CONTROL FLOW GRAPHS

- **Motivation:** language-independent and machine-independent representation of control flow in programs used in high-level and low-level code optimizers. The flow graph data structure lends itself to use of several important algorithms from graph theory.

Basic Blocks

- A straight line sequence of code
  - a maximal sequence of instructions that can be entered only at the first of them and exited only from the last of them.
- First instruction of basic block can be:
  - entry point of routine
  - target of branch
  - instruction immediately following branch - *leaders*
- extended basic block is maximal sequence that contains no join nodes other than first node.
  - Join node: if it has more than one predecessor
  - Branch node: if it has more than one successor

Control Flow Graph: Definition

A control flow graph CFG = (N_c ; E_c ; T_c) consists of

- N_c, a set of nodes. A node represents a straight-line sequence of operations with no intervening control flow i.e. a basic block.
- *E_c* Í *N_c x N_c x Labels*, a set of *labeled* edges.
- T_c , a node type mapping. T_c(*n*) identies the type of node *n* as one of: *START, STOP, OTHER.*

We assume that *CFG* contains a unique *START* node and a unique *STOP* node, and that for any node *N* in *CFG*, there exist directed paths from *START* to *N* and from *N* to *STOP*.

Dominators: Definition

Node *V* dominates another node $W \neq V$ of and only if every directed path from *START* to *W* in *CFG* contains V.

Define *dom* (*W*) = { *V* | *V* dominates *W* } , the set of dominators of node *W*.

Consider any simple path from *START* to *W* containing W 's dominators in the order $V_1, \ldots, V_k$. Then all simple paths from *START* to *W* must contain *W 's* dominators in the same order. The element closest to W, V_k = *idom* (*W*), is called the *immediate* dominator of *W*.

Control Flow Graphs

- From code can construct “flowchart”
- from flowchart to basic blocks
- edges come from the control flow information

Control Flow Analysis

- Use *dominators* to discover loops and simply note loops found for use in optimization
- alternate approach- *interval analysis*: Analyze overall structure of routine and decompose it into nested regions called intervals
  - nesting structure forms tree, useful in structuring and speeding up data flow analysis

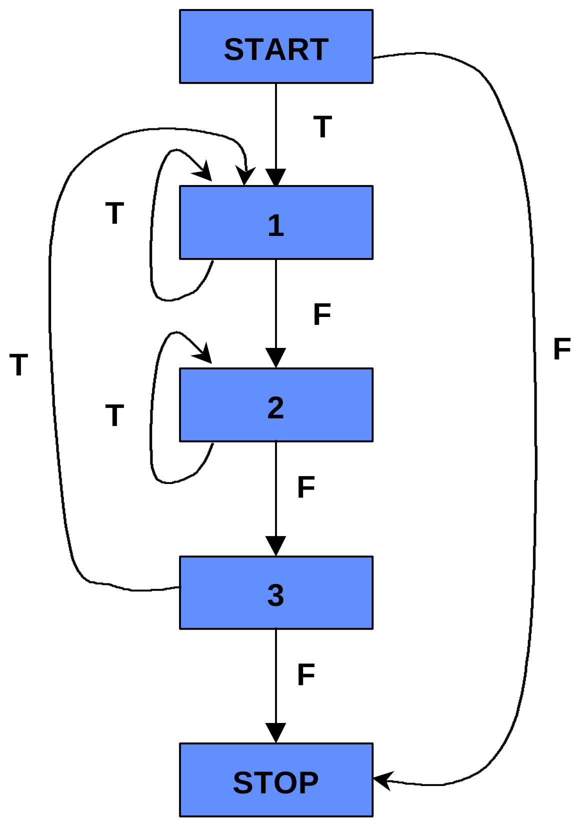
Dominators: Definition (Contd.)

The *idom* relation can be represented as a directed tree with root = *START*, and *parent*(*W*) = *idom* (*W*).

