Propagation of Uncertainty in a Simulation-Based Maritime Risk Assessment Model Utilizing Bayesian Simulation Techniques

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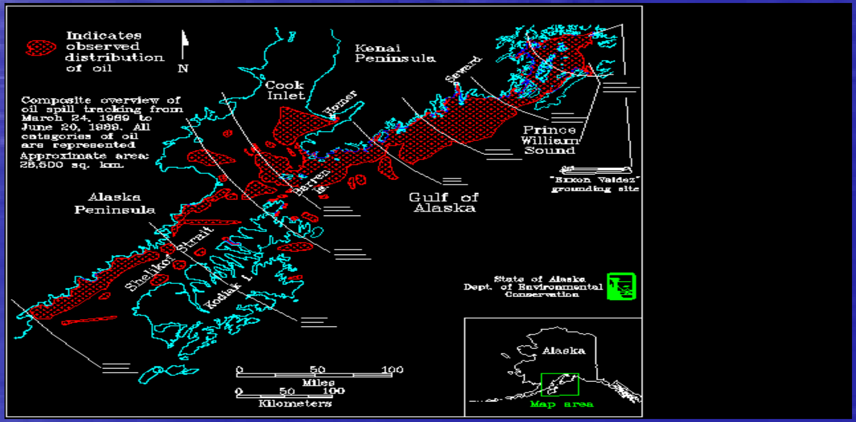
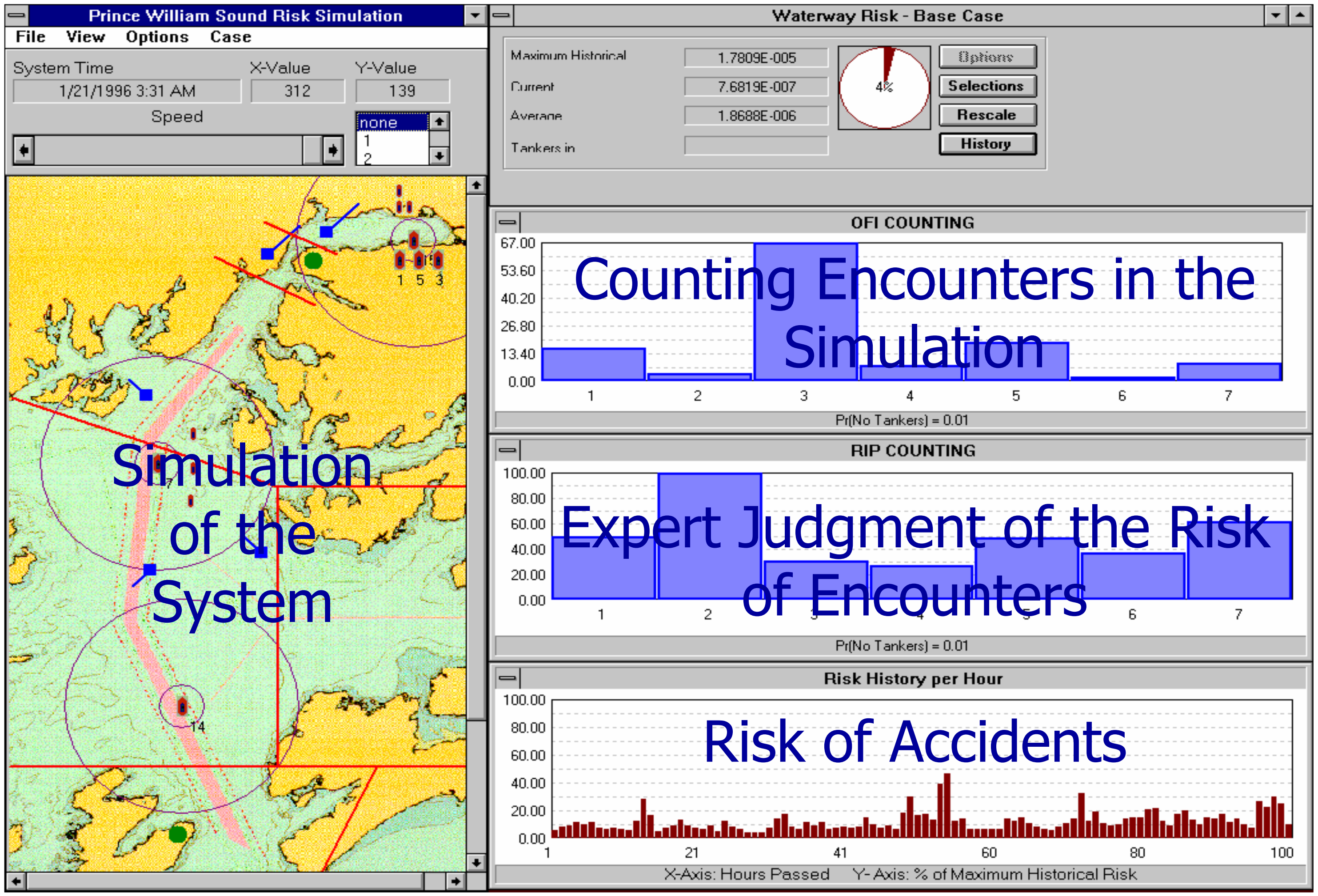
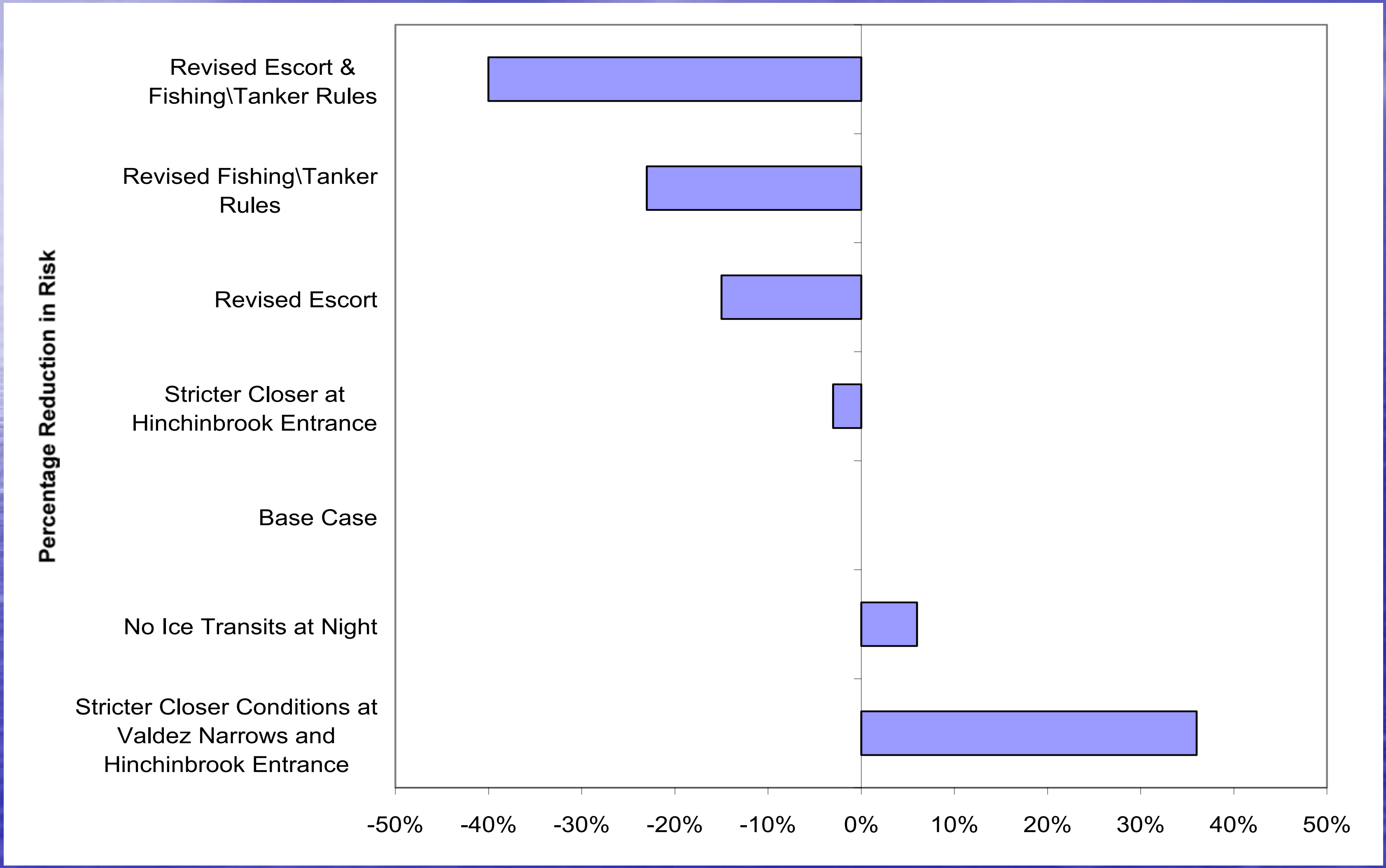
Prince William Sound Risk Assessment



