{ CSCI 6331 · 4331 | Lecture 6 }

Cryptography

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http://tinyurl.com/cryptogw/

Announcements

- Evaluation:

  10% In-Class/Piazza, 20% Final Presentation / Project

  30% Homework, 40% Final (Apr 25)

- Homework 3 is out

  due Feb 29 (Wed) in class

Message Authentication Codes

setting.

- both users generate and share a *secret* key k in advance

— runs key generation algorithm $\mathrm{k} \leftarrow \mathrm{Gen}(1^{\mathrm{n}})$

- to send message m, sender computes a MAC tag t and sends (m, t)

— runs tag generation algorithm $\mathrm{t} \leftarrow \mathrm{Mac}_{\mathrm{k}}(\mathrm{m})$

- upon receiving (m, t), receiver verifies whether t is a valid tag on m

— runs verification algorithm $\mathrm{Vrfy}(\mathrm{m}, \mathrm{t}) \in \{0, 1\}$ (1 being valid)

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syntax. *message authentication code* (MAC) is a triple of randomized algorithms $(\mathrm{Gen}, \mathrm{Mac}, \mathrm{Vrfy})$

- correctness. for every key k output by $\mathrm{Gen}(1^{\mathrm{n}})$, and every $\mathrm{m} \in \{0, 1\}^*$, we have $\mathrm{Vrfy}_{\mathrm{k}}(\mathrm{m}, \mathrm{Mac}_{\mathrm{k}}(\mathrm{m})) = 1$.

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1. Generate random key k using $\mathrm{Gen}(1^{\mathrm{n}})$
2. Adversary given 1^{n} and oracle access to $\mathrm{Mac}_{\mathrm{k}}(\cdot)$, eventually outputs (m, t). Let Q = set of queries
3. Wins if $\mathrm{Vrfy}_{\mathrm{k}}(\mathrm{m}, \mathrm{t}) = 1$ and $\mathrm{m} \notin \mathrm{Q}$.

definition. (t, ϵ)-secure if for all advesaries running in time t, winning probability bounded by ϵ.

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- fixed: given MAC("hello"), MAC("world"), hard to compute MAC("wello"); however, computing MAC("hello world") may be easy.

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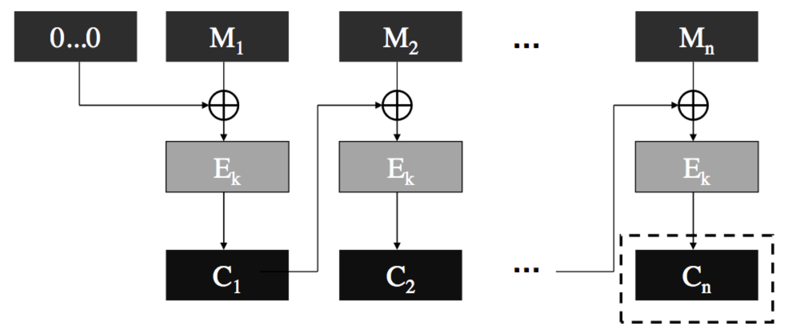

comparison with CBC-mode encryption.

- always use $\mathrm{IV} = 00\ldots0$ (or, no IV); CBC-mode encryption uses *random* IV.
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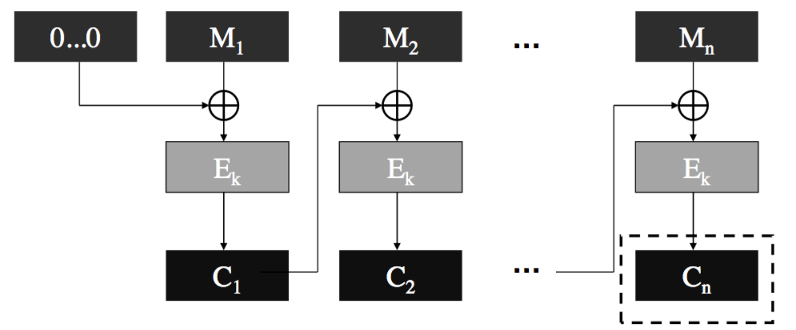

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- important distinction: many cryptography libraries provide a "CBC function"

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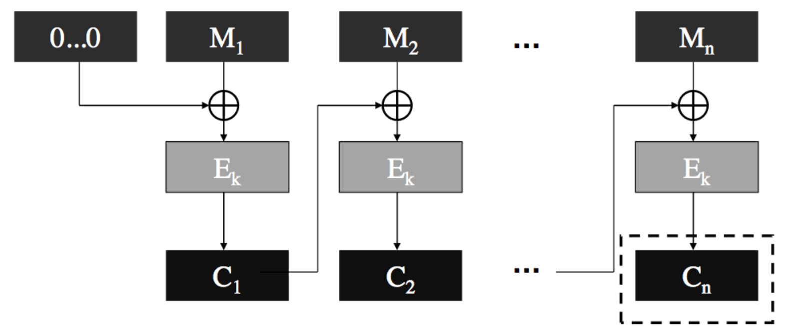