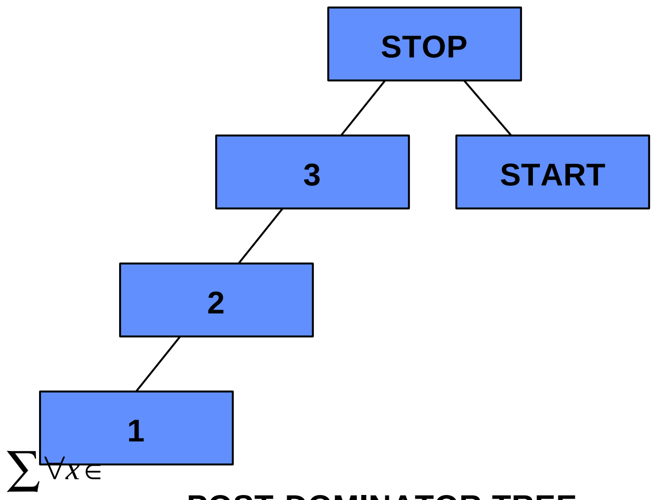
Postdominators: Definition

- Node W *postdominates* another node $V \neq W$ if and only if every directed path from *V* to *STOP* in *CFG* contains *W* .
- Define *pdom* (*V*) = { *W* | *W* postdominates *V*}, the set of *postdominators* of node V
- Consider any simple path from *V* to *STOP* containing *V*'s postdominators in the order $W_1, \ldots, W_k$. Then all simple paths from *V* to *STOP* must contain *V*'s postdominators in the same order. The element closest to *V*, W_1 = *ipdom*(*V*), is called the *immediate postdominator* of V.
- The *ipdom* relation can be represented as a directed tree with root = is *STOP* and *parent*(*V*) = *ipdom*(*V*)

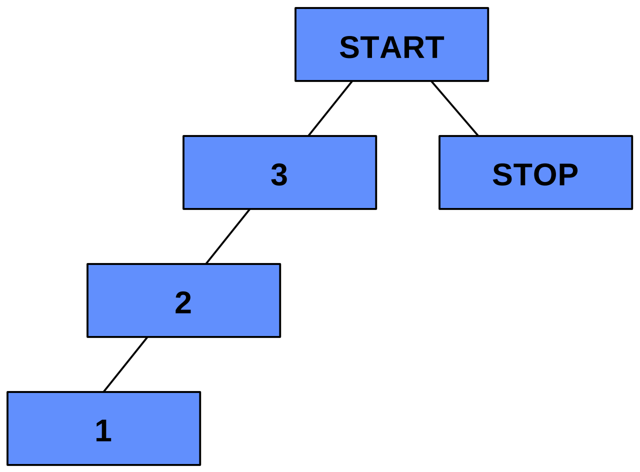
Example: Dominator and Postdominator Trees



CONTROL FLOW GRAPH



POST-DOMINATOR TREE



DOM INATOR TREE

Algorithms for computing Dominator/Postdominator

- [Purdom and Moore, 1972] *O(N x E)* execution time
- [Lengauer and Tarjan, 1979]

  Simple version: *O(E x logN*) execution time

  Sophisticated version: *O(E x α(E,N))* execution time
- [Harel, 1985] *O(N + E*) execution time

Loop Nesting Structure of a Control Flow Graph

- The loop nesting structure of a CFG is revealed by its *interval structure:*
- Edge e = (*x, h, l*) in CFG is called a *back edge* if *h Î dom (x); h is called a header* node, and *x* is called a *latch node*.
- The strongly connected region defined by back edge *e = (x, h, l)* is *STR*(*e*), which consists of the nodes and edges belonging to all paths from node *h* to node *x*, along with the back edge (*x,h, l*).
- The *interval with header h, I(h),* is defined as the union of *STR*(*e*) over all back edges e targeted to header node *h*.