Assessing Maritime Risk



Evaluating Risk Reduction



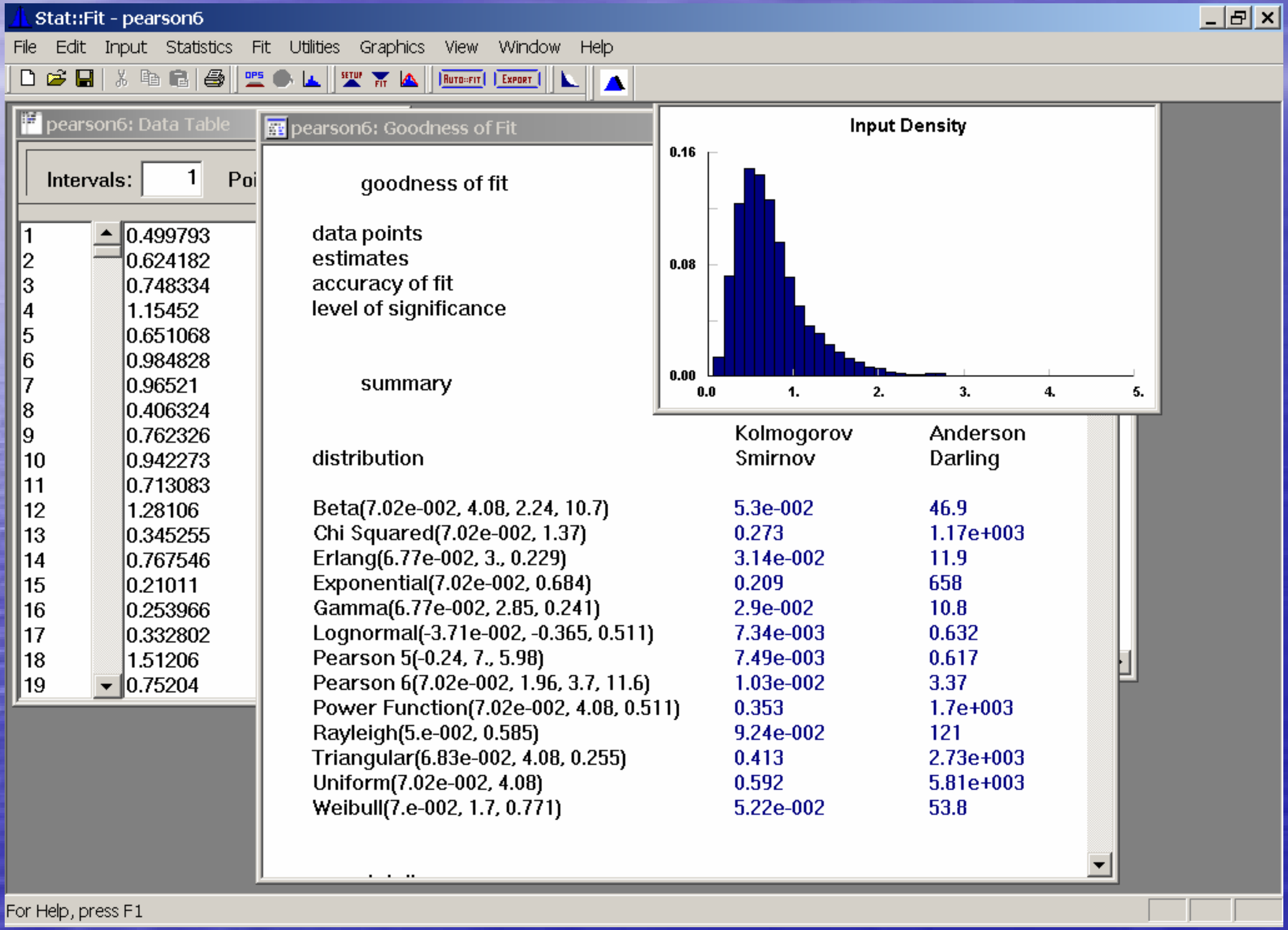
National Research Council Review

- *"The truth is that we are uncertain. The language of uncertainty is probability Therefore, speaking the truth means to develop analyses results in terms of probability curves rather than in terms of point estimates."*
  - Kaplan S. "The Words of Risk Analysis", *Risk Analysis*, 1997; 17(4) 407-417.

Modeling the Uncertainty

- Uncertainty in the System Simulation
  - Bayesian Simulation Model
- Uncertainty in the Expert Judgments
  - Bayesian pair-wise comparison model
- Propagate the Uncertainty throughout the Model
  - Monte Carlo methods
  - Grid Computing implementation

Classical Simulation Input Modeling



Bayesian Simulation Input Modeling

- Computations are simple with conjugate prior distributions

Probability Model	Conjugate Prior-Posterior
Exponential	Gamma
Log Normal	Normal-Gamma
Gamma	???
Weibull	???

- To sample inter-arrival times in the simulation
  - Sample from the posterior distribution of the parameters
  - Sample from the probability model given the sample parameters

Bayesian Inference

- Conjugate prior distributions are not always available
  - No close form solution
  - Multiple model parameters => multivariate posterior
- Answer - Gibbs Sampling
  - Sample from posterior, full-conditional distributions of each parameter repeatedly
  - This Markov Chain has the multivariate posterior distribution as its limiting, stationary distribution
  - Start at arbitrary values and iterate until warmed up

How do you choose the best probability model?

- Traditionally Bayes factors and posterior predictive densities
  - Deviance Information Criteria is a newer development

Bayesian Deviance	$D(\Theta_j^k) = -2\ln p(D^k \mid \Theta_j^k) + 2\ln f(D^k)$
Expected Deviance	$\overline{D} = E[D(\Theta_j^k) \mid D^k]$
Exp. Nos. Parameters	$p_D = E[D(\Theta_j^k) \mid D^k] - D(E[\Theta_j^k])$
DIC	$DIC = \overline{D} + p_D$

  - Spiegelhalter, D. J., N. G. Best, B. P. Carlin, A.van der Linde. 2002. Bayesian measures of model complexity and fit. *Journal of the Royal Statistical Society: Series B* **64**(4) 583-639.

