Mechanism and Machine Theory Vol. 19, No. 1, pp. 139–148, 1984
Printed in Great Britain.

0094-114X/84 $3.00 + .00
Pergamon Press Ltd.

THREE POSITION PRECESSING MECHANISM SYNTHESIS

CENGIZ OREN† and ROGER E. KAUFMAN‡
The George Washington University, Washington, DC 20052, U.S.A.

Abstract—This paper presents a complex number development of a three position geared five-bar precessing mechanism synthesis.

The methodology which is presented in this study is based on an extension of Burmester cycloidal circlepoint–centerpoint theory[2]. This paper presents the mathematical formulation of the synthesis process.

A structured FORTRAN computer program was developed on a VAX 11/780 computer in order to implement the synthesis process. The synthesis program is capable of presenting the synthesis results on a graphical display. Typical results are shown.

INTRODUCTION

Definition

The precessing mechanism under consideration is a geared five-bar linkage in which input and output links are pinned together at one point on the fixed reference plane. The mechanism traces paths which precess and are symmetrical about the single fixed pivot (Fig. 1). Typically, these patterns resemble a multi-leaf flower. The mechanism is capable of generating symmetrical motions with an arbitrary number of leaves. There are also three arbitrarily specified design or "precision" positions within each leaf.

A maximum of three prescribed precision position synthesis can be attained with the mechanism topology described[5]. More complex topologies would permit a larger number of prescribed positions.

The number of leaves in a motion pattern is determined by the velocity ratio of the input and the output crank rotation[7]. Applications of the geared precessing mechanism can be categorized as function generation, path generation and motion generation.

DEVELOPMENT OF THEORY

One can represent the particular geared five-bar precessing mechanism under consideration in this paper by two closed vector polygons[1] (Fig. 2).

(1) Central crank vector polygon (vectors **P, Q, r**)

(2) Precessing crank vector polygon (vectors **V, W, X, r**)

A typical link of the mechanism is represented vectorially as in $\mathbf{P}_{k,j}$ where k is the leaf number and j is the prescribed design position of the link within the kth leaf.

If the vectors were written as complex numbers[2];

$$\mathbf{P}_{k,j} = P_{k,j_x} + iP_{k,j_y} \qquad (1)$$

then the rotation of the link through an angle $\theta_{k,j}$ from its reference (1st position) to its jth position can be written,

$$\mathbf{P}_{k,j} = e^{i\theta_{k,j}} \mathbf{P}_{k,1} \qquad (2)$$

where $e^{i\theta_{k,j}}$ is called a "rotation operation"[3].

THE EFFECTS OF THE GEAR RATIOS

The gear ratio of the mechanism determines the rotational speed and the number of leaves it will generate. Since the mechanism is geared, it will require continuous rotation of both cranks.

The velocity ratio of the gears will determine the number of leaves directly since the output crank rotation is a function of input crank rotation, $\psi = f(\theta)$ or $\psi_{kj} = \theta_{k,j}/\Omega$ where Ω is the velocity ratio (The velocity ratio, Ω is one plus the geared ratio.)

For example, if the velocity ratio Ω is 5, this means that the traced path will have 4 symmetrical leaves with three prescribed positions of the moving body within each leaf.

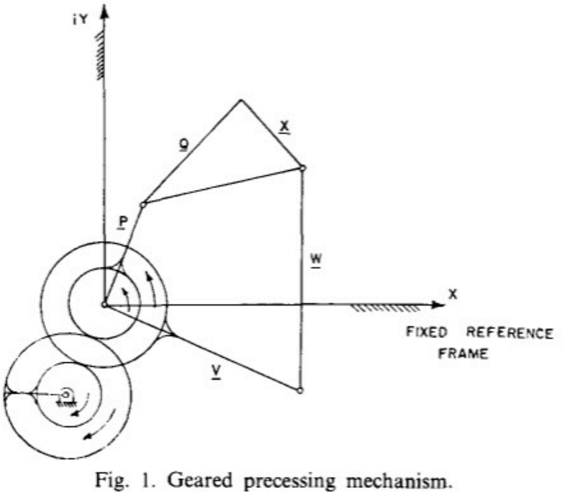


Fig. 1. Geared precessing mechanism.

†Based on the first author's Master of Science Thesis at the George Washington University.

‡Professor of Engineering.

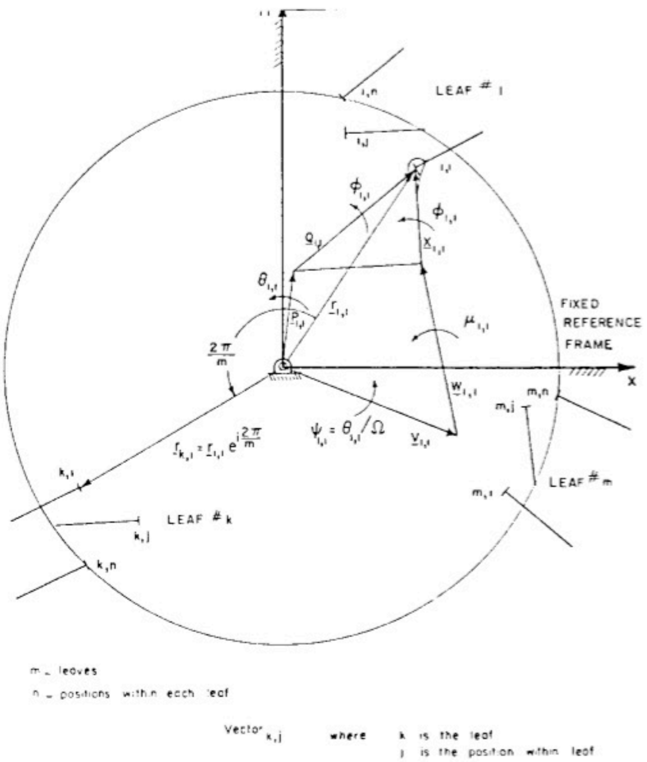


Fig. 2. Vectorial representation of the precessing mechanism.

Motion requirements

Every time the input crank (which is arbitrarily defined as the faster crank) revolves with respect to the slower crank (output crank), the mechanism will generate one leaf pattern.