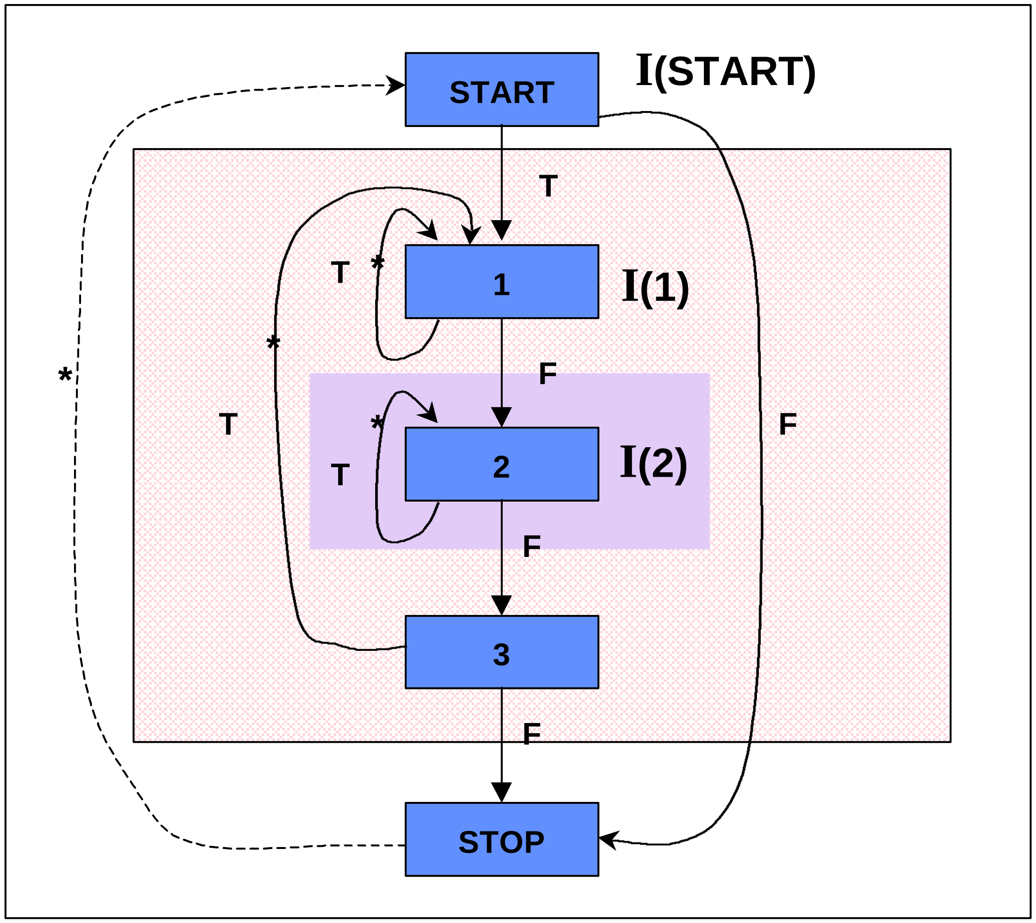
Loop Nesting Structure of a Control Flow Graph (Contd.)

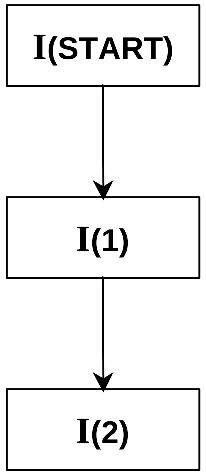
- Interval nesting is defined by the subgraph relationship. $I(h_1)$ is a subinterval of $I(h_2)$ if $I(h_1)$ is a subgraph of $I(h_2)$.
- The interval nesting relation can then be represented by a unique interval nesting tree (or forest of trees).
- It is convenient to add a pseudo-edge from *STOP* to *START* to make *START* a header node with *I(START)* = entire CFG, and thus force the interval relation to be a single tree rather

Reducible Control Flow Graphs

- A CFG is *reducible* if and only if the directed graph obtained by removing all back-edges is *acyclic*. This graph is referred to as the *forward control flow graph.*

