A Mathematical Method for Designing a Set of Colour Scanning Filters

Poorvi L. Vora and H. Joel Trussell,
Dept. of Electrical and Computer Engineering,
North Carolina State University,
Raleigh, NC 27695-7911
Lawrence Iwan
Imaging Concepts Laboratory
Eastman Kodak Company
Rochester, NY 14653

ABSTRACT

Accurate colorimetric scanning of a colour image is absolutely essential for good colour reproduction. This paper formulates the design of a set of three or more colour scanning filters as an optimization problem. The optimization criterion is the measure of goodness which was developed in previous work. The method can be incorporated into any scanning system for which the colorimetric responses can be defined. The total system, including the lamps, light path and sensor characteristics, is taken into account. Simulations and results from actual hardware demonstrate the utility of the method.

1 INTRODUCTION

The purpose of colour scanning measurements is to characterize the visual stimulus of a signal. This characterization is often used to reproduce a colour image on some display or printer. As the reproduction is viewed by a human observer, the measurements should allow the creation of an image which will appear the same as the original. The CIE has tabulated colour matching functions for this purpose. These functions are used as filters for the measurement of radiant sources. For reflective or transmissive sources, the functions together with an illuminant are used to specify the necessary data.

It is well known that the scanning filters need not be exact duplicates of the colour matching functions but need be only a nonsingular transformation of them. This fact is used in determining the filters used in television and other optical applications. While the CIE colour matching functions are well defined, it is not always possible to duplicate the filters or a linear combination of them with various filter materials. The inclusion of an illumination spectrum in the light path along with the sensor characteristics makes the fabrication of scanning filters more difficult. This paper addresses the problem of determining a 'good' set of filters that can be fabricated, assuming that the combined effect of the scanning illuminant, light path and sensor characteristics is known. The measure of goodness developed by the authors in previous work [1], [2], is used as an optimization criterion. This criterion is different from those used by other researchers [3], [4], [5], [6] in that it measures the set of filters as a whole and not the individual filters.

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This paper is organized as follows. Notation and preliminaries are addressed in Section 2. Section 3 presents the the motivation behind posing the filter design problem as one of constrained optimization, and some ways of parametrizing the problem. Section 4 presents the algorithm used, the optimal values obtained, and the designed and fabricated filters. Conclusions are presented in Section 5.

2 NOTATION AND PRELIMINARIES

Most current research in colour systems assumes that the visual frequency spectrum can be adequately represented by samples taken about 10 nm apart over the range 400-700 nm. Integrals are approximated by summations, and the infinite-dimensional Hilbert space of visible spectra with the usual 2-norm is reduced to an N-dimensional Hilbert space, where N is the number of samples (in this case N=31). A continuous function of wavelength is represented by an N-vector of its sampled values. Hence, visual spectra will be treated as vectors in an N-dimensional Hilbert space in this paper.

The notation in this paper follows that of [7], [1]. The set $\{\mathbf{a}_i\}_{i=1}^{3}$ denotes the set of CIE matching functions. The matrix $\mathbf{A}$ denotes the matrix of the CIE matching functions,

$$\mathbf{A} = [\mathbf{a}_1\ \mathbf{a}_2\ \mathbf{a}_3] \tag{1}$$

A reflectance spectrum whose colour stimulus is to be reproduced is denoted by $\mathbf{f}$. The characterization of colour must include the effect of the viewing illumination. If $\mathbf{L}$ is a diagonal matrix such that $\mathbf{L}_{ii} = \mathbf{l}(i)$, the tristimulus vector under the illuminant $\mathbf{l}$ is defined by:

$$\mathbf{t} = (\mathbf{LA})^T\mathbf{f} \tag{2}$$

Let the matrix $\mathbf{A}_L$ denote the matrix product $\mathbf{LA}$. Then,

$$\mathbf{t} = \mathbf{A}_L^T\mathbf{f} \tag{3}$$

Hence, the tristimulus vector of a reflectance spectrum $\mathbf{f}$ under an illuminant $\mathbf{l}$ provides a complete characterization of the visual stimulus of $\mathbf{f}$ under that illuminant [7], [8]. Thus the visual stimulus of a signal is determined uniquely by its projection onto the space spanned by the set of vectors, $\{\mathbf{La}_i\}_{i=1}^{3}$ [7], [8]. The subspace spanned by the set of vectors, $\{\mathbf{La}_i\}_{i=1}^{3}$ is defined as the Human Visual Subspace (HVSS) for the illuminant $\mathbf{l}$. If the range space of a matrix $\mathbf{X}$ is denoted by $R(\mathbf{X})$, the HVSS may be denoted by $R(\mathbf{LA})$.