Every time the output crank revolves with respect to the fixed reference plane, the mechanism will generate one complete pattern. If ratios are chosen so that both links get back to their starting positions, (i.e. integer ratio between links) then the pattern will repeat.

The above requirement will be carried out when

$$\theta_{k,j} = \Omega\psi_{k,j} \text{ or } \psi_{k,j} = \theta_{k,j}/\Omega. \tag{3}$$

In order to synthesize a precessing mechanism for n-leaved motion with three prescribed positions of the moving body within each leaf, a designer must choose values for the Ω and, for example, the ψ_{kj} rotation angles. Then, once rotations of the input crank have been calculated, the output crank rotations can be determined from eqn (3).

THE NUMBER OF PRECISION POSITIONS

Because the central crank vector polygon has a more limited number of parameters, it will determine the number of positions for which a precessing mechanism of this type can be synthesized. There are more parameters available in the precessing crank polygon, so some of them can be chosen to meet the needs of the central crank and there will be still enough parameters remaining to allow a solution for the precessing crank.

For the mechanism in the Jth design position the vector equation of closure can be written for the central crank vector polygon as follows

$$e^{i\theta_j}\mathbf{P} + e^{i\phi_j}\mathbf{Q} = \mathbf{r}_j. \tag{4}$$

From the above equation a table can be constructed for the number of positions permitted by the central crank (Table 1). This particular precessing mechanism can be synthesized for a maximum of three precision position synthesis within each leaf[6]. In doing this, the ϕ's can be arbitrarily picked.

For three design position synthesis a nonlinear compatibility relationship must be solved to determine the θ angles. For two precision position synthesis, solution of a compatibility relationship is not necessary.

MOTION CRITERIA

A precessing mechanism of the type under consideration has two cranks, namely the input and output cranks, but these are not the same as the double cranks conventionally referred to in the Grashof classification scheme. Nonetheless, the Grashof relationship gives important insight into the ability of the geared five-bar precessing mechanism's ability to move continuously[4].

The particular Grashof relation which is required in the synthesis of this precessing mechanism as follows;

(1) Either the input crank or the output crank must be the shortest link.

Table 1.

Precision position	Complex equ.	Real equ.	Unknown	= of real unknown	Arbitrary choice
1	1	2	**P**, **Q**	4	2
2	2	4	**P**, **Q**, θ_2	5	1
3	3	6	**P**, **Q**, θ_2, θ_3	5	0

(2) The sum of the lengths of the longest and the shortest links must be less than the sum of the lengths of the other two links, that is $L + s < p + Q$ (Fig. 3).

Also both input crank and output crank must be able to rotate continuously in the same direction. If one of the above conditions fails, the precessing mechanism cannot be moved continuously.

These constraints put many limits on the synthesis process.

THE SYNTHESIS PROCESS

In the development that follows, the precessing mechanism is defined by two separate vector polygons;

(a) The input vector polygon is called the "central crank polygon".

(b) The output vector polygon is called the "precessing crank polygon".

The above vector polygons can be represented in terms of complex equations. The solutions for the vector polygon equations are as follows.

Synthesis of the central crank

In Fig. 4 three positions within a single leaf are prescribed arbitrarily for the moving plane π. These arbitrary positions are specified relative to a coordinate system in the fixed plane. For simplicity only two arbitrary design positions are shown in Fig. 4. Vector $\mathbf{r}_{k,j}$ prescribes jth design position in kth leaf. $\phi_{k,j}$ is the displacement angle of the moving plane π. Adding up vectors around the polygon, $\mathbf{r}_{1,1}$ becomes

$$\mathbf{P}_{1,1} + \mathbf{Q}_{1,1} = \mathbf{r}_{1,1}. \quad (5)$$

As the moving plane goes from its 1st position to its jth position, the unknown vector $\mathbf{P}_{k,j}$ moves by the angle $\theta_{k,j}$. Since the unknown vector $\mathbf{Q}_{k,j}$ is attached to the moving plane, its rotation is the same as that of the moving plane.

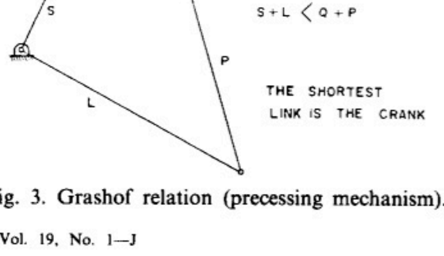


Fig. 3. Grashof relation (precessing mechanism).

For the prescribed positions, the eqn (5) can be written in the following form for three position synthesis

$$\mathbf{P} + \mathbf{Q} = \mathbf{r}_1 \quad (6)$$

$$e^{i\theta_2}P + e^{i\phi_2}\mathbf{Q} = \mathbf{r}_2 \quad (7)$$

$$e^{i\theta_3}P + e^{i\phi_3}\mathbf{Q} = \mathbf{r}_3. \quad (8)$$

The angles ϕ_2, ϕ_3 and the vectors $\mathbf{r}_j$'s are known from the input data. The angles θ_2, θ_3 are unknown along with vectors **P**, **Q**.

Complex vector equations can be written in matrix form, as follows

$$\begin{bmatrix} 1 & 1 \\ e^{i\theta_2} & e^{i\phi_2} \\ e^{i\theta_3} & e^{i\phi_3} \end{bmatrix} \begin{bmatrix} \mathbf{P} \\ \mathbf{Q} \end{bmatrix} = \begin{bmatrix} \mathbf{r}_1 \\ \mathbf{r}_2 \\ \mathbf{r}_3 \end{bmatrix}. \quad (9)$$

Since there are three complex equations and only two complex unknowns (**P**, **Q**), a solution for the compatibility equation will only exist if the augmented matrix of the system is of rank 2[3]. This compatibility condition can be expressed in the following compatibility equation[1].

$$\begin{vmatrix} 1 & 1 & \mathbf{r}_1 \\ e^{i\theta_2} & e^{i\phi_2} & \mathbf{r}_2 \\ e^{i\theta_3} & e^{i\phi_3} & \mathbf{r}_3 \end{vmatrix} = 0. \quad (10)$$

