Routing and Packet Scheduling for Throughput Maximization in IEEE 802.16 Mesh Networks

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***Abstract*— This paper considers the problem of maximizing the system throughput in IEEE 802.16 broadband access networks with mesh topology, and the following results are presented. We first consider a simplified linear network with only uplink traffic and provide an optimal scheduling algorithm and establish an analytical result on the length of the schedule. We then consider the problem of routing and packet scheduling in general topology, and show its NP-completeness. We also provide an ILP formulation for this problem. Based on our optimal algorithm for linear networks, we propose algorithms that find routes and schedules of packet transmissions in general mesh topologies. The performance of our proposed algorithms is analyzed using the NS-2 simulator. The results show that the suggested algorithms perform significantly better than other existing algorithms.**

Keywords: IEEE 802.16, Wireless Networks, Mesh Networks, Packet Scheduling, Multi-hop Routing

I. Introduction

The IEEE 802.16 protocol for wireless metropolitan area network (WMAN) has been recently standardized to meet the needs of wireless broadband access. The 802.16d, also known as WiMAX (Worldwide Interoperability for Microwave Access), supports a point-to-multipoint (PMP) topology and a mesh topology. In the PMP architecture, a base station (BS) is connected to the Internet and serves multiple subscriber stations (SSs) using some of the 802.16d standard. A subscriber station can, in turn, serve multiple end customers using some other protocols, such as 802.11 or 802.3. Besides PMP architecture, the IEEE 802.16d provides a multi-hop mesh, which can be deployed as a high speed wide-area wireless network. The mesh topology not only increases the wireless coverage, but also provides features such as lower backhaul deployment cost, rapid deployability and re-configurability.

The IEEE 802.16d MAC layer performs the standard MAC layer function of providing a medium-independent interface to the underlying 802.16 Physical (PHY) layer. The MAC protocol defines how and when a BS or SS may initiate transmission on the channel.

The 802.16 specification provides request mechanisms for bandwidth allocation, however, the detailed scheduling and reservation algorithms are not specified in the standard. Depending upon the varying channel conditions and traffic demands, certain scheduling algorithms may lead to more efficient bandwidth usage than others. This paper addresses the design of routing and packet scheduling with the objective of maximizing the throughput at the base station.

A. Background Work

Authoritative information on 802.16 mesh networks can be found in the official IEEE specification [1].

Algorithms for wireless mesh networks have been proposed in [4], [5] and [6]. While these results are not specifically within 802.16 framework, the insights they provide are helpful nevertheless. In [7], authors have presented the routing and centralized scheduling depending on different traffic models (i.e., CBR, VBR). The authors consider that the routing tree is fixed as a shortest path routing, and the routing tree is more effective in deciding the overall performance of the network. In [8], authors have presented an interference aware routing scheme. The authors considered a blocking metric of a route, which is defined as the sum of blocking metric of all nodes on that route. Blocking metric for a node is defined to be the number of nodes whose transmission would be blocked by that node. In [9], authors have considered the maximum parallelism in order to send as many packets as possible at one time minislot. Whenever a new node comes in the network, the routing tree is adjusted again. Even though, this is the enhancement of [8], our simulation results suggest that the performance may not improve significantly in terms of total transmission time.

In [10], authors have proposed the usage of 802.16 mesh networks as a backhaul for 3G wireless network, and consider the problem of minimizing the number of wireless links while still meeting the bandwidth demands. In [11], authors have presented QoS for IEEE 802.16 Mesh topology, and admission control scheme to ensure the throughput guarantee for the high priority nodes. However, that paper does not present how to schedule the packets to improve the overall throughput.

In [12], authors have presented a routing, channel and link scheduling (RCL) algorithm. In [13], authors have considered the effect of number of channels on the capacity of network and have established tight bounds in random networks. Also of relevance is [14], where the authors have presented that in multi-hop wireless mesh networks, there are many shortest paths that may have very poor performance in terms of throughput and delay. Thus, there is motivation to select the one which also considers interference information as well as bandwidth requirements. Besides the above resources, a survey of wireless mesh networks can be found in [15]. Theoretical foundations of capacity and maximum possible utilization in wireless networks have been presented in [16] and [17].

B. Structure of Paper

The rest of this paper is organized as follows. In Section II, we present an overview of IEEE 802.16 mesh mode followed by the problem statement. In Section III, we present an optimal scheduling algorithm for linear (chain) networks and establish some analytical foundations. These foundations are used in the later sections. In Section IV, we show that the routing and packet scheduling problem is NP-complete for general network topologies. In Section V, we present an ILP formulation for the problem. Following that, several routing and scheduling algorithms are presented in Section VI. Simulation results for our algorithms are presented in Section VII. We present our conclusions in Section VIII.

II. Overview of 802.16 Mesh Mode and Problem Statement

The IEEE 802.16 provides both point-to-multipoint (PMP) and mesh topologies. The main difference between the PMP and mesh modes is that in the PMP mode, traffic only occurs between the BS and SSs, while in the mesh mode traffic can be routed through other SSs and can occur directly between SSs. The algorithm for scheduling the transmission between the BS and SSs, and among the SSs may be done in a distributed manner, in a manner centralized by the BS, or as a combination of both.

Fixed wireless is the base concept for the metropolitan area networking (MAN), given in the 802.16 standard. In fixed wireless, a backbone of base stations is connected to a public network. Each of these base stations supports many fixed subscriber stations. These base stations use the media access control (MAC) layer, and allocate uplink and downlink bandwidth to SSs as per their individual needs on a real-time need basis.