Example: Interval Structure for a Reducible CFG



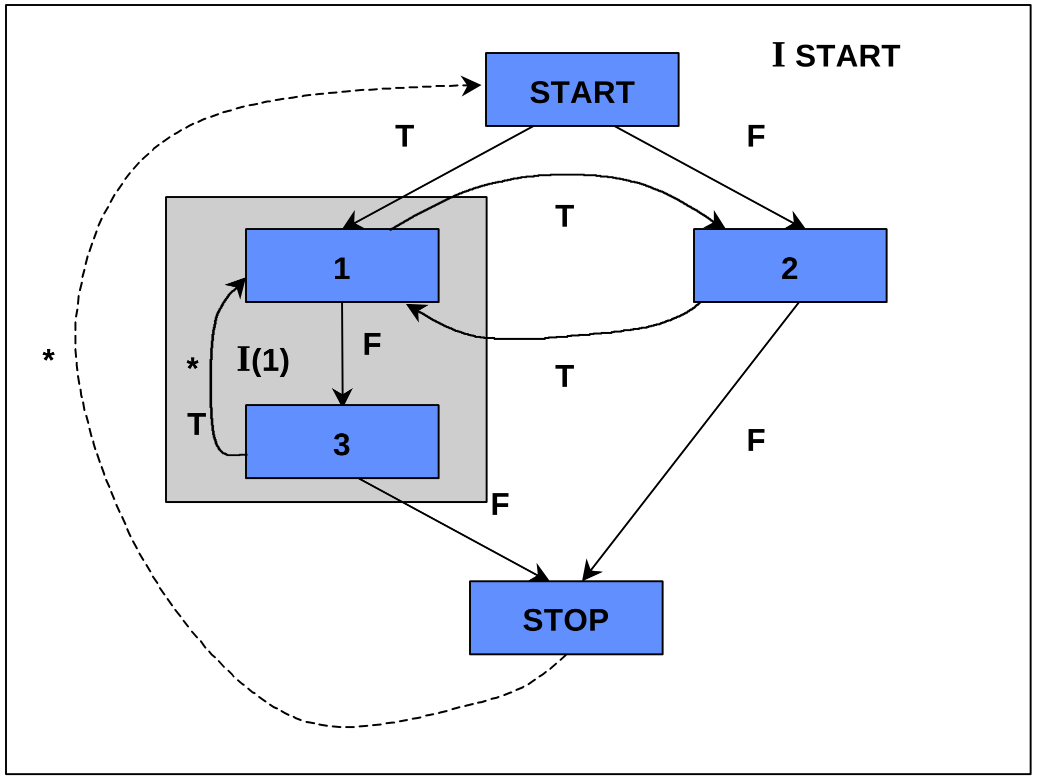


INTERVAL NESTING TREE

* Back edge

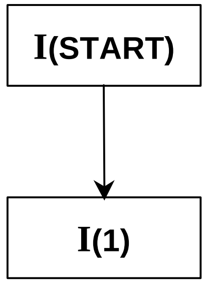
INTERVALS IN A CONTROL FLOW GRAPH

Example: Interval Structure for an Irreducible CFG



* Back edge

INTERVALS IN A CONTROL FLOW GRAPH



INTERVAL NESTING TREE

Data and Control Dependences

Motivation: identify only the essential control and data dependences which need to be obeyed by transformations for code optimization.

Program Dependence Graph (PDG) consists of

1. Set of nodes, as in the CFG
2. Control dependence edges
3. Data dependence edges

Together, the *control and data dependence edges dictate whether or not a proposed code transformation is legal.*

Control Dependence Analysis

We want to capture two related ideas with control dependence analysis of a CFG:

1. Node *Y* should be control dependent on node *X* if node *X* evaluates a predicate (conditional branch) which can control whether node *Y* will subsequently be executed or not. This idea is useful for determining whether node *Y* needs to wait for node *X* to complete, even though they have no data dependences.

Control Dependence Analysis (contd.)

2. Two nodes, *Y* and *Z*, should be identified as having identical control conditions if in every run of the program, node *Y* is executed if and only if node *Z* is executed. This idea is useful for determining whether nodes *Y* and *Z* can be made adjacent and executed concurrently, even though they may be far apart in the CFG.

Control Dependence: Definition

[Ferrante et al, 1987]

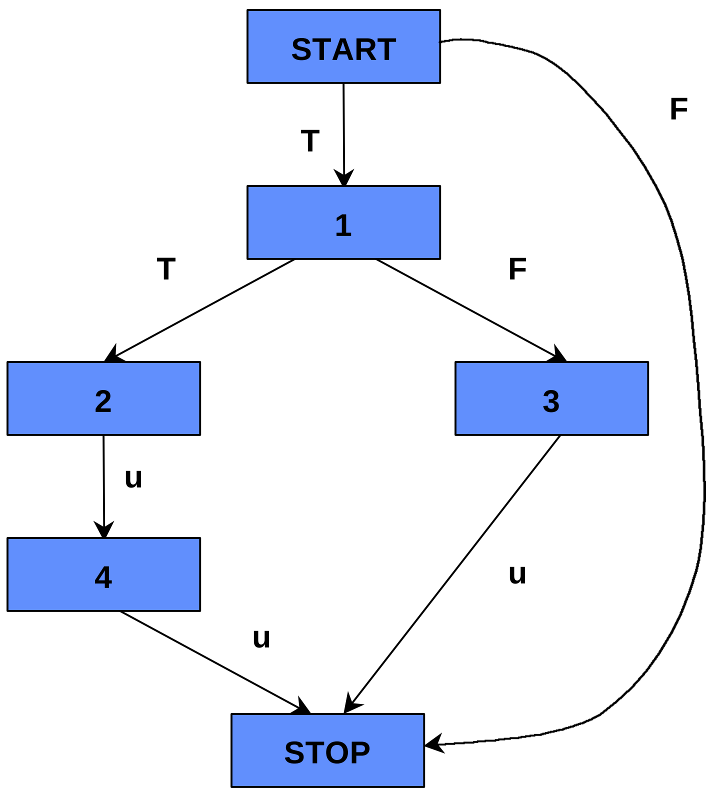
Node *Y* is *control dependent* on node *X* with label *L* in *CFG* if and only if

1. there exists a nonnull path *X* $\longrightarrow$ Y, starting with the edge labeled *L*, such that *Y* post-dominates every node, *W*, strictly between *X* and *Y* in the path, and