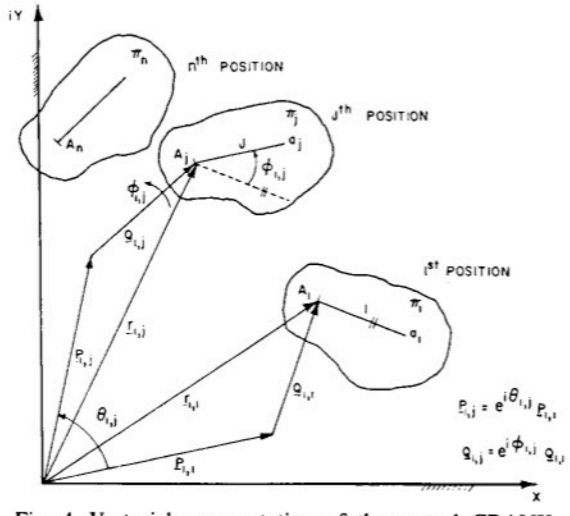


Fig. 4. Vectorial representation of the central CRANK vector polygon.

Expanding eqn (10) in terms of cofactors of the first column

$$\begin{vmatrix} e^{i\phi_2} & \mathbf{r}_2 \\ e^{i\phi_3} & \mathbf{r}_3 \end{vmatrix} - e^{i\theta_2}\begin{vmatrix} 1 & \mathbf{r}_1 \\ e^{i\phi_3} & \mathbf{r}_3 \end{vmatrix} + e^{i\theta_3}\begin{vmatrix} 1 & \mathbf{r}_1 \\ e^{i\phi_2} & \mathbf{r}_2 \end{vmatrix} = 0 \quad (11)$$

then

$$\underline{\Delta}_2 - e^{i\theta_2}\underline{\Delta}_3 + e^{i\theta_3}\underline{\Delta}_4 = 0 \quad (12)$$

where

$$\underline{\Delta}_2 = \begin{vmatrix} e^{i\phi_2} & \mathbf{r}_2 \\ e^{i\phi_3} & \mathbf{r}_3 \end{vmatrix} \quad (13)$$

$$\underline{\Delta}_3 = \begin{vmatrix} 1 & \mathbf{r}_1 \\ e^{i\phi_2} & \mathbf{r}_3 \end{vmatrix} \quad (14)$$

$$\underline{\Delta}_4 = \begin{vmatrix} 1 & \mathbf{r}_1 \\ e^{i\phi_2} & \mathbf{r}_2 \end{vmatrix}. \quad (15)$$

The (eqn 2) can be written as

$$e^{i\theta_2}\underline{\Delta}_3 - e^{i\theta_3}\underline{\Delta}_4 = \underline{\Delta}_2. \quad (16)$$

The equation above can be given the following graphical interpretation. $\underline{\Delta}_2, \underline{\Delta}_3, \underline{\Delta}_4$ are known complex numbers (Fig. 5 and 6). Vector $e^{i\theta_2}\underline{\Delta}_3$ is rotated through an unknown angle θ_2 and vector $e^{i\theta_3}\underline{\Delta}_4$ is rotated through an unknown angle θ_3. These vectors must be added to be equal to the known vector $\underline{\Delta}_2$. As can be seen from Fig. 6, the compatibility triangle can be assembled in two possible ways. One possibility is the rotation of θ_2 and θ_3 for the "First Choice" compatibility triangle. The other possibility is the rotation of $\tilde{\theta}_2$ and $\tilde{\theta}_3$ for the "Alternate Choice" compatibility triangle. The alternate choice of angles θ_2 and θ_3 are distinguished by means of the overscore as written $(\tilde{\theta}_2, \tilde{\theta}_3)$. The uniqueness of this triangle provides two sets of solutions for the unknown angles θ_2 and θ_3 which is a useful gain from a synthesis point of view.

The law of cosines can be used to determine the internal angles of the triangle (Fig. 7).

$$\gamma_2 = \cos^{-1}(\underline{\Delta}_3^2 + \underline{\Delta}_2^2 - \underline{\Delta}_4^2/2\underline{\Delta}_3\underline{\Delta}_2) \quad (17)$$

$$\gamma_4 = \cos^{-1}(\underline{\Delta}_4^2 + \underline{\Delta}_2^2 - \underline{\Delta}_3^2/2\underline{\Delta}_4\underline{\Delta}_2). \quad (18)$$

Once the internal angles of the triangle are calculated, solutions for the θ_2, $\tilde{\theta}_2$ and θ_3, $\tilde{\theta}_3$ become simpler.

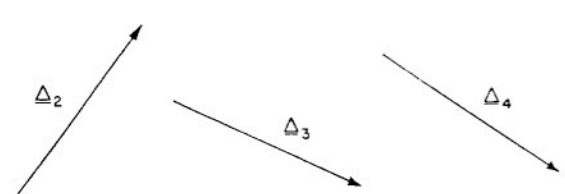


Fig. 5. Vector representation of $\underline{\Delta}_2$, $\underline{\Delta}_3$, $\underline{\Delta}_4$.

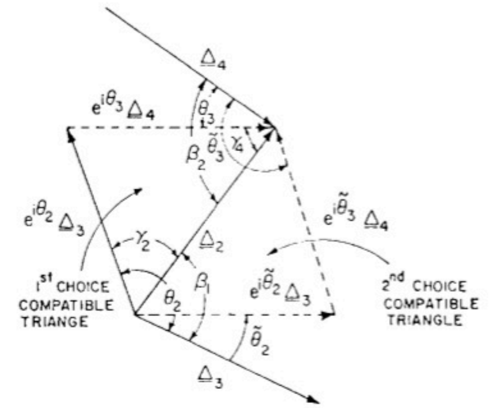


Fig. 6. Compatibility triangle.

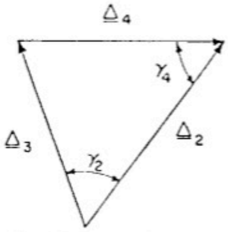


Fig. 7. Law of cosines.