A. Mesh Mode

The mesh mode supports two different physical layers, WirelessMAN-OFDM and WirelessHUMAN. Both of these use 256 point FFT OFDM TDMA/TDM for channel access. The standards also support adaptive modulation and coding where the burst profile of the link (i.e., modulation scheme and the coding rate) and the link rate is changed depending upon the channel conditions. The IEEE 802.16 has a range of up to 30 miles, and can deliver broadband at around 75 megabits per second and provides for non-line of sight access in low frequency bands like 2 - 11 GHz.

Contrary to the basic PMP mode, there are no separate downlink and uplink subframes in the Mesh mode. A mesh frame consists of a control and data subframe. The control subframe serves two basic functions. One is the creation and maintenance of cohesion between the different systems, termed "network control". The other is the coordinated scheduling of data-transfers between systems, termed "schedule control". The data frame is shared between centralized scheduling and distributed scheduling.

Figure 1 depicts the 802.16 mesh frame structure. In a network control subframe, network configuration (MSH-NCFG) and network entry (MSH-NENT) packets provide a basic level of communications for nodes to exchange network configuration information. In the schedule control subframe, the MSH-CSCH and MSH-CFCH packets are used for transmission bursts pertaining to centralized messages, and the remainder is allocated to transmission bursts containing MSH-DSCH packets for distributed scheduling. The data subframe consists of minislots. Minislots, with possible exception of the last minislot in the frame, consist of $\lceil(\text{OFDM symbols/frame} - \text{MSH-CTRL-LEN} \times 7)/256\rceil$ symbols, where MSH-CTRL-LEN is the length of the 802.16 mesh control frame. A scheduled allocation consists of one or more minislots.

B. Problem Statement

In this paper, we consider the problem of routing and scheduling packets based on centralized scheme, that is, the base station acts as a centralized scheduler for the entire network.

We use the following constraints for any transmission:
- A node cannot send and receive simultaneously.
- There may be only one transmitter in the neighborhood of a receiver.
- There may be only one receiver in the neighborhood of a transmitter.

The problem can then be stated as follows.
Routing and Packet Scheduling (RPS) Problem: We are given a graph $G = (V, E)$, where set V consists of base station v_0 and subscriber stations $\{v_1, v_2, \ldots v_n\}$, such that $(v_i, v_j) \in E$ if and only if v_i and v_j are within the transmission range of each other. SS v_i needs to send $w(v_i)$ packets to the base station. The objective is to find a feasible routing tree and a schedule for the packets such that the number of timeslots required is minimized.

III. Optimal Solution for Chain Network

We consider a chain topology IEEE 802.16 multihop network. We only need to consider scheduling, as the routing is fixed. We only consider uplink traffic (symmetric results can also be obtained when downlink only traffic is considered). Consider a chain network G in which the set of nodes and links are $V(G) = \{v_0, v_1, \cdots, v_n\}$ and

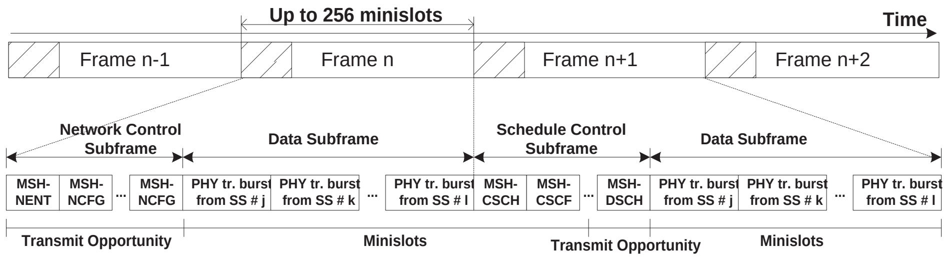

Fig. 1. 802.16 Mesh Frame Structure

$E(G) = \{(v_i, v_{i+1}) \mid 0 \leq i \leq n-1\}$, each node v_i $(1 \leq i \leq n)$ is assigned a positive integer w_i denoting the number of packets to be sent to the base station. Node v_0 denotes the base station. Let $m = \sum_{i=1}^{n} w_i$.

A. Optimal Schedule Length

Given an uplink demand function $W = (w_1, \cdots, w_n)$, we define $W' = (w'_1, \cdots, w'_n)$ such that $w'_n = w_n + 1$ and $w'_i = w_i$ for each i, $1 \leq i \leq n-1$. We then establish the following result.

Theorem 1: The optimal schedule length for W' is defined as: (i) $T_{opt}(W') = T_{opt}(W) + 3$ if $w_n > 0$ and (ii) $T_{opt}(W') = \max\{T_{opt}(W) + 3, n\}$ if $w_n = 0$.

Proof: Let $m = \sum_{i=1}^{n} w_i$ and assign a serial number $i \in \{1, \cdots, m\}$ to each packet such that numbers $1, \cdots w_1$ are assigned to packets originated from v_1, numbers $w_1 + 1, \cdots, w_1 + w_2$ are assigned to packets originated from v_2, and so on. Assume without loss of any generality that packets $1, \cdots, m$ arrive in the base station in the same order. Now, consider W' in which $w'_n = w_n + 1$, and assign number $m+1$ to this new packet.

Suppose $w_n > 0$. Then, for any optimal schedule for W', when packet m arrives in the base station in time $T_{opt}(W)$, packet $m+1$ arrives at node v_3. Hence, packet $m+1$ needs 3 more time slots to arrive in the base station.