Deviance Information Criterion

	Expected Deviance	Nos. Parameters	DIC
exponential	877.778	1.010	878.787
Weibull	838.202	1.750	839.952
Gamma	837.349	2.025	839.374
log normal	847.737	1.999	849.736

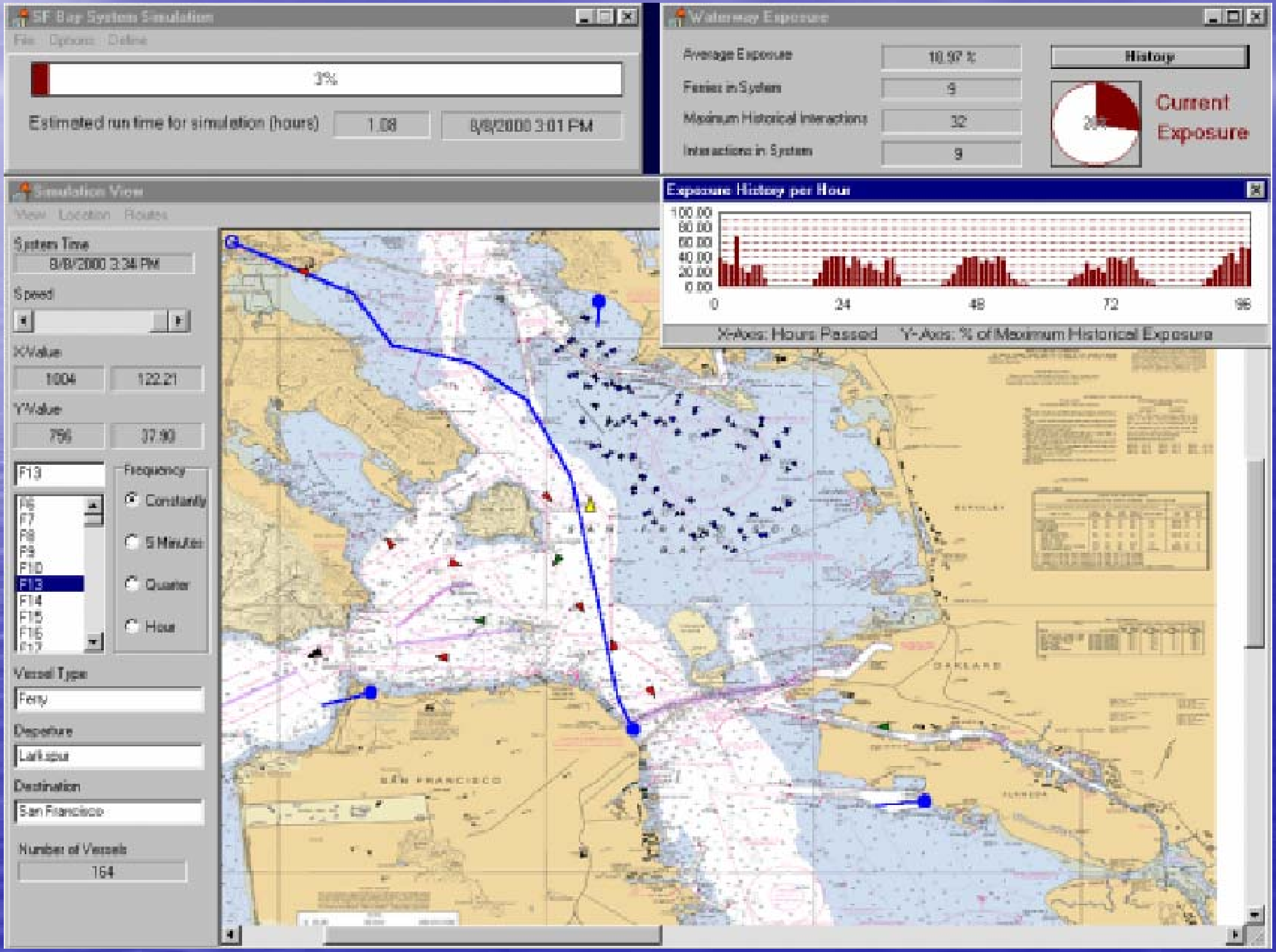
Bayesian Output Model

- Our output is a count of the number of encounters per year
  - Poisson distribution is a natural probability model
  - Gamma distribution is the conjugate prior distribution for the expected number of encounters per year
  - Prior shape → Prior Shape + Total Encounters
  - Prior scale → Prior Scale + Nos. Simulated Years
- We can run 50 years of simulation and collect the total number of simulated encounters!

