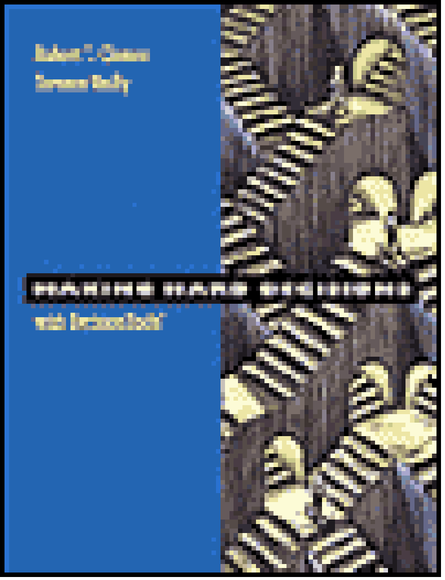

Making Hard Decisions

R. T. Clemen, T. Reilly

Chapter 5

Sensitivity Analysis

Draft: Version 1

Introduction

Sensitivity analysis is used through the entire modeling process

Purpose of sensitivity analysis:

- To analyze what **really matters** in the decision problem
- To construct a **requisite** decision model

Examples of sensitivity analysis techniques in DA:

- Determine if **deterministic** dominance or **stochastic** dominance is present
- Identifying the important variables through **tornado diagrams**
- Identify **interaction effects** between important variables
- Identify the importance of **probability assessments** (which are also variables).

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The Eagle Airline Case

Dick Carothers wants to expand his operation.

Mid West has to offer:

- An airplane @ price $95000.
  (He can probably by the plane for $85K-$90K)
- An option to buy the airplane a year later
  (Cost of the option $2.5 – $4k)

Currently:

- Eagle Airlines (=Dick Carothers) owns 3 plains
- 60% of flights are chartered flights and 40% are scheduled

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The Eagle Airline Case

Cost Data Mid West Plane:

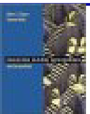
- New Engines, FAA Maintained
- Contains all equipment that Eagle Airlines needs
- Has 5 seats
- Operation Cost: $245 per hour
- Fixed Cost: $20k (=Yearly Insurance) + Finance Charges

Finance Charges:

- Borrow 40% of the price at 2% above the prime rate (=9.5%, but subject to change).

Revenue Data:

- Chartered Flights: $300 - $400 per hour
- Scheduled Flights: $100 per person per hours, plains are on average 50% full
- Expected number of hours flown with new plane 800-1000.



The Eagle Airline Case

Variables in control:
- The price he is willing to pay.
- The amount financed.

Variables not in control:
- Insurance Cost
- Operation Cost

Carothers could always invest his cash **$52,500 @8% yearly interest rate**, yielding an annual interest in the first year of: **$4200**

What should Dick Carothers do?

Sensitivity Analysis: Problem Identification Level

Are we solving the right problem?

Error of the 3rd kind: Solving the wrong problem.

How to avoid this error?

- Continue to be **skeptical** about the problem on the surface being the real problem

Eagle Airlines Case:

Carothers wants to expand his operation. The fact that he owns an airline company does not mean he has to expand by buying another plane. He could, for example, expand by **investing in computer industry.**

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Sensitivity Analysis: Problem Structure Level

Are any of pieces of the puzzle missing?
Is this a single or multiple objective problem?

Sensitivity Analysis: Dominance Considerations

- Ask whether one alternative could end up better than another. If not, ignore that alternative.

Eagle Airline Case:

"Buying the option" is considered never better than **"Buying the plane"** alternative since asking price a year from now will be adjusted to be **similar** and hence "Buying the option" simply **adds additional cost.**

(You are not learning any thing new by waiting in this case)

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Sensitivity Analysis: Importance of variables

Back to the Eagle Airlines Case:

Objective: Maximize Profit. Consider Annual Profit, Ignore Taxes

- **Annual Profit** =
  Annual Total Revenue – Annual Total Cost

- **Total Revenue** =
  Revenue from Charters + Revenue from scheduled flights

- **Total Cost** = Variable Cost + Fixed Cost

Sensitivity Analysis: Problem Structure Level

Revenue from Charters:

(Charter Ratio)*(Hours flown per year)*Charter Price

Revenue from Schedules Flights:

(1-Charter Ratio)*(Hours flown per year)*(Ticket price per hour)*(Number of Seats)*(Average Occupancy)

Fixed Cost:

Insurance + (Purchase Price)*(% Financed)*(Interest Rate)

Variable Cost:

(Hours flown per year)*(Operating Cost)

Step 1: Determine a range for every decision variable and a best guess (Low, Base, High) and calculate as a first cut sensitivity analysis, **the output variable** using first all the **low values** for **the input variables** and second all the **high values** for the **input variables**.