Now, assume that $w_n = 0$. In this case, packet $m+1$ arrives in the base station constrained by the previous packets or arrives independently at the earliest time unconstrained by other packets. In the first case, $T_{opt}(W') = T_{opt} + 3$. In the second case, $T_{opt} = n$. This complete the proof of the theorem. ∎

B. Optimal Algorithm **BGreedy**

In each time slot, select the lowest indexed node (say, v_i) which has non-zero uplink demand w_i, and schedule v_i to send one packet to v_{i-1}. Then, find the second lowest indexed node (say v_j) which can send data considering the interference constraints, and schedule that node. Continue until no other node can send data. To illustrate this idea, consider an example of a chain network G where $n = 5$ and the uplink demand function is given as $(w_1, w_2, w_3, w_3, w_4, w_5) = (1, 0, 1, 2, 1)$. Table I shows the load of each node in each time slot, where w_B denotes the number of packets received at the base station.

TABLE I
EXAMPLE: SCHEDULE USING BGREEDY

Time Slot	w_B	w_1	w_2	w_3	w_4	w_5
0	0	1	0	1	2	1
1	1	0	0	2	1	1
2	1	0	1	1	1	1
3	1	1	0	1	2	0
4	2	0	0	2	1	0
5	2	0	1	1	1	0
6	2	1	0	1	1	0
7	3	0	0	2	0	0
8	3	0	1	1	0	0
9	3	1	0	1	0	0
10	4	0	0	1	0	0
11	4	0	1	0	0	0
12	4	1	0	0	0	0
13	5	0	0	0	0	0

Theorem 2: BGreedy algorithm produces an optimal schedule for any uplink demand function $w_1, w_2 \ldots w_n$.

Proof: Let us assume that BGreedy algorithm schedules packets in time $f(w_1, w_2 \ldots w_n)$. We prove the optimality of BGreedy algorithm using induction on the total number of packets. Clearly, the claim is true if there is only one packet, as the schedule produced by the BGreedy algorithm matches the optimal schedule. Let us assume that the claim is true if the total number of packets in the system is at most N.
Now consider a system with total number of packets as $N+1$. Without loss of generality, we assume that node v_n has positive uplink demand. [Any higher numbered nodes that have 0 uplink demand can simply be ignored.]
We consider two cases: (i) $w_n = 1$, and (ii) $w_n > 1$. We observe that in the first case, the packet from v_n either arrives at Base Station independently, or is constrained by other packets. In the first case, the total number of timeslots is simply n. In the second case, the total number of timeslots is $f(w_1, w_2 \ldots w_n - 1) + 3$.

We observe that using the algorithm BGreedy, in the second case, the last packet labeled $N+1$ arrives in the base station

3 slots after packet labeled N arrives, and in the first case, packet $N+1$ arrives in n time slots. Thus, BGreedy is an optimal algorithm. ■

Using Theorem 2, the following result is also established.

Corollary 1: If $w_i = 0$ for all i, $1 \leq i \leq n-1$ and $w_n > 0$ such that $n \geq 3$, then $T_{opt}(W) = 3(m-1) + n$.

The scheduling in chain topology networks was also considered in [8], where an optimal solution was presented in the form of a linear programming solution. This clearly requires higher computational time than BGreedy algorithm which runs in $O(m)$ time for scheduling packets in each time slot. More importantly, the result presented in this paper is the first analytical result on this problem, and the ideas presented here are used in formulation of algorithms for general graphs. As we will see, these analytical results are also central in the proof of NP-completeness for the RPS problem in general graphs.

IV. NP-Completeness of the RPS Problem

To prove the NP-completeness of the RPS problem, we present a polynomial time transformation from the well known 3-PARTITION problem.

3-PARTITION Problem: We are given a finite set $A = \{a_1, \cdots, a_{3m}\}$ of $3m$ elements such that for each i, $1 \leq i \leq 3m$, $a_i \in Z^+$ and $B/4 < a_i < B/2$, where $B = (\sum_{i=1}^{3m} a_i)/m$. The following decision problem is called 3-PARTITION: Can A be partitioned into m disjoint sets $A_1, A_2, ..., A_m$ such that for $1 \leq j \leq m$, $\sum_{a_i \in A_j} a_i = B$?

Note that the above constraints on the element sizes imply that every such A_j must contain exactly three elements from A. We next proceed to show a polynomial time transformation from an instance A to the 3-PARTITION problem to an instance to the RPS problem.

An instance of the RPS has three components: a set of nodes (including the base station), a set of wireless links (a link exists between two nodes within the communication range), and a non-negative integer assigned to each node (denoting the number of packets to be sent in the next frame to the base station.) The set of nodes in the network G (see Figure 2 (a)) is defined as $N(G) = \{BS, x_0\} \cup V \cup Z \cup Y \cup I_Z \cup I_Y$, where BS denotes the base station, x_0 denotes a single node, and (i) $V = \{v_1, \cdots, v_{3m}\}$, (ii) $Z = \{z_1, \cdots, z_m\}$, (iii) $Y = \{y_1, \cdots, y_m\}$, (iv) $I_Z = \cup_{j=1}^{m} I_{z_j}$ where I_{z_j} denotes a set of $l_j - 1$ nodes where $l_1 = 1$ and $l_j = 3jB - 5B - j + 4$ for $2 \leq j \leq m$, and (v) $I_Y = \cup_{j=1}^{m} I_{y_j}$ where I_{y_j} denotes a set of $h_j - 1$ nodes where $h_j = 4jB + (m-2)B - j + 2$ for $1 \leq j \leq m$.