Example – Ferries in SF Bay

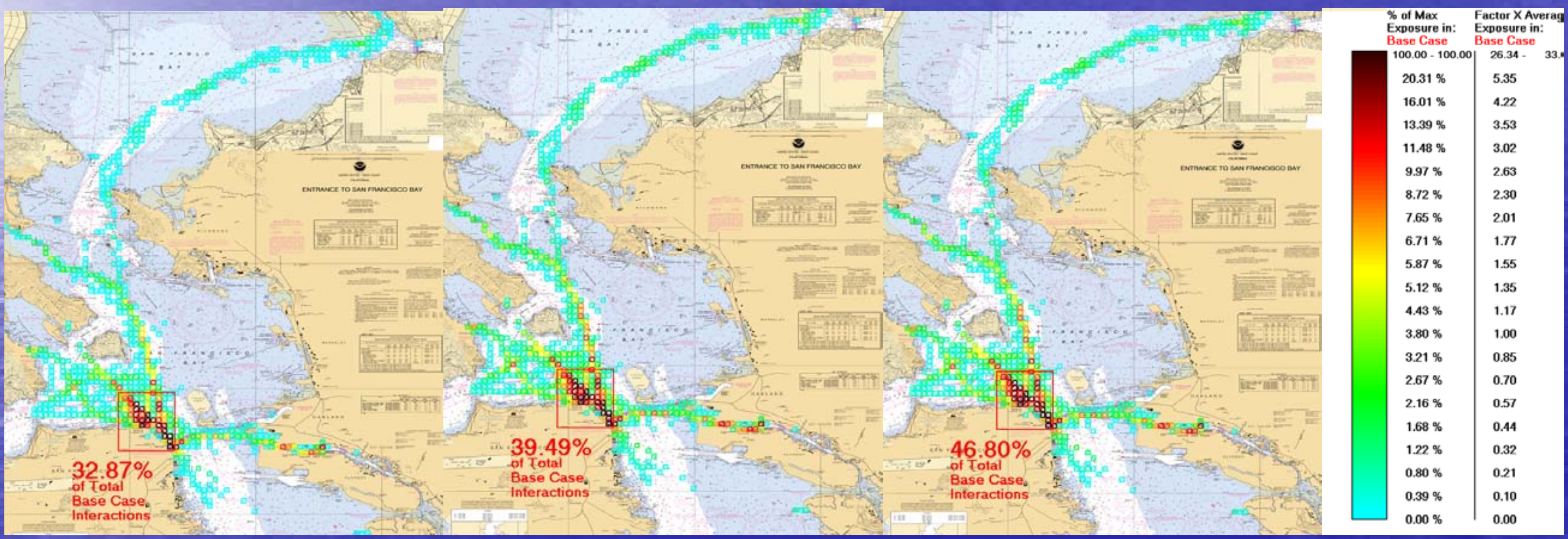
- Three proposed expansion alternatives
  - Alternative 3: Enhance Existing System
  - Alternative 2: Robust Water Transit System
  - Alternative 1: Aggressive Expansion
- These are to be compared to the existing ferry system operating in SF Bay