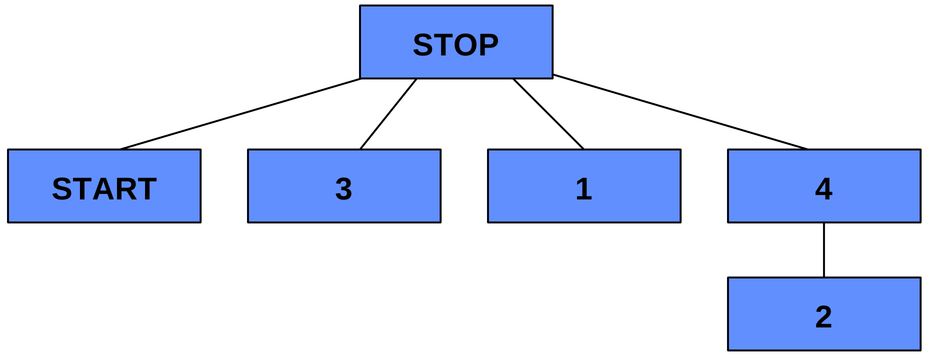
2. *Y* does not post-dominate *X*

Y is control dependent on *X* only if X can *directly* affect whether *Y* is executed or not; *indirect* control dependence can be defined as the transitive closure of control dependence

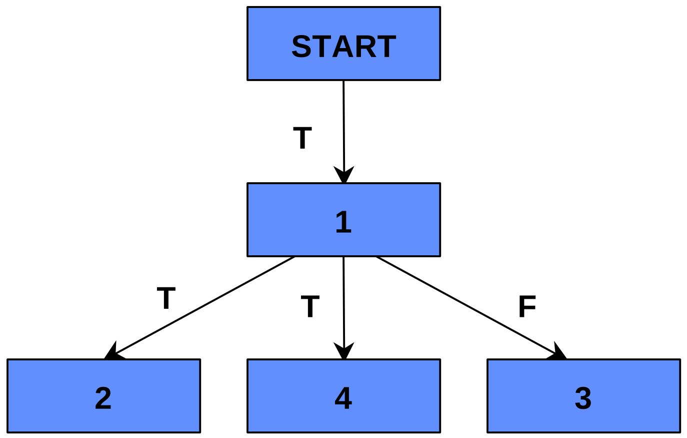
Example: acyclic CFG and its Control Dependence Graph