The set of links are defined as $L(G) = \{(x_0, BS)\} \cup E(V, Z) \cup E(Z, BS) \cup E(Y, BS) \cup E(x_0, y_m) \cup E(Y, y_m) \cup E(Z, y_m)\}$, where (i) $E(V, Z) = \{(v_i, z_j) \mid 1 \leq i \leq 3m \text{ and } 1 \leq j \leq m\}$, (ii) $E(Z, BS) = \cup_{j=1}^{m} E(z_j, BS)$ where $E(z_j, BS)$ denotes a set of links in a linear chain of length l_j connecting z_j to BS using $l_j - 1$ nodes in I_{z_j} as intermediate nodes, (iii) $E(Y, BS) = \cup_{j=1}^{m} E(y_j, BS)$ where $E(y_j, BS)$ denotes a set of links in a linear chain of length h_j connecting y_j to BS using $h_j - 1$ nodes in I_{y_j} as intermediate nodes, (iv) $E(x_0, y_m)$ denotes the set of links connection x_0 to all nodes in I_{y_m} except nodes in the sub-chain from y_m to b_0. (See Figure 2 (b)), (v) $E(Y, y_m) = \cup_{i=1}^{m-1} E(y_i, y_m)$ where $E(y_i, y_m)$ denotes the set of links connection y_i^1 (the last node in the chain connecting y_i to BS) to all nodes in I_{y_m} except nodes in the sub-chain from y_m to b_i. (See Figure 2 (c)), (vi) $E(Z, y_m) = \cup_{i=1}^{m-1} E(z_i, y_m)$ where $E(z_i, y_m)$ denotes the set of links connection z_i^1 (the last node in the chain connecting z_i to BS, where $z_i^1 = z_i$ as z_i is connected to BS) to all nodes in I_{y_m} except nodes in the sub-chain from y_m to d_i. (See Figure 2 (d)).

The uplink demand function X is defined such that $w(v_i) = a_i$ for $1 \leq i \leq 3m$, $w(x_0) = mB$, $w(y_i) = 1$ for $1 \leq i \leq m$, and weight zero is assigned to all other nodes.

We call this transformed RPS instance as *RSInstance*.

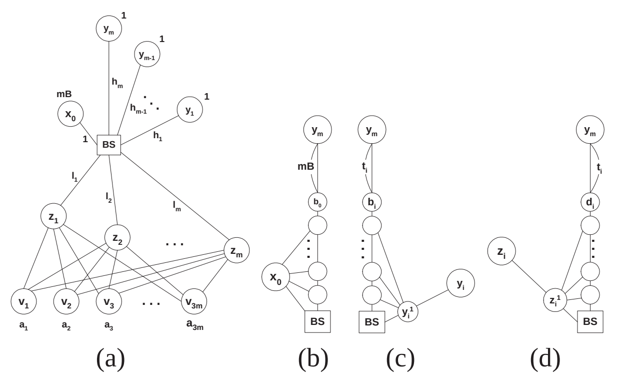

Fig. 2. Transformation from A to G

From the above discussion, we make the following observation (stated in Lemma 1) which will be used in proving the correctness of our transformation stated in Lemma 2. Here, we assume that packets from the same node (as an originator or an intermediate node) must follow the same route; hence, rounting must be tree-based.

Lemma 1: Consider a sub-network of G shown in Figure 2 that only includes nodes $z_1, \cdots, z_m$ and the BS. The length of the chain connecting z_i to BS is such that $l_1 = 1$ and $l_i = 3iB - 5B - i + 4$ for $2 \leq i \leq n$. Assume $w(z_i) = B$ for each i, $1 \leq i \leq m$. Let s_i and f_i denote the time slots in which the first and the last packets from z_i arrive at the BS. By applying our $BGreedy$ algorithm, we then have the following: (i) $s_1 = 1$, $f_1 = B$; (ii) $s_i = f_{i-1} + 2$, $l_i = s_i$, and $f_i = 3(B-1) + l_i$, for $2 \leq i \leq m$.

Proof: Conditions in (i) clearly hold. If there is no interference, then $s = l_i$ and our $BGreedy$ guarantees that $f_i = 3(B-1) + l_i$ from Corollary 1. Therefore, one can easily verify that $s_i = f_{i-1} + 2$ for each i, $2 \leq i \leq m$. ■

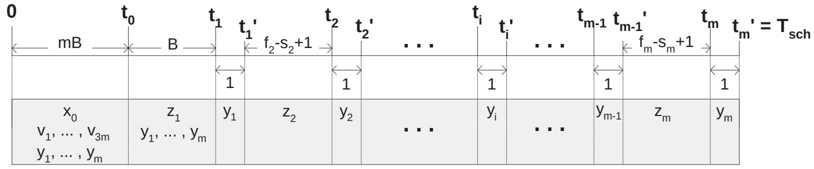

Fig. 3. Schedule for G: $T_{sch} = h_m = 5mB - 2B - m + 2$; $t'_i = h_i = 4iB + (m-2)B - i + 2$ for $1 \le i \le m-1$; $t_0 = mB$, $t_1 = (m+1)B$, and $t_i = mB + f_i$, where $f_i = 3(B-1) + l_i$ where $l_i = 3iB - 5B - i + 4$ for $2 \le i \le m$; and $s_i = t'_{i-1}$ and $f_i = t_i$.