SF Bay Simulation



Results with Uncertainty

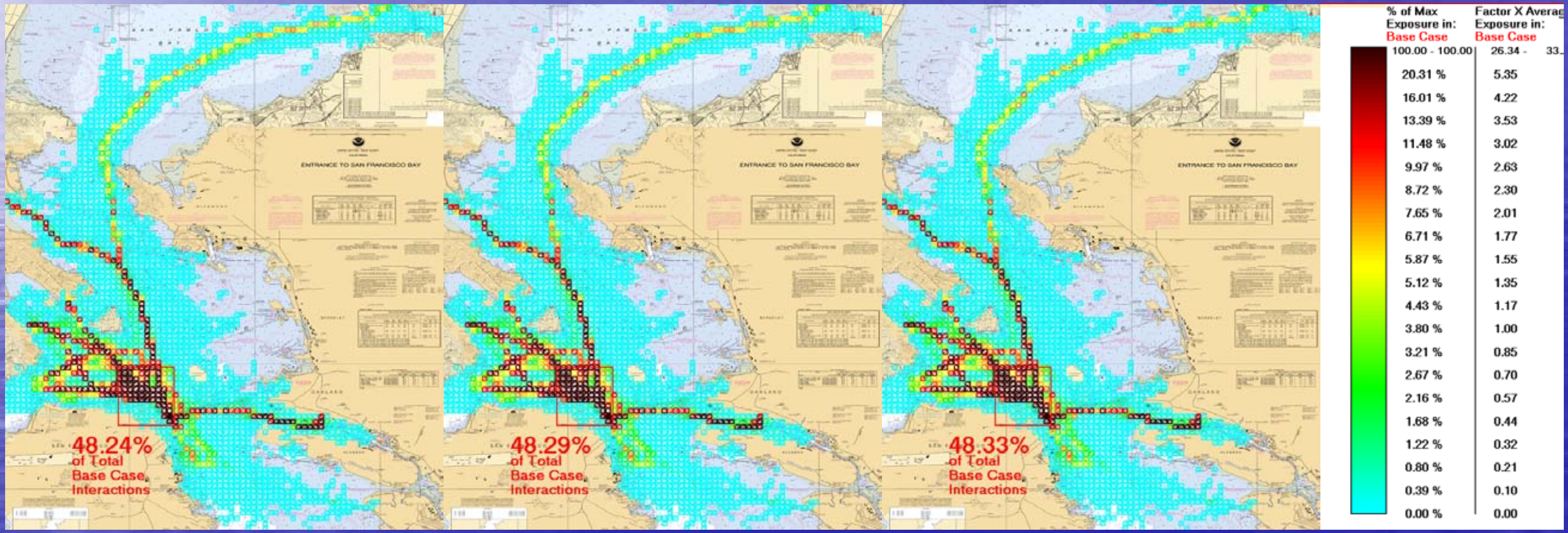
- After just one day of simulation



Posterior 5-th percentile | Posterior 50-th percentile | Posterior 95-th percentile