The set $\{\mathbf{m}_i\}_{i=1}^{r}$ denotes a set of r scanning filters. The matrix $\mathbf{M}$ denotes the matrix of the scanning filters,

$$\mathbf{M} = [\mathbf{m}_1\ \mathbf{m}_2\ .....\mathbf{m}_r] \tag{4}$$

An arbitrary scanning filter is represented by the vector $\mathbf{m}$. Let $\mathbf{f}$ be a reflectance spectrum to be scanned. If $\mathbf{f}$ is illuminated by a scanning illuminant $\mathbf{l}$ then the resulting spectrum is one whose i^{th} value is $\mathbf{l}(i)\mathbf{f}(i)$. Let $\mathbf{o}$ be a vector representing the optical path of the illuminated spectrum. The signal that reaches the scanning filters has i^{th} value $\mathbf{o}(i)\mathbf{l}(i)\mathbf{f}(i)$. The output of a scanning filter $\mathbf{m}$ for such a signal has i^{th} value $\mathbf{m}(i)\mathbf{o}(i)\mathbf{l}(i)\mathbf{f}(i)$. If $\mathbf{d}$ represents the detector response, then the output of the detector is $\sum_{i=1}^{N}\mathbf{m}(i)\mathbf{d}(i)\mathbf{o}(i)\mathbf{l}(i)\mathbf{f}(i)$. This can be represented as $\mathbf{m}^T\mathbf{Hf}$, where $\mathbf{H}$ is a diagonal matrix such that $\mathbf{H}_{ii} = \mathbf{d}(i)\mathbf{o}(i)\mathbf{l}(i)$. Hence, if $\mathbf{M}$ represents a set of scanning filters and $\mathbf{H}$ the combined effect of the scanning illuminant, the optical path and the detector response, $\mathbf{M}_H = \mathbf{HM}$ denotes the scanning system. Let the set $\{\mathbf{Hm}_i\}_{i=1}^{r}$ be defined as the set of 'effective' scanning filters. The set of linear combinations of the effective scanning filters may be denoted $R(\mathbf{M}_H)$. The output of the scanning system represented by $\mathbf{M}_H$ is:

$$\mathbf{c} = \mathbf{M}_H^T\mathbf{f} \tag{5}$$

A set of effective scanning filters which is exactly the set $\{\mathbf{La}_i\}_{i=1}^{3}$ will give output values $\mathbf{t}$. Given the set $\{\mathbf{La}_i\}_{i=1}^{3}$ it may not be possible to construct exactly a set of filters $\{\mathbf{m}_i\}_{i=1}^{r}$ such that $\mathbf{Hm}_i = \mathbf{La}_i$. A method that seeks to construct exactly the vectors $\{\mathbf{La}_i\}_{i=1}^3$ does not allow flexibility in the incorporation of limitations of the fabrication process or the fact it is the space $R(\mathbf{A}_L)$ that is important.

Notice, however, that any invertible linear combination of the tristimulus values can be manipulated to give the tristimulus values. Hence any such transformation of the tristimulus values will also serve to characterize the visual stimulus of the signal. If $\mathbf{X}$ represents an invertible transformation, it is enough to obtain a set of readings $\mathbf{M}_H^T\mathbf{f}$ such that

$$\mathbf{M}_H^T\mathbf{f} = \mathbf{X}\mathbf{A}_L^T\mathbf{f} \tag{6}$$

This is possible when

$$\mathbf{M}_H = \mathbf{A}_L\mathbf{X}^T \tag{7}$$

and

$$R(\mathbf{M}_H) = R(\mathbf{A}_L) \tag{8}$$

This is not a necessary condition, however. For example, it is possible that 3 filters whose span is equal to the HVSS cannot be constructed, but four filters whose span includes the HVSS can be. See [2] for an example. In such a case, there exists a linear transformation $\mathbf{Y}$, such that

$$\mathbf{Y}\mathbf{M}_H^T\mathbf{f} = \mathbf{A}_L^T\mathbf{f} \tag{9}$$

where $\mathbf{Y}$ is not necessarily square nor invertible. This implies that

$$\mathbf{M}_H\mathbf{Y}^T = \mathbf{A}_L \tag{10}$$

and

$$R(\mathbf{M}_H) \supseteq R(\mathbf{A}_L) \tag{11}$$

It is noted that perfect characterization of colour is possible only if the space spanned by the colour scanning filters includes the HVSS for the illuminant $\mathbf{l}$ [7], [8]. Hence a scanning system may use any set of vectors whose span includes the HVSS and it is not necessary to use only the vectors $\{\mathbf{La}_i\}_{i=1}^3$.