Lemma 2: There exists a desired partition of A if and only if there exists a schedule for *RSInstance* with schedule length $T_{sch} = 4mB - 2B - m + 2$.

Proof: Suppose there exists a desired partition of A. We then construct a schedule as shown in Figure 3 with schedule length $T_{sch} = 4mB - 2B - m + 2$ as follows.
- During the first mB time slots, packets from v_i, $1 \le i \le 3m$, are transmitted to z_i's such that each z_i has received exactly B packets. (Note that nodes v_i's are all interfering each other.) During the same duration, mB packets from x_0 are transmitted to the BS, and the packet in each y_i, $1 \le i \le m$ is also transmitted toward the BS. (Note that no interference exists among these packets during this period, so each packet can continuously move.)
- During the next B slots, B packets from z_1 are transmitted to the BS. (Recall the $l_1 = 1$, so z_1 is one-hop away from the BS.) In this period, packets from $y_1, \cdots, y_m$ are also continuously transmitted toward the BS.
- In time slot t_i for $1 \le i \le m$ as shown in Figure 3, the packet from y_i is transmitted to the BS.
- The remaining packets from $z_2, \cdots, z_m$ are continuously transmitted toward the BS such that the last packet from z_i, $2 \le i \le m$, arrives in the BS in time slot t_i.

Using Lemma 1, one can easily verify that the above schedule is feasible and with schedule length equal to $4mB - 2B - m + 2$.

Now, assume that there exists a schedule for the *RSInstance* with schedule length $4mB - 2B - m + 2$. First, we note that this is a lower bound on any schedule due to the length of the chain from y_m to the base station. In order to achieve this bound, the packet from y_m has to travel to the BS without being interrupted. This imposes restrictions that the packet from each y_i, for $1 \le i \le m-1$, cannot travel after time slot t_i due to the possible interferences with the packet from y_m; hence, it must arrive in the BS by the time slot t_i, which is also its lower bound. Also, packets from x_0 must be transmitted only during the first mB time slots as otherwise they will interfere with the packet from y_m. Finally, we observe that at most B packets can be transmitted from z_i for each i, $1 \le i \le m$, since otherwise additional packets will interfere with the packet from y_m. This establishes a desired 3 partition of A, which completes the proof. ■

The following theorem is now established.

Theorem 3: Routing and Packet Scheduling problem for general graphs is NP-complete.

V. ILP Formulation for Maximum Throughput

The mesh network is composed of one BS and several SSs. Let node v_0 denote BS and node v_i $(1 \le i \le n)$ denote each SS, where n is number of SSs. We assume that the adjacency matrix of the graph is given by $\mathbf{E}$, where $E_{ij} = E_{ji} = 1$ (i.e., links are bidirectional) if and only if nodes v_i and v_j are connected, i.e., within the transmission range, and 0 otherwise. The number of packets to be transferred from node v_i to BS through uplinks is denoted by w_i. We assume that the routing tree is rooted at BS and remains fixed for the duration of one frame length. We further assume that an upper bound on total transmission time, U is known.

1) Variables: We introduce the following variables to represent the routing tree: R_{ij} is 1, if v_j is parent of v_i in routing tree, and 0, otherwise.

As the routing tree can only be a subgraph of the given graph, we enforce the following constraints: $R_{ij} \le E_{ij}$ $\forall i \in \{1 \ldots n\}$, $\forall j \in \{0 \ldots n\}$.

Note that $R_{ij} = 1$ means that node v_j is on the path between the BS and node v_i. Since each node can have only one parent, we have $\sum_{j=0}^{n} R_{ij} = 1$ $\forall i \in \{1 \ldots n\}$.

We introduce binary variables X_{ijt}, where $1 \le i \le n$, $0 \le j \le n$ and $1 \le t \le U$. Each binary variable X_{ijt} takes values defined as: X_{ijt} is 1, if v_i sends a packet to v_j via an uplink at time t, and 0, otherwise.

Since there exits only one uplink at each node, we introduce the following constraints: $X_{ijt} \le R_{ij}$, where $\forall i \in \{1 \ldots n\}$, $\forall j \in \{0 \ldots n\}$, $\forall t \in \{1 \ldots U\}$.

Suppose edges (i_1, j_1) and (i_2, j_2) interfere in the specified graph. In that case, these two edges cannot be part of the simultaneous transmission. The set of these pairwise interfering edges can be deduced from the given graph, and does not depend on any other variables. The interference constraints can be mathematically captured as follows for all pair of edges $(i_1, j_1), (i_2, j_2)$ that interfere: $X_{i_1 j_1 t} + X_{i_2 j_2 t} \le 1$.

Suppose w_{it} represents the number of uplink packets that are to be transferred to BS and is yet remaining in the queue of node v_i at the end of the t timeslot.

Next, we observe that if the node v_i transmits a packet to v_j in timeslot t, then the number of packets at v_i decreases by 1, and the number of packets at v_j increases by 1. Thus, for each node v_j, the packet flow constraints can be specified as: $w_{jt} = w_{j(t-1)} + \sum_i X_{ijt} - \sum_k X_{jkt}$.

Suppose that A_t denotes the total number of packets that have not yet reached root or destination nodes at the end of timeslot t. Therefore A_t may be represented as follows. $A_t = \sum_{i=1}^{n} w_{it}$.

Then the problem is to find a scheduling such that t is minimized where $A_t = 0$. We introduce U more binary variables Y_t, for $1 \leq t \leq U$, and add the following constraints. $\sum_{t=1}^{U} Y_t = 1$ and $A_t \leq A_0(1 - Y_t)$ $\forall\, t \in \{1 \ldots U\}$.