Figs. 6 and 8 shows how the rotation angles θ_2 and $\tilde{\theta}_2$ are calculated. Finally the angle θ_2 becomes

$$\theta_2 = \beta_1 + \gamma_2 \quad (19)$$

and

$$\tilde{\theta}_2 = \beta_1 + \gamma_2 \quad (20)$$

From Figs. 6–9 θ_3 and $\tilde{\theta}_3$ can be calculated.

$$\theta_3 = \beta_2 - \gamma_4 \quad (21)$$

$$\tilde{\theta}_3 = \beta_2 + \gamma_4 \quad (22)$$

These compatibility values of θ_2, θ_3 satisfy eqn (10); therefore any two equations from eqn (9) can be solved to obtain **P**, **Q**

$$\mathbf{P} + \mathbf{Q} = \mathbf{r} \quad (23)$$

$$e^{i\theta_2}\mathbf{P} + e^{i\phi_2}\mathbf{Q} = \mathbf{r}_2 \quad (24)$$

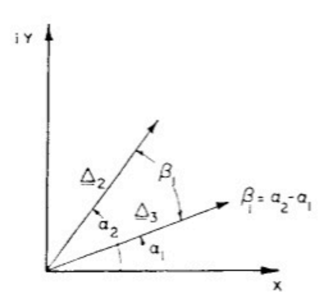


Fig. 8. Determining β_1 between vectors $\underline{\Delta}_2$ and $\underline{\Delta}_3$.

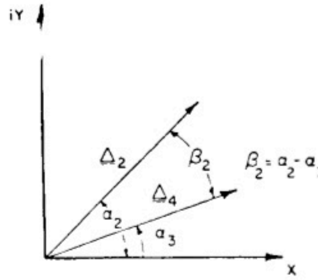


Fig. 9. Determining β_2 between vectors $\underline{\Delta}_2$ and $\underline{\Delta}_4$.

or, in matrix form

$$\begin{bmatrix} 1 & 1 \\ e^{i\theta_2} & e^{i\phi_3} \end{bmatrix} \begin{bmatrix} \mathbf{P} \\ \mathbf{Q} \end{bmatrix} = \begin{bmatrix} \mathbf{r}_1 \\ \mathbf{r}_2 \end{bmatrix} \tag{25}$$

now, by using Cramer's rule the unknown vectors **P** and **Q** can be solved as follows;
determinant

$$D = \begin{vmatrix} 1 & 1 \\ e^{i\theta_2} & e^{i\phi_2} \end{vmatrix} \tag{26}$$

vector **P**

$$\mathbf{P} = \frac{\begin{vmatrix} \mathbf{r}_2 & 1 \\ \mathbf{r}_2 & e^{i\phi_2} \end{vmatrix}}{D} \tag{27}$$

and vector **Q**

$$\mathbf{Q} = \frac{\begin{vmatrix} 1 & \mathbf{r}_1 \\ e^{i\theta_2} & \mathbf{r}_2 \end{vmatrix}}{D}. \tag{28}$$

The alternate choice rotation angles $\tilde{\theta}_2$, $\tilde{\theta}_3$ will give an alternate vector solution for the central crank.

One important point must be mentioned that, in some cases if either

$$\underline{\Delta}_2 > \underline{\Delta}_3 + \underline{\Delta}_4 \tag{29}$$

or

$$\underline{\Delta}_2 < |\underline{\Delta}_3 - \underline{\Delta}_4| \tag{30}$$

conditions occur, the compatibility triangle cannot be assembled. Therefore, no synthesis can be completed. Such conditions could play a very important role in the synthesis process.

Synthesis of the precessing crank

In the precessing vector polygon (Fig. 10), the moving plane and the described design positions $\mathbf{r}_{k,j}$ and the displacement angles of $\phi_{k,j}$ of the moving plane are same as those in the synthesis of the central crank.

For simplicity only two of the described design positions are shown in Fig. 10.

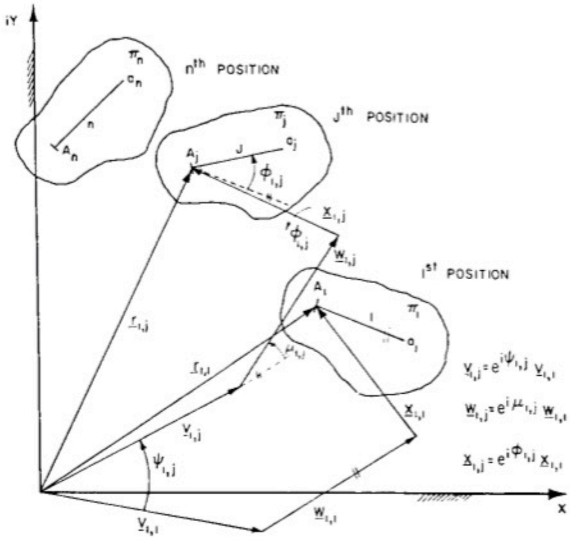


Fig. 10. Vectorial representation of the precessing CRANK vector polygon.

Adding up the vectors around the vector polygon yields

$$\mathbf{V} + \mathbf{W} + \mathbf{X} = \mathbf{r}_1. \tag{31}$$

As the moving body goes from its 1st position to its jth position within a leaf, the unknown vector $\mathbf{V}_{k,j}$ moves by the angle $\psi_{k,j}$, where $\psi_{k,j} = \theta_{k,j}/\Omega$ (Ω is the velocity ratio).

The unknown vector **W** moves by moves by the angle $\mu_{k,j}$. Since the unknown vector $\mathbf{X}_{k,j}$ is on the moving plane π, it will have the same angle of rotation, angle $\phi_{k,j}$.