CONTROL FLOW GRAPH

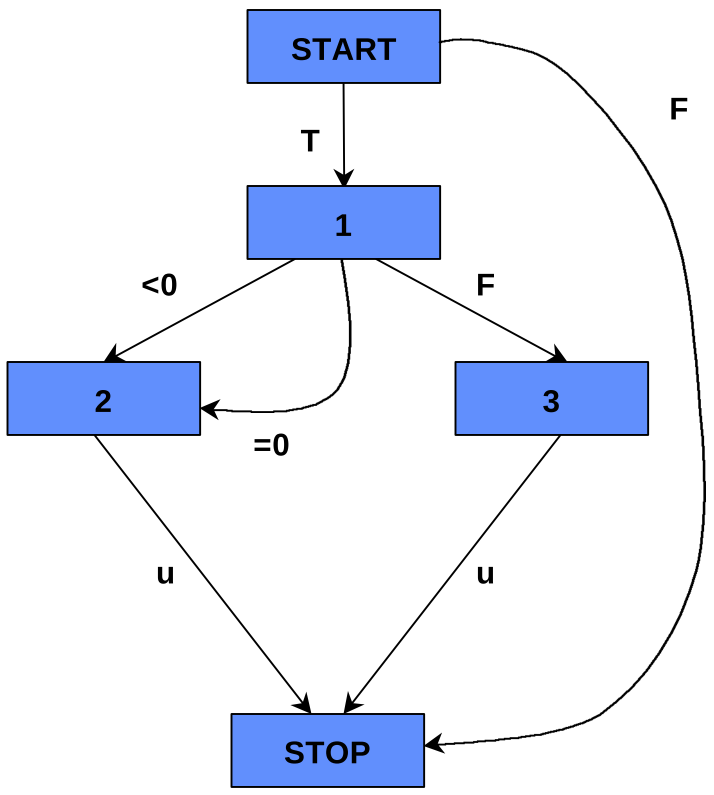


POSTDOMINATOR TREE

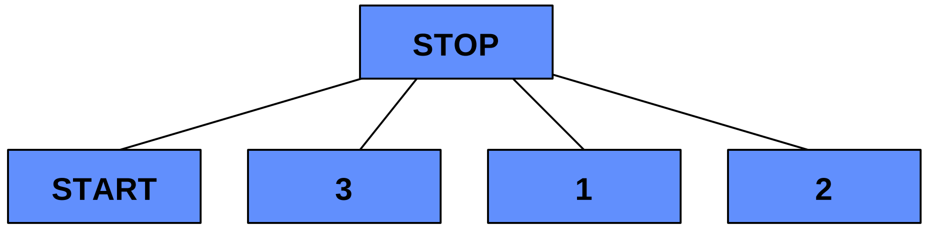


CONTROL DEPENDENCE GRAPH

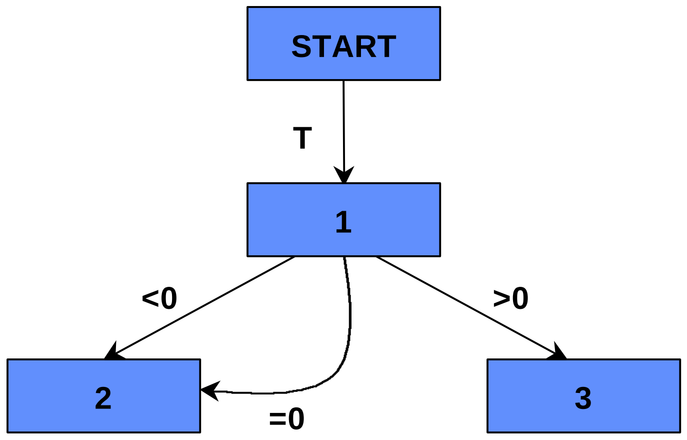
Example: CFG with multi-way branch and its CDG



CONTROL FLOW GRAPH



POSTDOMINATOR TREE



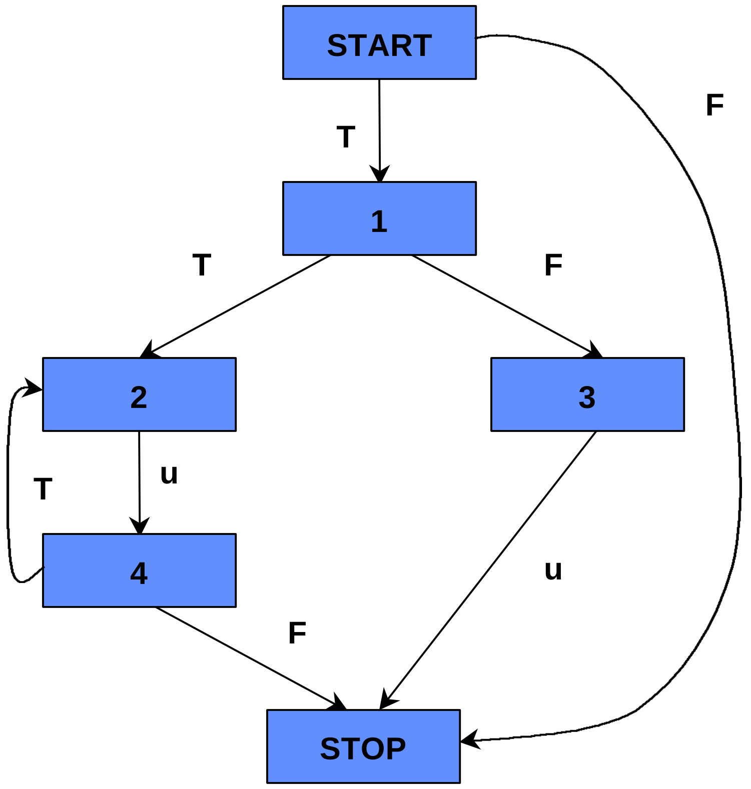
CONTROL DEPENDENCE GRAPH

Algorithm for Computing Control Dependence

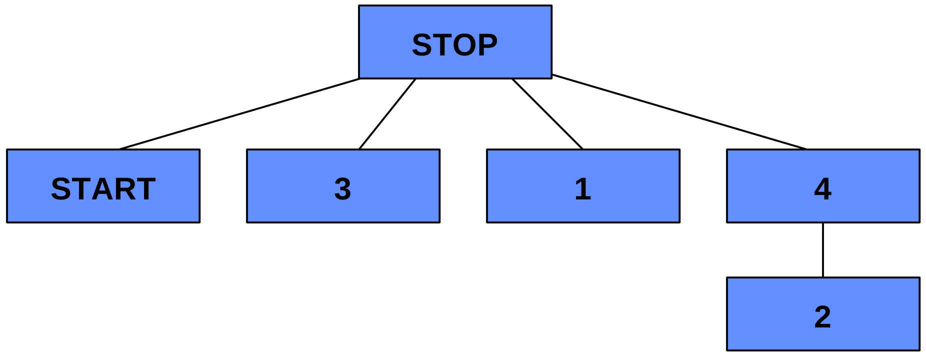
Given node *X* and branch label *L*, all control dependence successors can be enumerated as follows:

```
1. $Z \longleftarrow$ CFG successor of node $X$ with label $L$

2. while $Z \neq ipdom(X)$ do

     (a) /* $Z$ is control dependent on $X$ with label
           L ——— process Z as desired */

     (b) $Z \longleftarrow ipdom(Z)$

   end while
```