These two equations together imply that exactly one $Y_t = 1$, and that must happen for some timeslot for which $A_t = 0$.

2) Objective Function: To find the smallest such timeslot, we set the objective function to: Minimize $\sum_{t=1}^{U} t\, Y_t$.

3) Complexity Analysis: We observe that the number of variables, as well as the number of constraints depends on U, the upper bound on the number of time slots. As a first order estimate, U may be set to the time of any feasible solution. Obviously, $U \leq \sum_{i=1}^{n} w_i h_i$, where h_i is the shortest distance from node v_i to base station, since one possible schedule is to randomly select one packet in each timeslot, and schedule only that packet in that timeslot. It can be verified that the complete problem instance consists of $O(n^2\, U)$ variables and $O(n^4\, U)$ constraints.

VI. Algorithms for Mesh Networks

In this section, we focus on routing and scheduling algorithms for the RPS problem in general graphs. We propose two novel routing algorithms, and two novel scheduling algorithms. For sake of comparison with other recent works, we also briefly mention a few known algorithms and their generalized versions. Our goal is to present new algorithms, and cover the landscape of known algorithms so that a fair comparison can be presented using simulation results.

A. System Architecture

In our system model, the routing tree (scheduling tree) is constructed in two conditions. First, when a new node enters the network, the scheduling tree is updated according to broadcasting messages (MSH-NCFG and MSH-NENT) from the incoming node. Then, the mesh BS recalculates the routing tree and reconfigures the network by broadcasting MSH-CSCH message to the subscriber stations. Secondly, the BS also periodically recomputes the routing tree by considering updated throughput requirements, and changing the routing tree if required.

B. Routing Algorithms

We present two novel routing algorithms (Maximum Parallelism Routing and Min Max Degree BFS Tree), and also present a generalized version of a known algorithm. To adequately compare these routing algorithms, we also present two known algorithms which have been proposed for 802.16 mesh networks very recently, as well as default routing algorithm specified in IEEE 802.16 standard.

1) Maximum Parallelism Routing: This novel routing algorithm uses the following motivation. It is desirable to maximize the parallelism, and that parallelism must take the number of packets into account as well. By constructing a breadth first tree, we obtain a layered graph. We focus on the set of edges between two consecutive layers. From this set, we can establish the pairs of edges that are interfering, and the pairs of edges that are non-interfering. Further, an edge can be assumed to be "weighted" by the number of packets on the sender node. We select the set of edges, such that considering pair-wise non-interfering edges in that set, the sum of weights on the edges is maximized. Formally, suppose U_i is the set of nodes on i-th layer and U_{i+1} is the set of nodes on $(i+1)$-th layer, and the set of edges between the two layers is $E = \{e_j :\ 1 \leq j \leq m\}$. We select the set $S \subseteq E$, such that objective function $f(S)$ is maximized, where $f(S) = \sum_{e_j,\, e_k \in S} \{w(e_j) + w(e_k) \mid e_j, e_k$ non-interfering $\}$.

We also enforce the implicit constraint that the set of edges uses each vertex from set U_{i+1} exactly once, but it can use a vertex from set U_i more than once. This corresponds to the constraint that a node can have more than one child, but can have only one parent node in the routing tree.

We observe that maximum parallelism routing as defined above has the nice property that it takes both the interference graph and the traffic conditions into consideration. Consequently, the maximum parallelism routing can change on the basis of change in traffic conditions, even if the interference graph does not change. We believe this observation is vital, as in initial implementations of 802.16, interference graph is expected to remain fairly static, though of course traffic conditions vary as part of end user's activities. Thus, it is important for routing tree to adjust based on the traffic conditions.

2) Min Max Degree BFS Tree: In this routing algorithm, we consider a breadth first tree, such that the maximum degree of that tree is minimized. The motivation of this algorithm is to combine the benefit of the shortest path (breadth first tree) with having least bottlenecks.

We observe that the problem of finding a BFS tree such that maximum tree is minimized can be solved by treating each pair of adjacent layers as independent. Consider adjacent layers U_i and U_{i+1}, minimum degree subgraph that matches each vertex from U_{i+1} has a degree between $\lceil \frac{|U_{i+1}|}{|U_i|} \rceil$ and $|U_{i+1}|$. Thus, it can be found efficiently using $O(\log(|U_{i+1}|))$ invocations of the flow maximization problem.

3) Interference Aware Routing (Wei et al.): For comparison purposes, we include the interference aware routing scheme presented by Wei et al in [8]. In that paper, the authors considered a blocking metric of a route, which is defined as the number of nodes whose transmission would be blocked by that route. Please refer to [8] for more details.

4) Concurrent Transmission (Tao et al.): Also, for comparison purposes, we include the routing scheme presented by Tao et al in [9]. In that paper, the authors attempt to maximize the parallelism as well as adjust the routing tree upon change in network configuration. In the sense of adjustment, this work is an enhancement of [8], which does not adjust the routing tree that has already been created.

5) Improved Interference Aware Routing (Generalization of Wei et al.): We extend the interference aware routing scheme presented by Wei et al in [8]. In that paper, the authors considered a blocking metric of a route, which is defined as the sum of blocking metric of all nodes on that route. Blocking metric for a node is defined to be the number of nodes whose transmission would be blocked by that node. We extend this idea, and also take the number of packets into account by defining the blocking metric of the node v to be the number of blocked nodes multiplied by the number of packets at the node v. Formally, $B(P) = \sum_{v \in P} B(v)$.