The closure of the vector polygon equations for three positions is expressed as follows

$$\mathbf{V} + \mathbf{W} + \mathbf{X} = \mathbf{r}_1 \tag{32}$$

$$e^{i\theta_2/\Omega}\mathbf{V} + e^{i\mu_2}\mathbf{W} + e^{i\phi_2}\mathbf{X} = \mathbf{r}_2 \tag{33}$$

$$e^{i\theta_3/\Omega}\mathbf{V} + e^{i\mu_3}\mathbf{W} + e^{i\phi_3}\mathbf{X} = \mathbf{r}_3. \tag{34}$$

These complex vector equations can be written in matrix form

$$\begin{bmatrix} 1 & 1 & 1 \\ e^{i\theta_2/\Omega} & e^{i\mu_2} & e^{i\phi_2} \\ e^{i\theta_3/\Omega} & e^{i\mu_3} & e^{i\phi_3} \end{bmatrix} \begin{bmatrix} \mathbf{V} \\ \mathbf{W} \\ \mathbf{X} \end{bmatrix} = \begin{bmatrix} \mathbf{r}_1 \\ \mathbf{r}_2 \\ \mathbf{r}_3 \end{bmatrix}. \tag{35}$$

Since the angles μ_2 and μ_3 were already prescribed during the solution for the central crank, solution of these equations becomes a trivial task.

First choice precessing crank determinant

$$\mathbf{D} = \begin{vmatrix} 1 & 1 & 1 \\ e^{i\theta_2/\Omega} & e^{i\mu_2} & e^{i\phi_2} \\ e^{i\theta_3/\Omega} & e^{i\mu_3} & e^{i\phi_3} \end{vmatrix} \tag{36}$$

vector **V**

$$\mathbf{V} = \frac{\begin{vmatrix} \mathbf{r}_1 & 1 & 1 \\ \mathbf{r}_2 & e^{i\mu_2} & e^{i\phi_2} \\ \mathbf{r}_3 & e^{i\mu_3} & e^{i\phi_3} \end{vmatrix}}{D} \tag{37}$$

vector **W**

$$\mathbf{W} = \frac{\begin{vmatrix} 1 & \mathbf{r}_1 & 1 \\ e^{i\theta_2/\Omega} & \mathbf{r}_2 & e^{i\phi_2} \\ e^{i\theta_3/\Omega} & \mathbf{r}_3 & e^{i\phi_3} \end{vmatrix}}{D} \tag{38}$$

vector **X**

$$\mathbf{X} = \frac{\begin{vmatrix} 1 & 1 & \mathbf{r}_1 \\ e^{i\theta_2/\Omega} & e^{i\mu_2} & \mathbf{r}_2 \\ e^{i\theta_3/\Omega} & e^{i\mu_3} & \mathbf{r}_3 \end{vmatrix}}{D}. \tag{39}$$

The alternate set of input rotation angles $\tilde{\theta}_2$ and $\tilde{\theta}_3$ of the vector **P** in the central crank will give an alternate choice vector solution for the precessing crank. Since the rotation angles of $\psi_{k,j}$ are function of $\theta_{k,j}$, therefore replacing $\theta_{k,j}$'s with the alternate set of $\tilde{\theta}_{k,j}$ one can obtain the alternate set of vector solutions in the same manner.

Finding the solutions for the unknown vectors of **P**, **Q** in the central crank vector polygon and the vectors of the **V**, **W**, **X** in the precessing crank vector polygon completes the synthesis process of the precessing mechanism.

THE SYNTHESIS PROGRAM

The precessing mechanism synthesis process was developed with the structured FORTRAN programming language on the VAX/VMS computer. The synthesis program CRANK has 28 subroutines with a main program and a declaration subroutine which is externally linked.

Program crank consists of two parts;

(a) Mechanism synthesis.

(b) Graphical representation of the synthesized mechanism.

(a) *Mechanism synthesis*

Mechanism synthesis mainly solves for the augmented matrix and the compatibility triangle. Once the input and the output crank rotations are calculated, it solves for the unknown vectors of the central crank vector polygon and precessing vector polygon. The alternate choice compatibility triangle solutions are also calculated. Program checks for the singularity condition in matrix operations are incorporated into these calculations.

As mentioned earlier, the precessing mechanism must have continuous input and output crank rotations in the same direction. The program checks for this and if the test for continuous rotations fails, the synthesis process will be interrupted.

(b) *Graphical representation*

The graphical representation of the mechanism takes place on a Printronix dot matrix output device.

The synthesis program is equipped with various plotting capabilities.

(1) Plotting of the vector solutions for the mechanism.

(a) First leaf, three prescribed positions mechanism plot (Fig. 11).

(b) One full rotation of the mechanism plot (Fig. 12).

(2) Plotting of the coupler curve (Fig. 13).

(3) Plotting of the successive motion of the prescribed positions, as the mechanism makes one full rotation (Fig. 14).

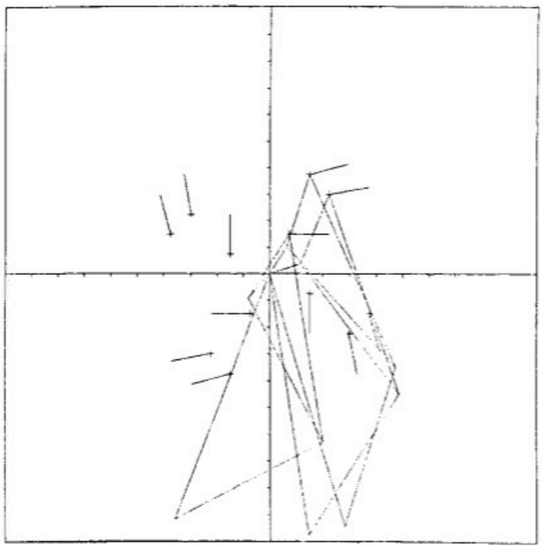


Fig. 11. Precessing mechanism plotting in first leaf with three prescribed design positions.

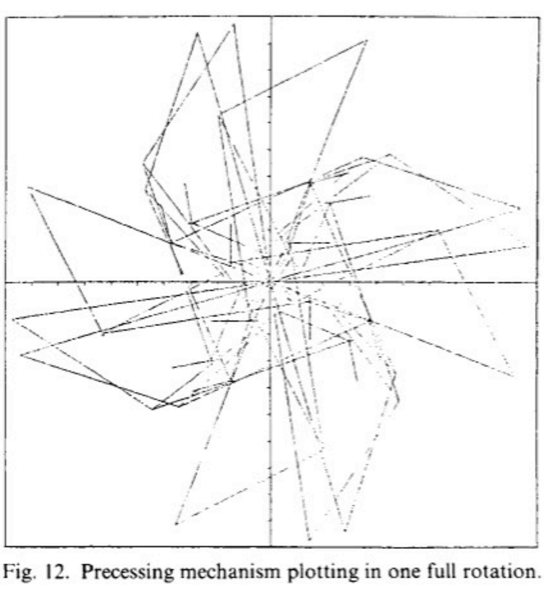


Fig. 12. Precessing mechanism plotting in one full rotation.