A number of problems arise in the implementation of an 'optimal' scanning system that obtains the required tristimulus values (or a transformation of them as mentioned above). In particular, it is difficult to construct a designed scanning filter exactly, and any errors in filter construction will change the space spanned by the filters, resulting in an error in the measurement of the required projection. This error will lead to an error in the reproduction. Note that this error will occur even if the measurements are noise-free in all other regards. Most of the literature in the design of colour scanning filters reports optimization routines that minimize a norm of the difference between each constructed filter and the corresponding vector in $\{\mathbf{La}_i\}_{i=1}^3$ [5], [3], [4], or maximize the q-factor (defined by Neugebauer [9]) of the individual scanning filters [6]. To the knowledge of the authors, there has been no reported research on the use of a measure of the entire set of colour scanning filters as an optimization criterion.

Previous work by the authors in the area of characterization of scanning filters has resulted in a measure of goodness of a set of colour scanning filters with respect to the kind of error mentioned above [1]. This measure of goodness is related to the projection of the space defined by the filters onto the HVSS. The basis of this measure is the relation between the vector space defined by the human visual system determined from the CIE colour matching functions and the vector space defined by the scanning filters. Let the matrix $\mathbf{V}$ represent the space to be spanned, i.e. $R(\mathbf{V})$ is the space to be spanned. For example, $R(\mathbf{V})$ could be the HVSS for an illuminant $\mathbf{l}$, and $\mathbf{V} = \mathbf{A}_L$. Let an orthonormal basis for $R(\mathbf{V})$ be defined by $\mathbf{N} = [\mathbf{n}_1\ \mathbf{n}_2 ....\mathbf{n}_\alpha]$ such that $R(\mathbf{N}) = R(\mathbf{V})$ and $\mathbf{N}^T\mathbf{N} = \mathbf{I}$. Such a basis may be obtained by the Gram-Schmidt orthogonalisation procedure [10]. The number of orthonormal vectors, α, is the rank of $\{\mathbf{v}_i\}_{i=1}^s$ and α equals s iff $\{\mathbf{v}_i\}_{i=1}^s$ is a linearly independent set. Similarly, define an orthonormal basis for $R(\mathbf{M}_H)$ by $\mathbf{O} = [\mathbf{o}_1\ \mathbf{o}_2 ....\mathbf{o}_\beta]$ such that $R(\mathbf{O}) = R(\mathbf{M}_H)$ and $\mathbf{O}^T\mathbf{O} = \mathbf{I}$. Again, notice that β is the rank of $\{\mathbf{Hm}_i\}_{i=1}^r$ and β equals r iff $\{\mathbf{Hm}_i\}_{i=1}^r$ is a linearly independent set. The orthonormal bases **N** and **O** need not represent realisable filters. Then, $\nu(\mathbf{V}, \mathbf{M}_H)$, a measure of goodness, is defined as

$$\nu(\mathbf{V}, \mathbf{M}_H) = \frac{\sum_{i=1}^{\alpha} \lambda_i^2(\mathbf{O}^T\mathbf{N})}{\alpha} \tag{12}$$

where $\lambda_i(\mathbf{O}^T\mathbf{N})$ denotes the i^{th} singular value of $\mathbf{O}^T\mathbf{N}$. Let $P_{M_H}(.)$ represent the orthogonal projection operator onto $R(\mathbf{M}_H)$ in N-space. Similarly, let $P_V(.)$ represent the orthogonal projection operator onto $R(\mathbf{V})$ in N-space. Define $P_V(\mathbf{O}) = [P_V(\mathbf{o}_1)\ P_V(\mathbf{o}_2).......P_V(\mathbf{o}_\beta)]$ and likewise $P_{M_H}(\mathbf{N}) = [P_{M_H}(\mathbf{n}_1)\ P_{M_H}(\mathbf{n}_2).......P_{M_H}(\mathbf{n}_\alpha)]$. Some manipulation of (12) gives