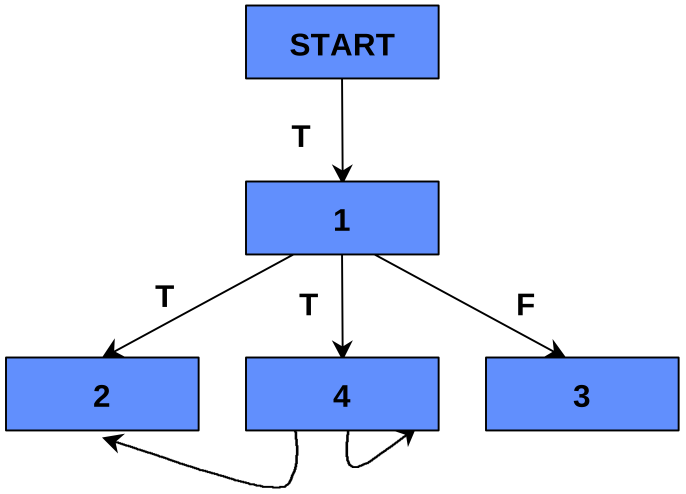
Example: cyclic CFG and its Control Dependence Graph



CONTROL FLOW GRAPH

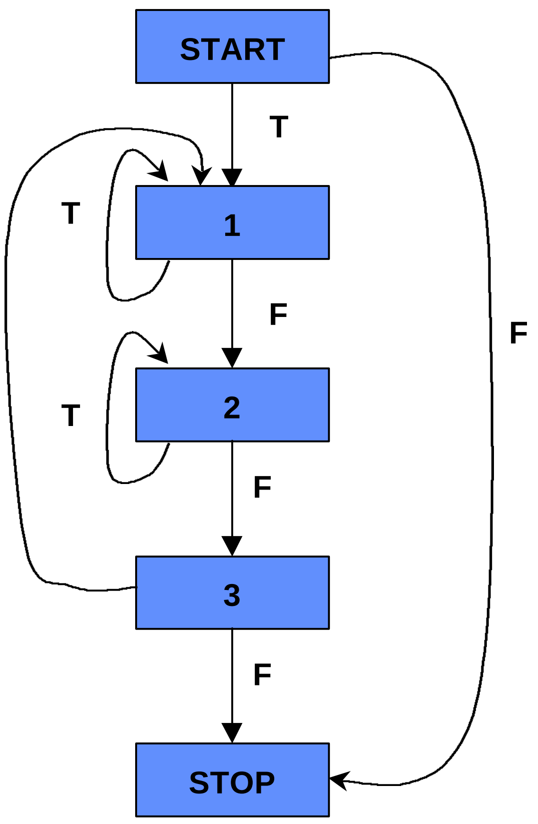


POSTDOMINATOR TREE

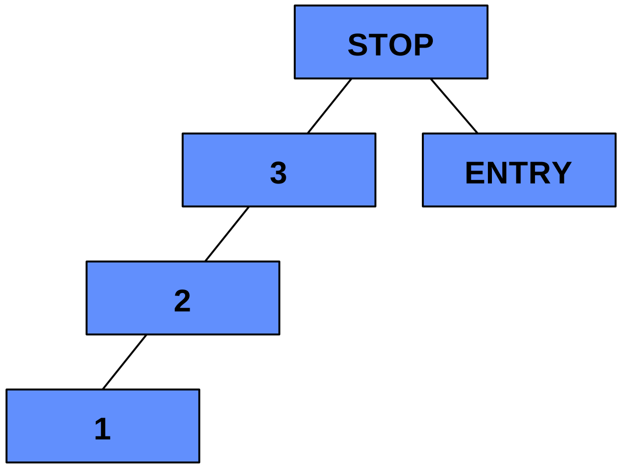


CONTROL DEPENDENCE GRAPH

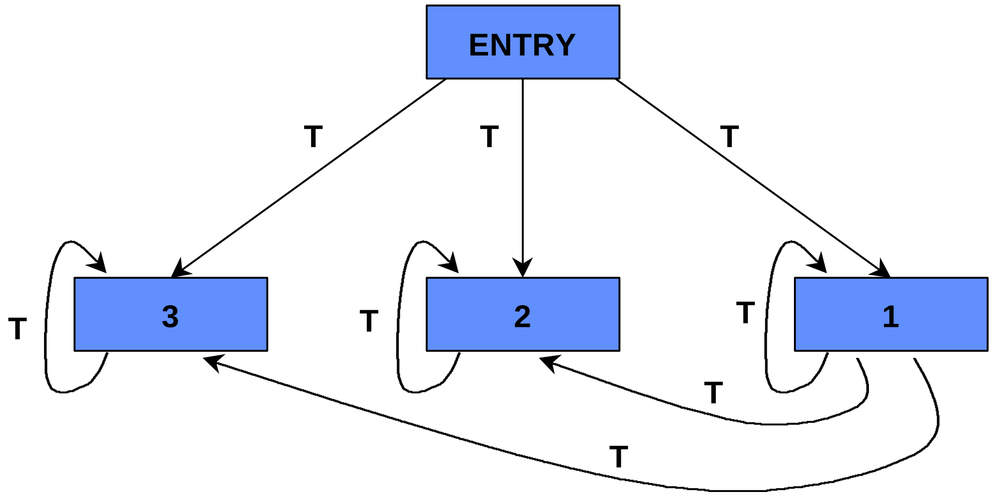
Example: another cyclic CFG and its CDG



CONTROL FLOW GRAPH



POST-DOMINATOR TREE



CONTROL DEPENDENCE GRAPH

Properties of Control Dependence

- CDG is a *tree* $\rightarrow$ CFG is *structured*
- CDG is *acyclic* $\rightarrow$ CFG is *acyclic*
- CDG is *cyclic* $\rightarrow$ CFG is *cyclic*

The *control conditions of node Y* is the set,

$CC(Y) = \{(X,L) | Y$ is control dependent on X with label $L\}$

Two nodes, *A* and *B*, are said to be *identically control dependent* if and only if they have the same set of control conditions i.e. $CC(A) = CC(B)$

Data Dependence Analysis

If two operations have potentially interfering data accesses, data dependence analysis is necessary for determining whether or not an interference actually exists. If there is no interference, it may be possible to reorder the operations or execute them concurrently.

The data accesses examined for data dependence analysis may arise from array variables, scalar variables, procedure parameters, pointer dereferences, etc. in the original source program.

Data dependence analysis is conservative, in that it may state that a data dependence exists between two statements, when actually none exists.

Data Dependence: Definition

A *data dependence*, $S_1 \rightarrow S_2$, exists between CFG nodes S_1 and S_2 with respect to variable X if and only if

1. there exists a path P: $S_1 \rightarrow S_2$ in *CFG*, with no intervening write to X, and

2. at least one of the following is true:

   (a) **(flow)** X is written by S_1 and later read by S_2, or

   (b) **(anti)** X is read by S_1 and later is written by S_2 or