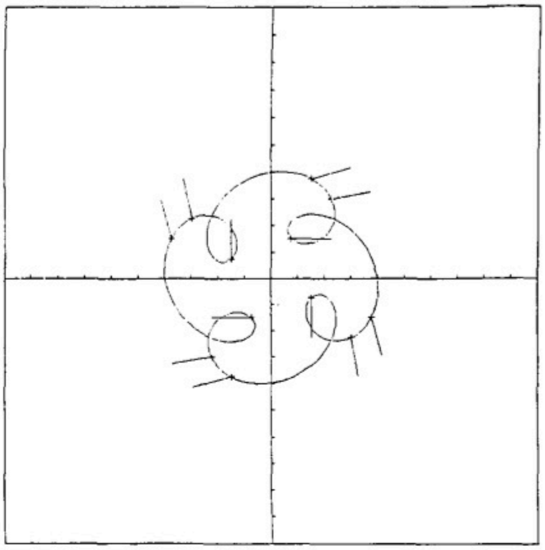


Fig. 13. Coupler curve plotting.

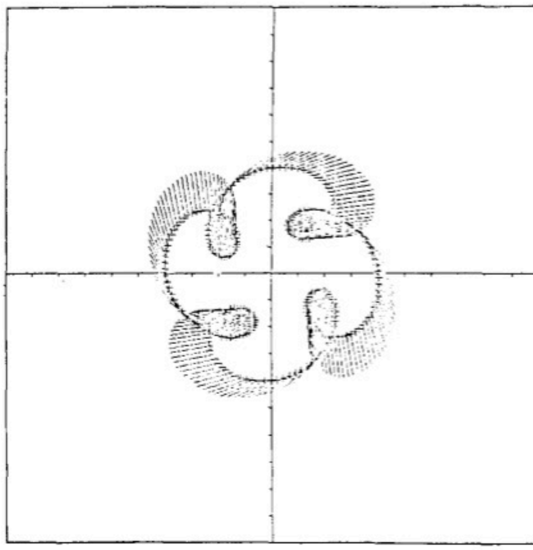


Fig. 14. Successive motion plotting of the prescribed position which moves with moving plane.

Program crank also provides a hard copy printout of the solutions for the vectors, accompanied with their rotation angles in three prescribed design positions (see Appendix 1).

Input data set

The position vectors are read in as pairs of real numbers in the form of a real part followed by an imaginary part separated by a comma. Second and third position vectors are accompanied with rotation angles ϕ_j's of the moving plane in degrees. Rotation angles μ_j's of the unknown vector **W** are also read, since they are also prescribed (Table 2).

COMMAND INSTRUCTIONS

Crank is equipped with a set of command instructions so that the designer has plotting alternatives[1]. This allows him to better understand the motion characteristics of the mechanism. (Table 3).

Command usage

LINK: Command LINK passes control of the program to subroutine DRAW–LINK which sets the coordinate system and the prescribed design positions. Two plotting choices are available.

CURV: Command CURV passes control of the

Table 2.

```
ENTER X & Y COORD. OF FIRST POSITION
1, 1
ENTER X & Y COORD. OF SECOND POSITION AND PH12
2, 1, 15
ENTER X & Y COORD. OF THIRD POSITION AND PH13
2, 5, 5
ENTER MU2, MU3
30, 45
ENTER VELOCITY RATIO (OMEGA). IT MUST BE AN
INTEGER.
5
```

Table 3.

```
SUBROUTINE COM...BLOCK IS EQUIPPED WITH COMMAND
INSTRUCTIONS.
  COMMAND DESCRIPTION:
  LINK: DRAW LINKAGE
  CURVE: DRAW COUPLER CURVE
  MOVE: DRAW SUCCESSIVE MOTION OF POSITION
  REST: RESULTS OF THE VECTOR CALCULATIONS
  QUIT: QUIT COMMAND MODE
```

program to subroutine COUPLER–CURV which sets the coordinate system, the prescribed positions and plots the motion of the moving plane or the coupler link.

MOVE: Command Move passes control of the program to subroutine MOV–POS which plots the successive incrementation of the moving design position.

REST: Command REST passes control of the program to subroutine RESULTS which prints a hard copy of the synthesis results (see Appendix 1).

QUIT: Command QUIT takes control of the program from subroutine COM–BLOCK where all above operations take place, so that the program can be utilized for other designs.

CONCLUSION

The synthesis technique which is illustrated in this study is an extension of Burmester circlepoint–centerpoint theory. This synthesis is useful in function, path and motion generation applications which can be used for designing linkages capable of generating symmetric motions. In this synthesis an arbitrary number of leaves along with three arbitrarily specified positions within each leaf must be present.

Because of the compatibility condition (see Synthesis of the central crank), there are two alternatives for each synthesis solution. With this choice of alternatives, a designer can study both solutions and choose one which better suits the design requirements. However two solutions don't always exist for design.

The geared five-bar precessing mechanism could possibly be utilized in applications such as indexing or decorative stitching. A few possible designs for such precessing mechanisms have been presented in this study. As an extension, it is possible to synthesize a whole family of geared precessing mechanisms for higher order arbitrarily chosen design positions.

A suggestion for further work is to extend the techniques developed in this thesis to encompass a wider variety of other types of precessing mechanisms based around 6, 8 or 10 bar chains.

Acknowledgements—The first author would like to express his appreciation to Professor Roger E. Kaufman for his guidance throughout the graduate study program and especially for his encouragement in the development of the precessing mechanism synthesis. He also would like to express his appreciation to the Computer Graphics Laboratory of the School of Engineering and Applied Science of the George Washington University. A special thanks to Ziggy for her unrelenting interest in these studies.