$$\nu(\mathbf{V}, \mathbf{M}_H) = \frac{trace(P_{M_H} P_V)}{\alpha} \tag{13}$$

which can be shown to be

$$\nu(\mathbf{V}, \mathbf{M}_H) = \frac{trace\ (\mathbf{M}_H(\mathbf{M}_H^T\mathbf{M}_H)^{-1}\mathbf{M}_H^T\mathbf{V}(\mathbf{V}^T\mathbf{V})^{-1}\mathbf{V}^T)}{\alpha} \tag{14}$$

It can be shown that this is [1]

$$\nu(\mathbf{V}, \mathbf{M}_H) = \frac{\sum_{i=1}^{\beta} qfactor(\mathbf{o}_i)}{\alpha} \tag{15}$$

where the set $\{\mathbf{o}_i\}_{i=1}^{\beta}$ represents an orthonormal basis that spans $R(\mathbf{M}_H)$. The term q-factor is a generalization of the term defined by Neugebauer [9] and is used here in a general sense to mean the norm of the projection of the normalised vectors $\mathbf{o}_i$ onto the α -dimensional $R(\mathbf{V})$. Neugebauer's original definition of the q-factor implies the specific case where this space is the three-dimensional HVSS. The use of this measure to define a constrained optimization problem is discussed in Section 3.

3 PARAMETRIZATION OF FILTER CHARACTERISTICS

In the specific case where **V** represents the HVSS for an illuminant **l** the measure in Equation (14) may be rewritten as:

$$\nu(\mathbf{A}_L, \mathbf{M}_H) = \frac{trace\ (\mathbf{HM}(\mathbf{M}^T\mathbf{H}^T\mathbf{HM})^{-1}\mathbf{M}^T\mathbf{H}^T\mathbf{LA}(\mathbf{A}^T\mathbf{L}^T\mathbf{LA})^{-1}\mathbf{A}^T\mathbf{L}^T)}{\alpha} \tag{16}$$

Notice that all matrices except the matrix **M** are known. Without any other restrictions, the measure is a function of rN parameters, each parameter being the transmissivity of a filter at a particular wavelength. The filter design problem may be interpreted as an optimization problem where the goal is maximization of the measure with respect to each of the rN parameters.

The first way to attack this problem would be to design a set of three scanning filters (r = 3) $\{\mathbf{m}_i\}_{i=1}^{3}$ such that

$$\mathbf{m}_i(k) = \kappa_i \frac{\mathbf{l}(k)\mathbf{a}_i(k)}{\mathbf{h}(k)} \qquad i = 1,\ 2,\ 3 \tag{17}$$

where κ_i is a normalizing constant, so that the maximum transmissivity for each scanning filter is unity. For a real set of scanner characteristics, **h**, shown in Fig. 1, $\{\mathbf{a}_i\}_{i=1}^{3}$ the three CIE colour matching functions, and a uniform illuminant, the three scanning filters defined by Equation (17) are shown in Fig. 2. Notice that the scanner characteristic is far from uniform, and that this is likely to present problems in filter design. Hence, as expected, the filters designed according to Equation (17) have a large dynamic range (of the order of 10^4). This is because of the large variation in values of **h**(k). This leads to an ill-conditioned problem and high susceptability to small errors in the fabrication process. Further, smoothness of filter transmissivity curves is an important restriction in the filter fabrication process, and Fig.2 indicates that filters designed according to Equation (17) will not be easy to fabricate exactly. This makes it clear that each constructable filter does not possess N degrees of freedom and that expressing the measure as a function of 3N independent variables will not necessarily result in optimal *realizable* filters.