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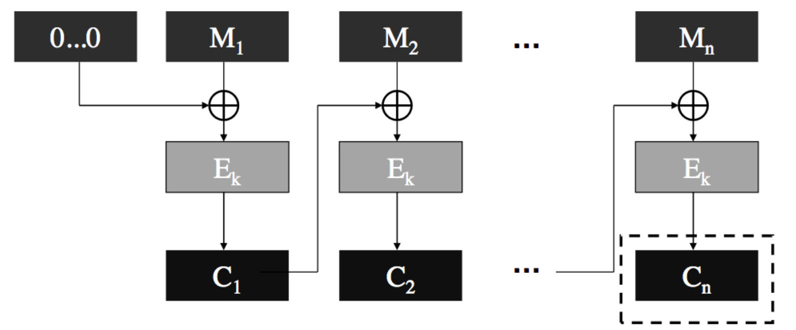

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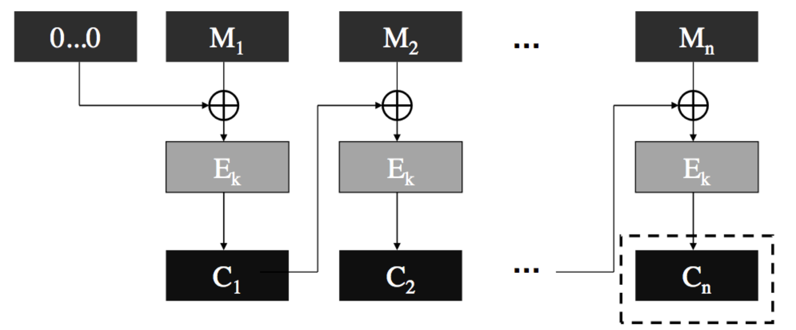

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- does getting MAC("world") help?
- how about getting MAC("adymx")?

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Handling variable-length messages.

- Method 1. Apply PRF to $|m|$ to obtain k'. Compute basic CBC-MAC using k'.
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- Method 2. Prepend message with $|m|$, encoded as n-bit string, compute basic CBC-MAC on resulting message
  - appending length to *end* of message is *not* secure.
- Method 3. choose two different keys (k_1, k_2) as MAC key. Let t := basic CBC-MAC on m using k_1; output tag $\hat{t} = F_{k_2}(t)$
  - advantage: can be used for streaming data with unknown length

Collision-Resistant Hash Functions

"hash functions" used in data structures, e.g. $\mathrm{H} : \{0,1\}^* \to \{0,1\}^{128}$

- take arbitrary-length strings and *compress* them into shorter strings
- given $(\mathrm{name}, \mathrm{record})$, store record in cell $\mathrm{H}(\mathrm{name})$
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- H is collison-resistant if it is infeasible to find collision in H
- only interested in H with input length $>$ output length
- MAC for variable-length message — hash-then-MAC

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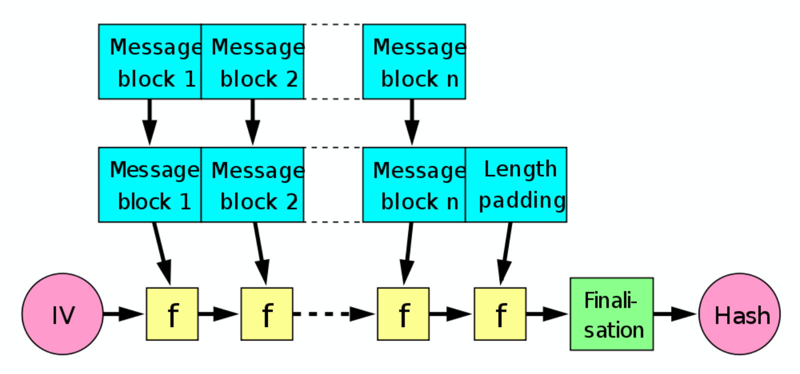
- examples: $\mathrm{MD5} : \{0,1\}^* \to \{0,1\}^{128}$ broken with 2^{64} computations; $\mathrm{SHA1} : \{0,1\}^* \to \{0,1\}^{160}$ broken with 2^{80} computations

Merkle-Damgård transform



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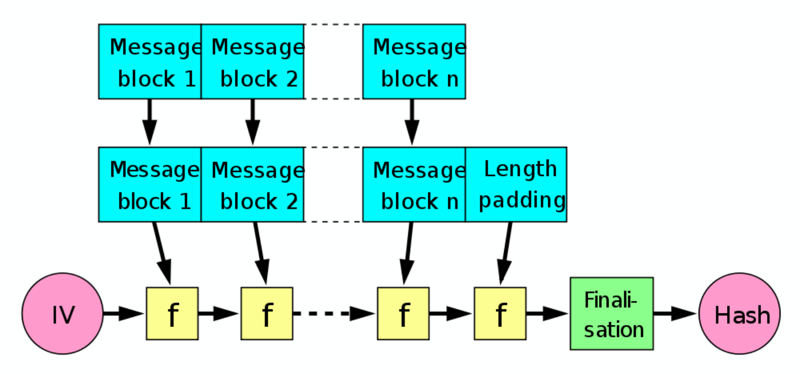

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— intuition: if two strings x, x' collide, then there must be distinct intermediate values that collide