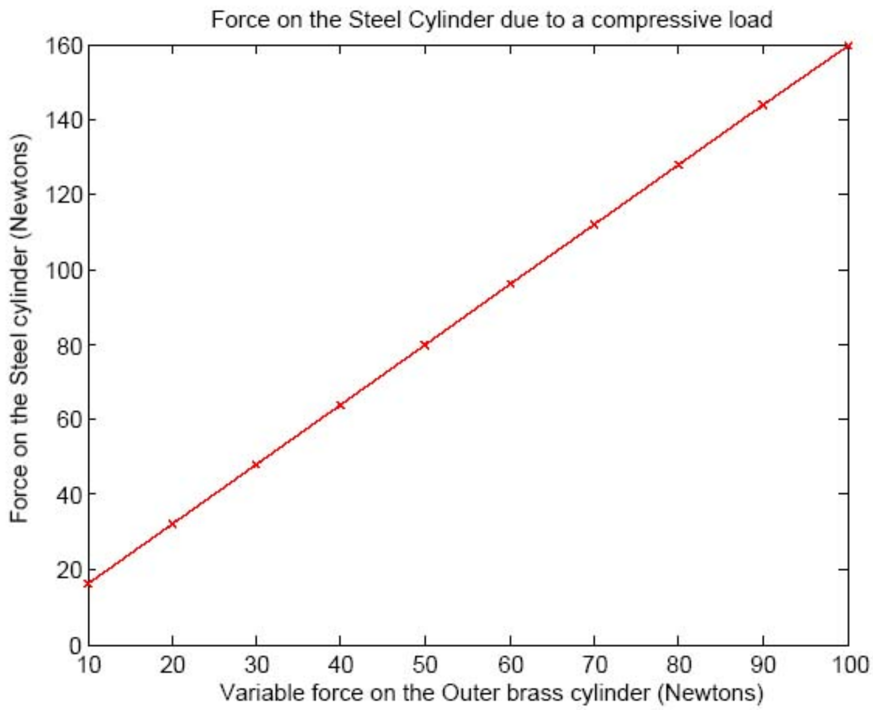
Since we are already aware by the problem description that Young's Modulus of Steel is *twice* that of brass i.e., $2E_b = E_s$. We now have,

$$P_s = \frac{2P_b A_s}{A_b} \tag{10}$$

$$P_s = 2P_b \frac{\pi r^2}{\frac{9}{4}\pi r^2 - \pi r^2} \tag{11}$$

$$\Rightarrow P_s = \frac{8}{5} P_b \tag{12}$$

Now that the analytical solution is given, the student is expected to write a small program in MATLAB to show the variation is P_s with P_b.



4 Question

Fix P_b at 10 Newtons. Fix the radius of the inner Steel cylinder at 1 meter. Vary the outer radius of the hollow brass cylinder between 1.5 meters to 2.5 meters by increments of 0.1 meters. Calculate P_s for each value of radius of the outer hollow brass cylinder. Use MATLAB to plot P_s vs. Radius of Outer cylinder.
Again, the commands you are most likely to use are `plot, title, xlabel` and `ylabel`.

4.1 Solution

We begin solving Eq.(10) which can be rewritten as

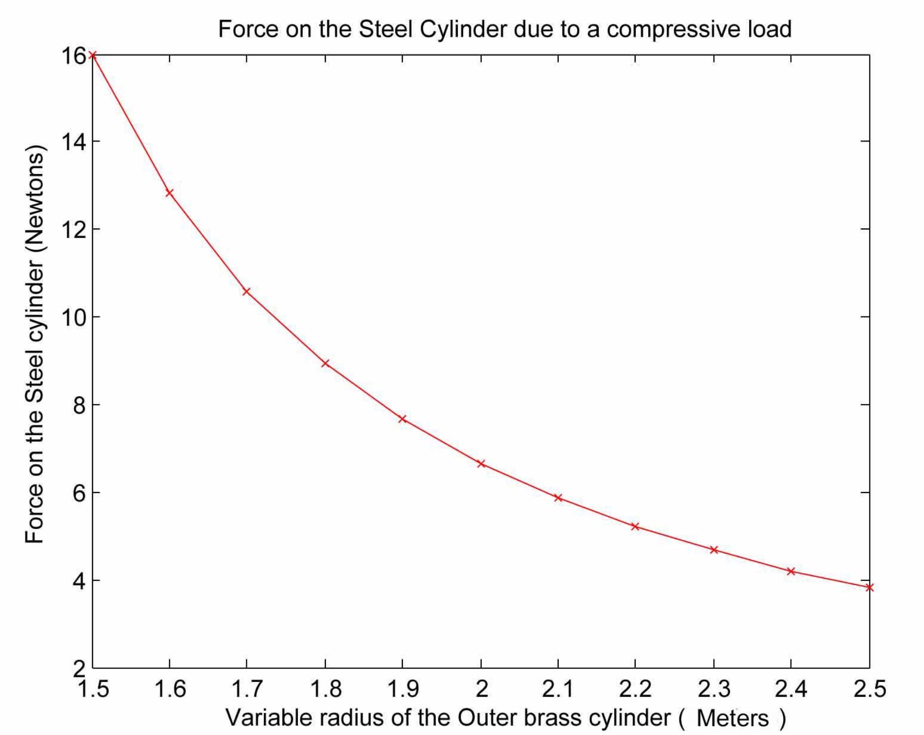
$$P_s = P_b \frac{2\pi r_s^2}{\pi r_b^2 - \pi r_s^2} \tag{13}$$

Note that $r_s = 1$ meter (fixed).
Also note that $P_b = 10$ Newtons (fixed).

$$P_s = (10) \frac{2\pi (1)^2}{\pi r_b^2 - \pi (1)^2} \tag{14}$$

Now that the analytical solution is given, the student is expected to write a small program in

MATLAB to execute Eq.(14) and plot P_s vs. r_b.



References

[1] http://www.uwgb.edu/dutchs/structge/stress.htm

[2] http://www.rwc.uc.edu/koehler/biophys/2f.html

[3] http://physics.uwstout.edu/StatStr/statics/Stress/strs31.htm#Topic%203:%20Stress,%20Strain%20&%20Hooke's%20Law

[4] http://www.geology.sdsu.edu/visualstructure/vss/htm_hlp/

[5] Potter, Merle C., 1999. *Fundamentals of Engineering - FE/EIT A.M. and General P.M. Review*, Great Lakes Press Inc.