One way of incorporating a manageable dynamic range and smoothness of the filters into the optimization algorithm is by modelling each filter in terms of known, smooth, non-negative mathematical functions. One such mathematical function is the gaussian function. A gaussian function with mean μ and standard deviation σ is:

$$p(x) = \frac{1}{\sqrt{2\pi}\sigma} exp(-\frac{(x - \mu)^2}{2\sigma^2})$$

If each filter $\mathbf{m}_i$ is modelled as a gaussian function of mean μ_i and standard deviation σ_i the normalized filter vectors $\mathbf{m}_i$ are

$$\mathbf{m}_i(k) = exp(-\frac{(\lambda_k - \mu_i)^2}{2\sigma_i^2}) \tag{18}$$

where λ_k depends on the sampling of the spectra. Each filter is a function of 2 independent variables and the measure is a function of 6 independent variables. The resulting 'optimal' filters will be gaussians and hence easier to fabricate. The results for the single-gaussian filter model and the scanner characteristic in Fig. 1 are presented in Section 4. While the smaller number of parameters makes finding the optimal values mathematically tractable, the resulting filters may not perform well.

One way of allowing more freedom in filter design is to extend the single-gaussian model to a sum-of-gaussians model. Each filter may be modelled as the weighted sum of two gaussians. Each filter is then a function of 5 parameters (2 means, 2 variances and 1 weighting factor). This results in vectors $\mathbf{m}_i$ of the form

$$\mathbf{m}_i(k) = exp(-\frac{(\lambda_k - \mu_{i1})^2}{2\sigma_{i1}^2}) + \alpha_i exp(-\frac{(\lambda_k - \mu_{i2})^2}{2\sigma_{i2}^2}) \tag{19}$$

The function $\nu(\mathbf{A}_L, \mathbf{M}_H)$ is now a function of 15 variables. Standard optimization routines can provide values of the 15 parameters such that the measure is maximum, or close to maximum. Thus, one can find an optimal set of filters for particular scanner characteristics and viewing illuminant such that each filter is modelled as the sum of two gaussians. Section 4 presents the results obtained for the particular value of $\mathbf{h}$ plotted in Fig. 5.

Given a set of possible filters that may be used as scanning filters, the measure may be optimized by an exhaustive search taking the filters three at a time, each filter representing a separate scanning filter. For example, given the set of Kodak Wratten filters, which are gelatin filters whose transmissivities are known, one can evaluate the measure for all possible combinations of three Wratten filters. The set with the highest value of the measure is the 'best' possible set from the Wrattens for this particular application. Given a set of existing filters, the fabrication process is bypassed. Results for this procedure given the set of Wratten filters to choose from, a uniform viewing illuminant and the scanner characteristic of Fig. 1 are presented in Section 4. Hardware implementation results are also presented for this particular case.

In the above mentioned case of choosing three filters from a given set of filters, it is possible to construct a scanning filter by cascading more than one filter from the given set of filters. The transmissivity of the resulting filter would be the product of the transmissivities of the individual filters. An exhaustive search of all such possibilities is much more computationally expensive. Reports exist in the literature of attempts to form scanning filters in such a manner [3], [4], [5]. Again, these attempts use a different optimization criterion as mentioned earlier.

4 EXPERIMENTAL RESULTS

For the particular combined response of illuminant, light path and scanner sensor response in Fig. 1, the sum-of-gaussian model and the exhaustive search suggested in Section 4 were implemented to find 'optimal' scanning filters. The viewing illuminant was assumed uniform, i.e. $\mathbf{L} = \mathbf{I}$. An exhaustive search was used to find the 'best' combination of Wratten filters. For both the single-gaussian model and the sum-of-gaussian model the MATLAB [11] function 'fmins' was used to find optimal filters. Very clearly, the measure does not have one global maximum, as, for example, the filters in a different order will give a different set of parameters but the same value of the measure. It is also likely that the measure has many local maxima. This implies that a particular solution is a function of the starting point and not necessarily the best possible set of filters. To minimize this effect several different initial points were used. The resulting filters gave varying but similar results. From the results in Sections 4.1 and 4.2 it is clear that the filters obtained are very good.

To test the performance of the filters, the scanning process was simulated using a 64 sample set of spectra of Munsell chips. Given a set of colour measurements for an ensemble with known tristimulus values, it is common to derive the 3x3 matrix that premultiplies the measurements to give a minimum mean square estimate of the tristimulus values. This correction is clearly dependent on the particular ensemble. Data correction is commonly performed in colorimetry when the scanning filters are to be used on a well-characterized data set. As demonstrated in [1], the correction reduces $L^*a^*b^*$ errors considerably. The corrected scanning filter data is

$$\mathbf{c} = (\mathbf{A}^T\mathbf{R}\mathbf{M}_H(\mathbf{M}_H^T\mathbf{R}\mathbf{M}_H)^{-1})(\mathbf{M}_H^T\mathbf{f}) \quad (20)$$

where $\mathbf{R} = E[\mathbf{f}\mathbf{f}^T]$ is the sample correlation matrix of the ensemble. The average ΔE_{Lab} error was calculated for the corrected data.