The second author would like to acknowledge support of his work through the National Science Foundation, grant number DAR—7910176 Physical, Mathematical, Engineering Applications Program.

REFERENCES

1. C. Oren, *Three Position Precessing Mechanism Synthesis*. Master's Thesis, The George Washington University (1981).
2. R. E. Kaufman, Singularity solutions in Burmester theory. *Trans. ASME, Paper No. 72-Mech*-23 (1972).
3. R. E. Kaufman, Introduction to Burmester theory. Unpublished notes. Section 1—Three position synthesis. Section 2—Four position synthesis.
4. Hartenberg and Denavit, *Kinematic Synthesis of Linkages*, Chap. 3. McGraw-Hill, New York (1964).
5. G. N. Sandor and R. E. Kaufman, Kinematic synthesis of geared linkages. *J. Mechanisms* **5**, 59–87 (1970).
6. G. N. Sandor, On the existence of a cyclodial Burmester theory in planar kinematics. *J. Appl. Mechanisms* **31**, *Trans. ASME*, Series E, **86**, 694–699 (1964).
7. R. E. Kaufman and G. N. Sandor, The Bicycloidal Crank—a new four-link mechanism. *ASME Paper No. 68- Mech* 14. *Presented at the 10th ASME Mechanism Conf.*, Georgia Institute of Technology (1968). Published in the *J. Engng Ind* 91–96 (1969).

APPENDIX

Appendix 1 illustrates synthesis of a precessing mechanism with five leaves. The sample synthesis shows the capabilities of the computer program CRANK.

The following synthesis consists of below steps:

(a) Input data (the prescribed positions and angles with velocity ratio OMEGA).

(b) Design results (Vector dimensions and their rotation angles).

(c) The mechanism plot in first leaf.

(d) The mechanism plot in one complete rotation.

(e) The coupler curve plot.

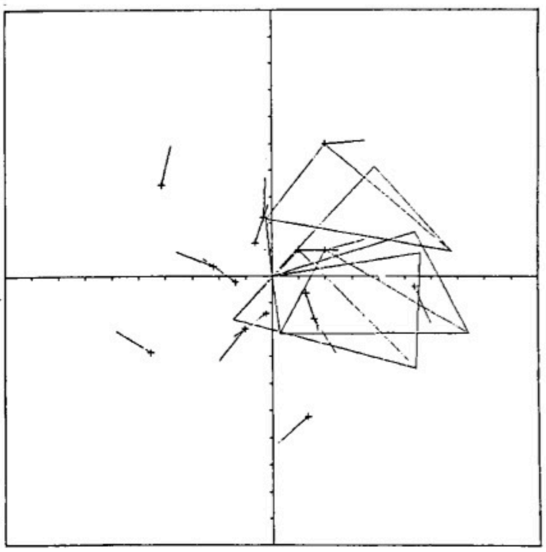
(f) The successive notion of the design position.

(g) Checking for second choice synthesis. If doesn't exist, the synthesis is completed.

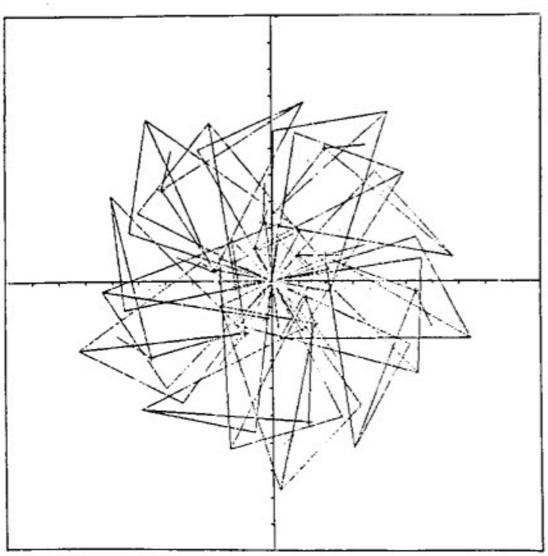
```
 ENTER X$Y COORD.OF FIRST POSITION
1,1
 ENTER X$Y COORD.OF SECOND POSITION AND PHI2
2,1,15
 ENTER X$Y COORD.OF THIRD POSITION AND PHI3
2,5,5
 ENTER MU2,MU3
30,45
 ENTER VELOCITY RATIO (OMEGA).IT MUST BE AN INTEGER.
5
 *** THE INPUT VALUES ***
 ........................
 POSITION VECTORS
 R1 (  1.000000     ,  1.000000     )
 R2 (  2.000000     ,  1.000000     ) PHI2   15.00000
 R3 (  2.000000     ,  5.000000     ) PHI3   5.000000
 ROTATION ANGELS OF THE VECTOR W
 MU2   30.00000      MU3   45.00000
 VELOCITY RATIO OMEGA            5
 ........................
```

```
 * COMMAND DESCRIPTION:
            LINK: DRAW LINKAGE
            CURV: DRAW COUPLER CURVE
            MOVE: DRAW SUCCESSIVE MOTION OF POSITION
            REST: RESULTS OF THE VECTOR CALCULATIONS
            QUIT: QUIT COMMAND MODE
 ENTER COMMAND ...
 COMMAND ?
REST
 *** DESIGN RESULTS OF THE PRECESSING MECHANISM ***
 **************************************************
 * CENTRAL CRANK *
 VECTOR P (INPUT CRANK)= ( -1.472980     , -1.618635     )
 INPUT CRANK ROTATIONS (PHI) OF P:
 THETA (2ND POSITION)=   49.88768      DEGREES
 THETA (3RD POSITION)=   228.4770      DEGREES
 VECTOR Q= (  2.472980     ,  2.618635     )
 ANGLE OF ROTATION (PHI) OF VECTOR Q:
 PHI (2ND POSITION)=   14.99717      DEGREES
 PHI (3RD POSITION)=   4.999056      DEGREES
 * PRECESSING CRANK *
 VECTOR V (OUTPUT CRANK)= (  5.582345     , 0.8903628     )
 OUTPUT CRANK ROTATION (PSI) OF V:
 PSI (2ND POSITION)=   8.314612      DEGREES
 PSI (3RD POSITION)=   38.07950    · DEGREES
 VECTOR W= (-0.1672265     , -4.325319     )
 ANGLE OF ROTATION (MU) OF VECTOR W:
 MU (2ND POSITION)=   29.99434      DEGREES
 MU (3RD POSITION)=   44.99151      DEGREES
 VECTOR X= ( -4.415119     ,  4.434957     )
 ANGLE OF ROTATION (PHI) OF VECTOR X:
 PHI (2ND POSITION)=   14.99717      DEGREES
 PHI (3RD POSITION)=   4.999056      DEGREES
 * COMMAND DESCRIPTION:
            LINK: DRAW LINKAGE
            CURV: DRAW COUPLER CURVE
            MOVE: DRAW SUCCESSIVE MOTION OF POSITION
            REST: RESULTS OF THE VECTOR CALCULATIONS
            QUIT: QUIT COMMAND MODE
 ENTER COMMAND ...
 COMMAND ?
```