   (c) **(output)** X is written by S_1 and later written by S_2

Def/Use chaining for Data Dependence Analysis

A *def-use chain* links a definition D (i.e. a write access of variable *X* to each use *U* (i.e. a read access), such that there is a path from *D* to *U* in *CFG* that does not redefine *X*.

Similarly, a *use-def chain* links a use *U* to a definition *D*, and a *def-def chain* links a definition *D* to a definition *D’* (with no intervening write to *X* in all cases).

Def-use, use-def, and def-def chains can be computed by data flow analysis, and provide a simple but conservative way of enumerating flow, anti, and output data dependences.

Static single assignment (SSA) form

- Static single assignment (SSA) form provides a more efficient data structure for enumerating def-use, def-use and def-def chains.
- SSA form requires that *each use be reached by a single def* (when representing def-use information; analogous requirements are enforced for representing use-def and def-def information). Each def is treated as a new “name” for the variable.
- Each variable is assumed to have a dummy definition at the *START* node of the CFG.
- A ϕ function is used to capture the merge of multiple reaching definitions

Dealing with Merge Points

```
If Cond
Then X <---  4
Else  X <---  6
         o
         o
         o
Use variable X several times
```

- Tricky situation since both defs can reach all subsequent uses; exact reaching def depends on whether Cond evaluated to true or not
- Keeping track of true and false cases separately is complicated and intractable (in the presence of nested conditionals)

The SSA approach

```
If Cond
Then $X_1$ <---  4
Else  $X_2$ <---  6
      $X_3$ <---  $\phi$ (X1,X2)   <------ Add this line
       o
       o
       o
Use variable X3 several times
```

- The SSA solution is to add a special ∅ function at each merge point

- The new ∅-def *X3* captures the merge of *X1* and *X2*