Results with Uncertainty

- After 50 years of simulation



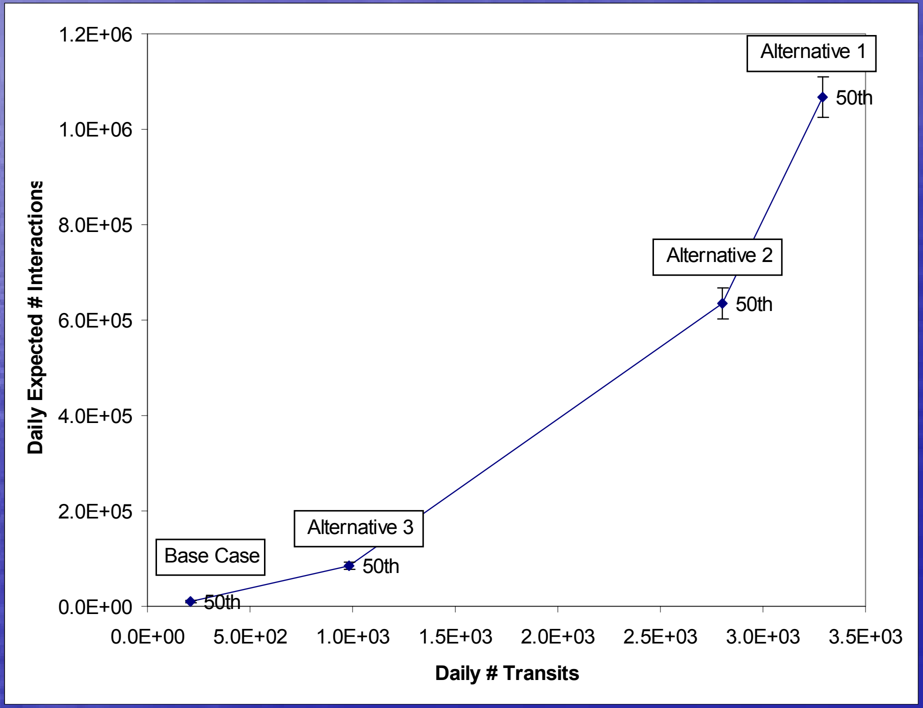
Posterior 5-th percentile

Posterior 50-th percentile

Posterior 95-th percentile

Results with Uncertainty

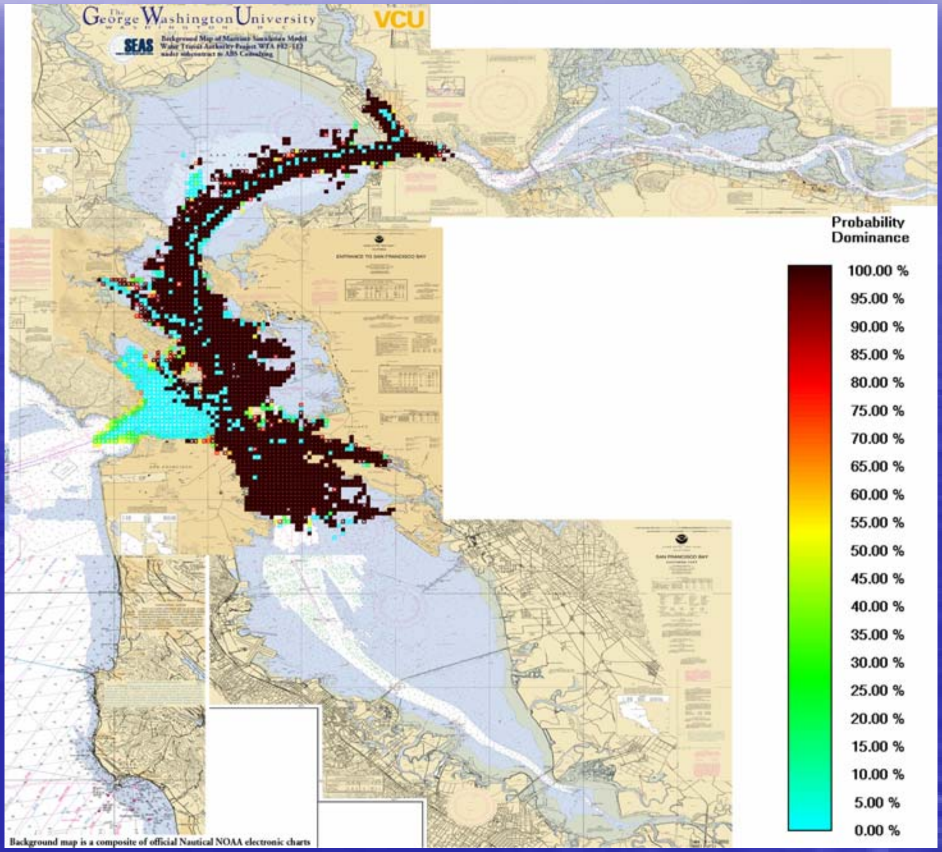
- After 1 day of simulation



  - After 50 years of simulation the credibility interval bars are not visible

Results with Uncertainty

- What is the probability that there will be more interactions in Alternative A than in Alternative B across the study area?
  - A = Alternative 3
  - B = Current Ferry System



Results with Uncertainty

- What is the probability that there will be more interactions in Alternative A than in Alternative B across the study area?
  - A = Alternative 1
  - B = Alternative 2