The blocking metric for a node v used in [8] is: $B(v) =$ Number of nodes blocked by v. We suggest the modified blocking metric as: $B(v) =$ (number of nodes blocked by v) $\times$ (number of packets at v).

Finally, the path with the minimum blocking metric is selected, as $P' = \arg\min B(P)$.

6) Random Routing: For comparison purposes alone, we also include the random routing that is the default implementation mentioned in the IEEE 802.16 standard. In this routing, a random spanning tree rooted at the base station is selected. In the results presented by [8] and [9], random routing has been used as a comparison point.

C. Scheduling Algorithms

In this section, we present scheduling algorithms, that are used to transmit the data using the routing tree created using one of the algorithms presented in Section VI-B. All our scheduling algorithms consider one simplification that in one timeslot, only uplink or downlink packets are considered.

1) Fair Queuing (Max Total Weight): Instead of randomly picking up nodes, this algorithm considers number of packets information in scheduling. It schedules the maximum weighted(number of packets) schedulable node from the entire network, in turn, schedules the next maximum schedulable node, and etc, until there is no more schedulable node. The algorithm is presented in Table II.

TABLE II

MAX TOTAL WEIGHT ALGORITHM

```
For each time slot t
  S ← φ
  while(true)
    n ← maximum weighted schedulable node
    if n is NULL then break
    Add n to S
  endwhile
  Transmit packets from all nodes in S
endfor
```

2) Max Weight: This algorithm extends Fair Queuing algorithm further by considering the distance of the node to the BS. It resembles *BGreedy* algorithm, and schedules the layer closest to BS first, then schedule the following layers in turn. The maximum weighted node in the same layer is scheduled first, and then the second maximum weighted schedulable node is scheduled, until there is no schedulable node in that layer. The algorithm is presented in Table III.

TABLE III

MAX WEIGHT SCHEDULING ALGORITHM

```
For each time slot
 S ← φ
 for each layer from top to bottom
  while (true)
   n ← maximum weighted schedulable node
   if n is NULL then break
   Add n to S
  endwhile
 endfor
 Transmit packets from all nodes in S
endfor
```

3) Line Scheduling: We further extended above algorithm by considering fairness of each node. Since maximum weighted nodes are always scheduled first, lower weighted nodes may have to wait long time to be scheduled. This algorithm schedules the network by scanning the entire network using lines. The algorithm applies *BGreedy* algorithm to the path to achieve optimal result. For the remaining nodes not in the path, the algorithm applies Max Weight algorithm to maximize the parallelized transmission. The algorithm is presented in Table IV.

4) Random Scheduling: Random scheduling algorithm is presented solely for the purpose of comparison, as it is the default implementation mentioned in the IEEE 802.16 standard. Further, this algorithm has been used as the benchmark in two recent papers [8] and [9]. Random scheduling randomly chooses one schedulable node in the network, schedules it and considers another schedulable node, until there is no schedulable node that has not been considered. The exact algorithm is presented in Table V.

TABLE IV

LINE SCHEDULING ALGORITHM

```
For each time slot t
 S ← φ
 l ← number of leaf nodes in routing tree
 //For each leaf node, there is a unique path to BS
 p ← t modulo l
 Add all schedulable nodes in p-th path
   to S according to BGreedy algorithm
 Apply Max Weight algorithm to add the
   remaining unscheduled nodes to S
 Transmit packets from all nodes in S
endfor
```

TABLE V

RANDOM SCHEDULING ALGORITHM

```
For each time slot
   S ← φ
       While (there is schedulable node)
          Add the schedulable node to S
       endwhile
       Transmit packets from all nodes in S
endfor
```

VII. SIMULATION RESULTS

We have evaluated the performance of our algorithms by comparing the ratio of timeslots over the throughput (i.e., the total number of time slots used in the scheduling divided by the total number of packets received at BS). We extended NS-2 simulator by implementing simplified 802.16 MAC layer protocol. In the simulation, we ignored control subframe part of the protocol. We used a shared data structure, that is used by nodes to share the scheduling information. We assumed that exactly one packet is transmitted in one time slot, and all packets are sent at same channel rate. We conducted simulations with five different scenarios. The topologies that we have considered are random mesh networks with 50 nodes that are randomly located in the test area. Since the positions of nodes are randomly generated, there exist some nodes that are not connected with BS even through other nodes.

In the simulation, we assume that the packet arrivals at each node follow Poisson distribution, and the mean interval between two consecutive packet arrivals for each node is randomly chosen from $0.004s$ and $1s$, since it is reasonable to assume that some nodes may not have data to send at some period of time.

Comparison of various routing algorithms is presented in Figure 4 (a). Here, the scheduling algorithm used is line scheduling, presented in Section VI-C.3. To provide a comparison that is independent of our scheduling algorithms, we also present results of various routing algorithms using random scheduling in Figure 4 (b). We observe that the Maximum Parallelism Routing algorithm performs significantly better than other known routing algorithms. It performs slightly better than other routing algorithms presented in this paper. Min Max Degree BFS tree routing algorithm also performs better than previously known routing algorithms, and only slightly worse than Maximum Parallelism Routing algorithm.

Comparison of various scheduling algorithms, using maximum parallelism routing is presented in Figure 4 (c). We observe that the line scheduling algorithms outperforms other scheduling algorithms. Specifically, the three suggested algorithms: Line Scheduling, Max Weight Scheduling and Max Total Weight Scheduling all perform significantly better than random scheduling algorithm.