4.1 Single-gaussian Model

The parameters for the single-gaussian model of Equation(18) were calculated using the MATLAB function 'fmins' for the scanner characteristic shown in Fig. 1. The parameter values for an optimal set of filters are

$$\begin{aligned} \mu_1 &= 438.3 & \sigma_1 &= 26.4 \\ \mu_2 &= 548.0 & \sigma_2 &= 37.6 \\ \mu_3 &= 607.0 & \sigma_3 &= 19.1 \end{aligned}$$

These parameter values result in a filter set of measure 0.9556 with the scanner characteristic of Fig. 1. The average ΔE_{Lab} error for the corrected data corresponding to the 64 sample set of Munsell chips is 1.74.

The parameters for a single-gaussian were also calculated for the scanner characteristic in Fig. 3. These parameter values were

$$\begin{aligned} \mu_1 &= 459.1 & \sigma_1 &= 22.0 \\ \mu_2 &= 557.7 & \sigma_2 &= 40.3 \\ \mu_3 &= 592.9 & \sigma_3 &= 35.8 \end{aligned}$$

These parameter values result in a filter set of measure 0.9485, and an average ΔE_{Lab} error of 0.84 with the 64 sample set of Munsell chips and the scanner characteristic of Fig. 3. Barr Associates, a filter manufacturer, provided an estimate of the closest filters they could manufacture given the specifications above. This filter set had a measure of 0.9478 and an average ΔE_{Lab} error of 0.86.

4.2 Sum-of-gaussian Model

Using the MATLAB function 'fmins' the following parameter values for the filter model of Equation (19) were obtained to give a filter set with measure 0.9928 for a uniform viewing illuminant and the scanner characteristic of Fig. 1.

$$\begin{aligned} \mu_{11} &= 442.6963 & \mu_{12} &= 428.1607 & \sigma_{11} &= 24.6126 & \sigma_{12} &= 7.0225 & \alpha_1 &= 0.3022 \\ \mu_{21} &= 593.6498 & \mu_{22} &= 539.8501 & \sigma_{21} &= 14.6686 & \sigma_{22} &= 29.0348 & \alpha_2 &= 0.5281 \\ \mu_{31} &= 601.2296 & \mu_{32} &= 638.1470 & \sigma_{31} &= 11.2386 & \sigma_{32} &= 49.389 & \alpha_3 &= 0.3969 \end{aligned}$$

These parameter values result in the filter responses plotted as dotted lines in Figs. 4-6. The measure of goodness of this set of filters is 0.9928. The filters Barr Associates are able to manufacture are indicated by the solid lines in Figs. 4-6. The measure of this filter set with respect to the scanner it was designed for is 0.9900. The designed filter set produced an average ΔE_{Lab} error of 0.45. The average ΔE_{Lab} error for the fabricated set was 0.50.

4.3 Exhaustive Search

An exhaustive search of the Wrattens was performed to find the best set of three filters, given the scanner characteristic in Fig. 1, and a uniform illuminant. The set of the filters Nos. 23A, 48A and 52 was found to be optimal with respect to the measure. The measure of this set was 0.912. The average simulated ΔE_{Lab} was calculated to be 2.04. The filters were installed in the scanner at the Imaging Concepts Laboratory, Eastman Kodak, Rochester, NY. The scanning system was then used to scan the 64 sample set of Munsell chips. The resulting ΔE_{Lab} error of the corrected data was 3.02. The discrepancy may be attributed to several sources which are presently under investigation. The results obtained by the optimization should be compared to a ΔE_{Lab} of 4.5 obtained with the previously installed set designed by using the standard q-factor [12].

5 CONCLUSIONS

The measure of goodness of a set of colour filters developed in earlier work may be used to define an optimization criterion for a scanning system. Simple physical constraints may be incorporated to frame a constrained optimization problem. This problem may be satisfactorily solved by standard minimization (or maximization) routines to give filters with fairly high measures in the specific case of modelling the filters as sums of two gaussians with a particular scanning illuminant and a uniform viewing illuminant. Hardware implementation indicates that this method is very useful for designing filters for colorimetric applications.

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