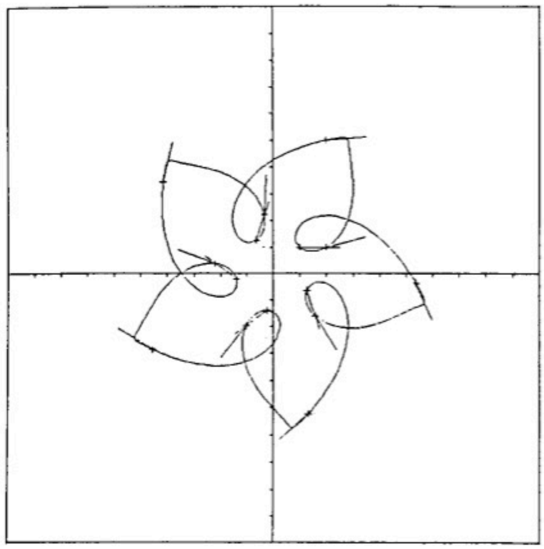
```
* COMMAND DESCRIPTION:
          LINK: DRAW LINKAGE
          CURV: DRAW COUPLER CURVE
          MOVE: DRAW SUCCESSIVE MOTION OF POSITION
          REST: RESULTS OF THE VECTOR CALCULATIONS
          QUIT: QUIT COMMAND MODE
 ENTER COMMAND ...
 COMMAND ?
LINK
 DO YOU WANT THE ROTATION OF THE MECHANISM ? (Y/N)
N
```



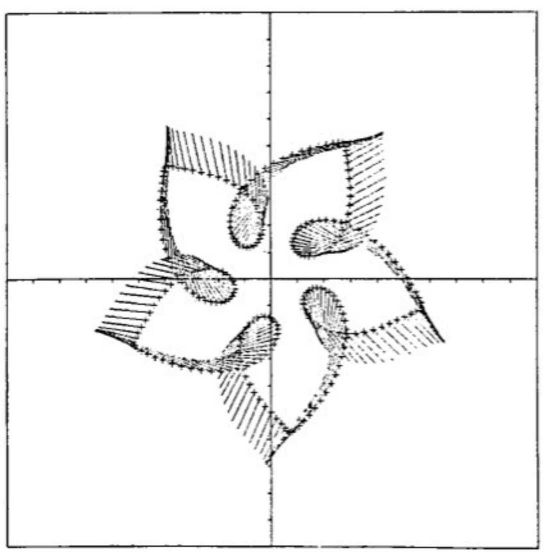
```
* COMMAND DESCRIPTION:
          LINK: DRAW LINKAGE
          CURV: DRAW COUPLER CURVE
          MOVE: DRAW SUCCESSIVE MOTION OF POSITION
          REST: RESULTS OF THE VECTOR CALCULATIONS
          QUIT: QUIT COMMAND MODE
 ENTER COMMAND ...
 COMMAND ?
LINK
 DO YOU WANT THE ROTATION OF THE MECHANISM ? (Y/N)
Y
```



```
* COMMAND DESCRIPTION:
          LINK: DRAW LINKAGE
          CURV: DRAW COUPLER CURVE
          MOVE: DRAW SUCCESSIVE MOTION OF POSITION
          REST: RESULTS OF THE VECTOR CALCULATIONS
          QUIT: QUIT COMMAND MODE
 ENTER COMMAND ...
 COMMAND ?
CURV
```



```
* COMMAND DESCRIPTION:
          LINK: DRAW LINKAGE
          CURV: DRAW COUPLER CURVE
          MOVE: DRAW SUCCESSIVE MOTION OF POSITION
          REST: RESULTS OF THE VECTOR CALCULATIONS
          QUIT: QUIT COMMAND MODE
 ENTER COMMAND ...
 COMMAND ?
MOVE
```



```
* COMMAND DESCRIPTION:
          LINK: DRAW LINKAGE
          CURV: DRAW COUPLER CURVE
          MOVE: DRAW SUCCESSIVE MOTION OF POSITION
          REST: RESULTS OF THE VECTOR CALCULATIONS
          QUIT: QUIT COMMAND MODE
 ENTER COMMAND ...
 COMMAND ?
QUIT
 *** MESSAGE 4 ***
 SECOND CHOICE CENTRAL CRANK CANNOT BE SYNTHESIZED
 THE SYNTHESIS PROCESS INTERRUPTED.
FORTRAN STOP
*
```

LA SYNTHESE D'UN MECANISME A PRECEDER EN TROIS POSITIONS

C. Oren et R.E. Kaufman

Résumé - Cet exposé présente une méthode de synthèse en nombres complexes d'un mécanisme à cinq barres avec engrenages et trois positions spécifiées.

La méthode est basée sur une extension de la théorie de cercle-point-centre-point cycloïdaux de Burmester. Cette étude présente la formulation mathématique du processus de synthèse.

Un programme structuré d'ordinateur employant le FORTRAN a été développé sur l'ordinateur VAX 11/780 pour implémenter le processus de synthèse. Le programme est capable de présenter les résultats graphiquement. Des résultats typiques sont donnés.