Comparison of combinations various scheduling algorithms is presented in Figure 4 (d). We observe that the combination of line scheduling and maximum parallelism routing algorithms achieves the best results.

To analyze the optimality of proposed algorithms, we have implemented the ILP formulation presented in Section V in the CPLEX solver. Due to the high time complexity of the ILP formulation, we run the CPLEX using eight SSs with total number of packets 138 in which the routing tree has 3 layers (i.e., depth 3). The optimal number of timeslots from the ILP formulation is 156 while other algorithms requires 174-243 timeslots. The results are shown in Figure 4 (e) and (f).

VIII. CONCLUSIONS

In this paper, we have considered the problem of maximizing the system throughput in IEEE 802.16 broadband access networks with mesh topology. We firstly consider a linear chain network and provided an optimal scheduling algorithm. We then showed that finding an optimal packet scheduling for a general topology is NP-complete even for a tree-based routing. To the best of our knowledge, these two results are the first analytical results for this problem, though various interesting heuristic algorithms have been proposed in the past.

We have presented routing and packet scheduling algorithms, and have reasoned as to how they can provide better performance than other known algorithms. Firstly, the algorithms that we have presented take both interference and traffic conditions into account. Secondly, the algorithms use known optimal solutions for the chain topology. We have also presented detailed simulation results using NS-2 simulator. The results show that our algorithm performs significantly better than other existing algorithms.

A key conclusion that has not escaped our attention is that considering only interference is not sufficient for good routing and scheduling algorithms. The traffic conditions must also be utilized, and the routing should be recomputed even if there is no change in the network configuration.

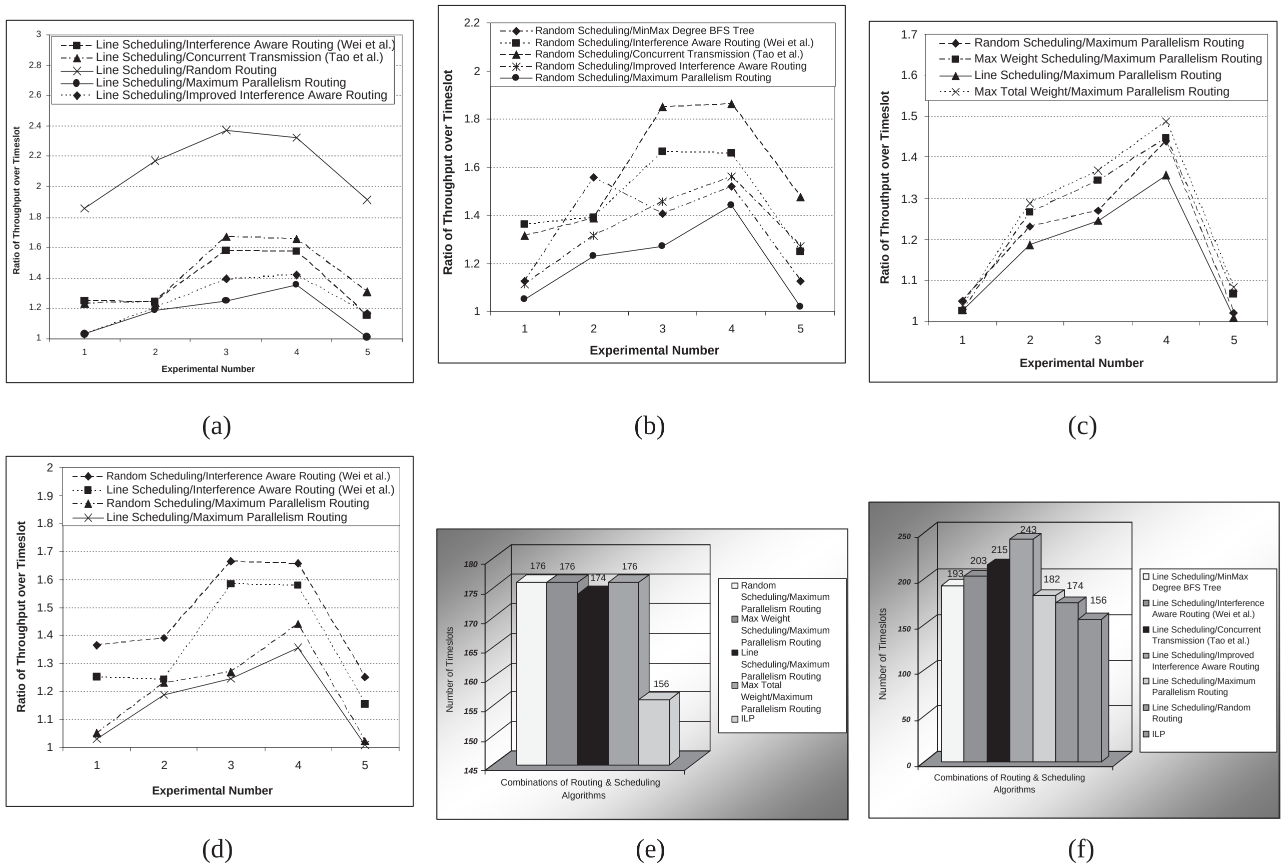

Fig. 4. (a) comparison of routing algorithms, using Line Scheduling, (b) comparison of routing algorithms, using random scheduling, (c) comparison of scheduling algorithms, using Maximum Parallelism Routing, (d) comparison of combination of routing and scheduling algorithm, (e) & (f) optimality analysis.

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