Sensitivity Analysis: Problem Structure Level

	Low	Base	High
Hours Flown	500	800	1000
Charter Price per hour	$300.00	$325.00	$350.00
Ticket Price per hour	$95.00	$100.00	$108.00
Occupancy Rate on Scheduled Fligh	40.00%	50.00%	60.00%
% of Charter Flights	45.00%	50.00%	70.00%
Operating Cost per Hour	$230.00	$245.00	$260.00
Insurance	$18,000.00	$20,000.00	$25,000.00
Proportion Financed	30.00%	40.00%	50.00%
Interest Rate	10.50%	11.50%	13.00%
Purchase Price	$85,000.00	$87,500.00	$90,000.00

Revenue From Charters	$67,500.00	$130,000.00	$245,000.00
Revenue From Scheduled Flight	$52,250.00	$100,000.00	$97,200.00
Fixed Cost	$20,677.50	$24,025.00	$30,850.00
Variable Cost	$115,000.00	$196,000.00	$260,000.00

	Using Low Values	Using Base Values	Using High Values
Total Revenue	$119,750.00	$230,000.00	$342,200.00
Total Cost	$135,677.50	$220,025.00	$290,850.00
Total Profit	-$15,927.50	$9,975.00	$51,350.00

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Is this a worst case – best case analysis?

One-Way Sensitivity Analysis

STEP 2:

1. Select a particular variable (= free variable)
2. Set all other variables to their best guesses (=base values)
3. Set free variable to its lowest value and calculate payoff
4. Set free variable to its highest value calculate payoff
5. Set free variable to some intermediate values and calculate payoff
6. Draw results in a one way sensitivity analysis graph

Eagle Airlines Case: Fix all variables, except hours flown.

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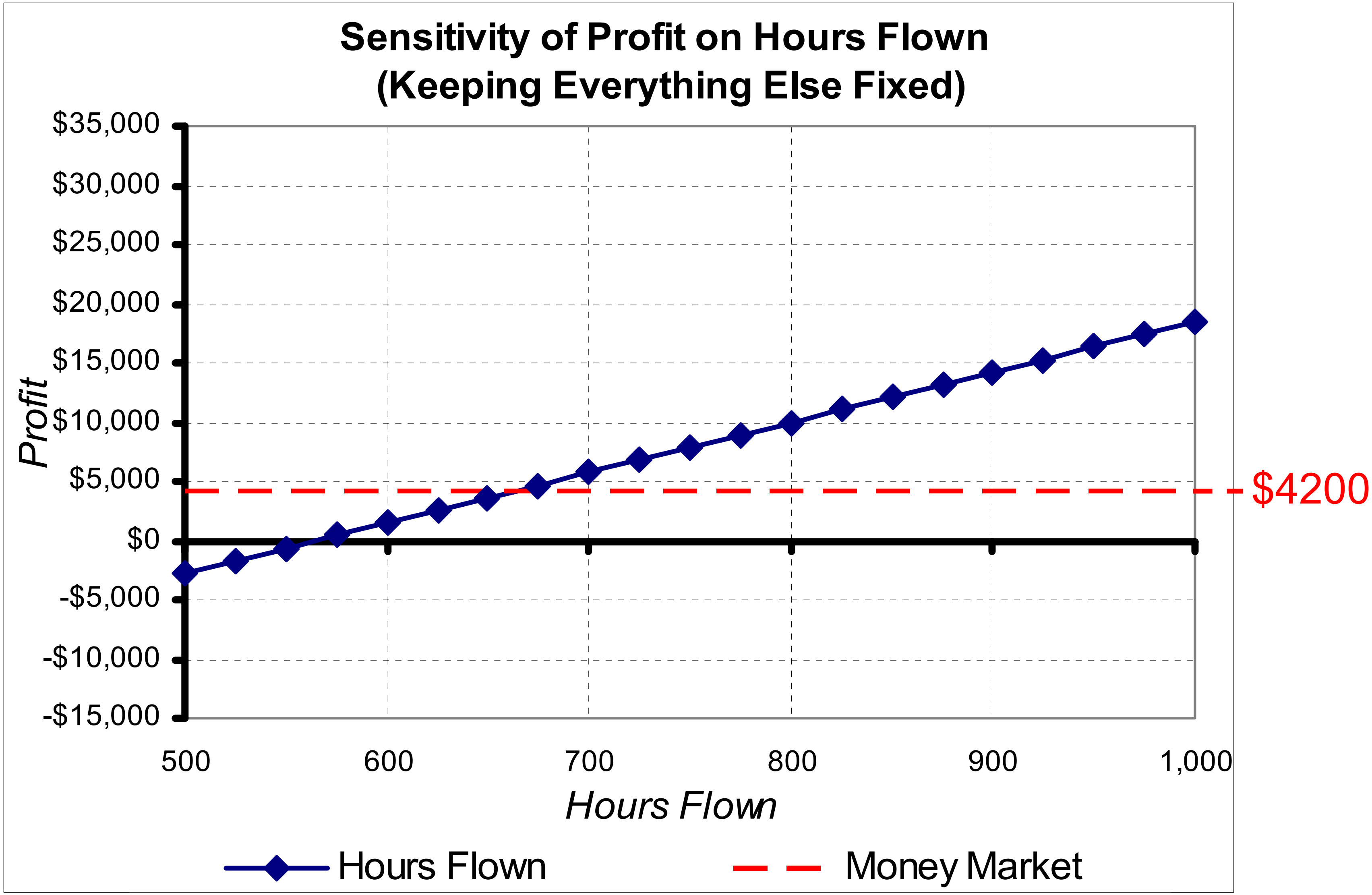
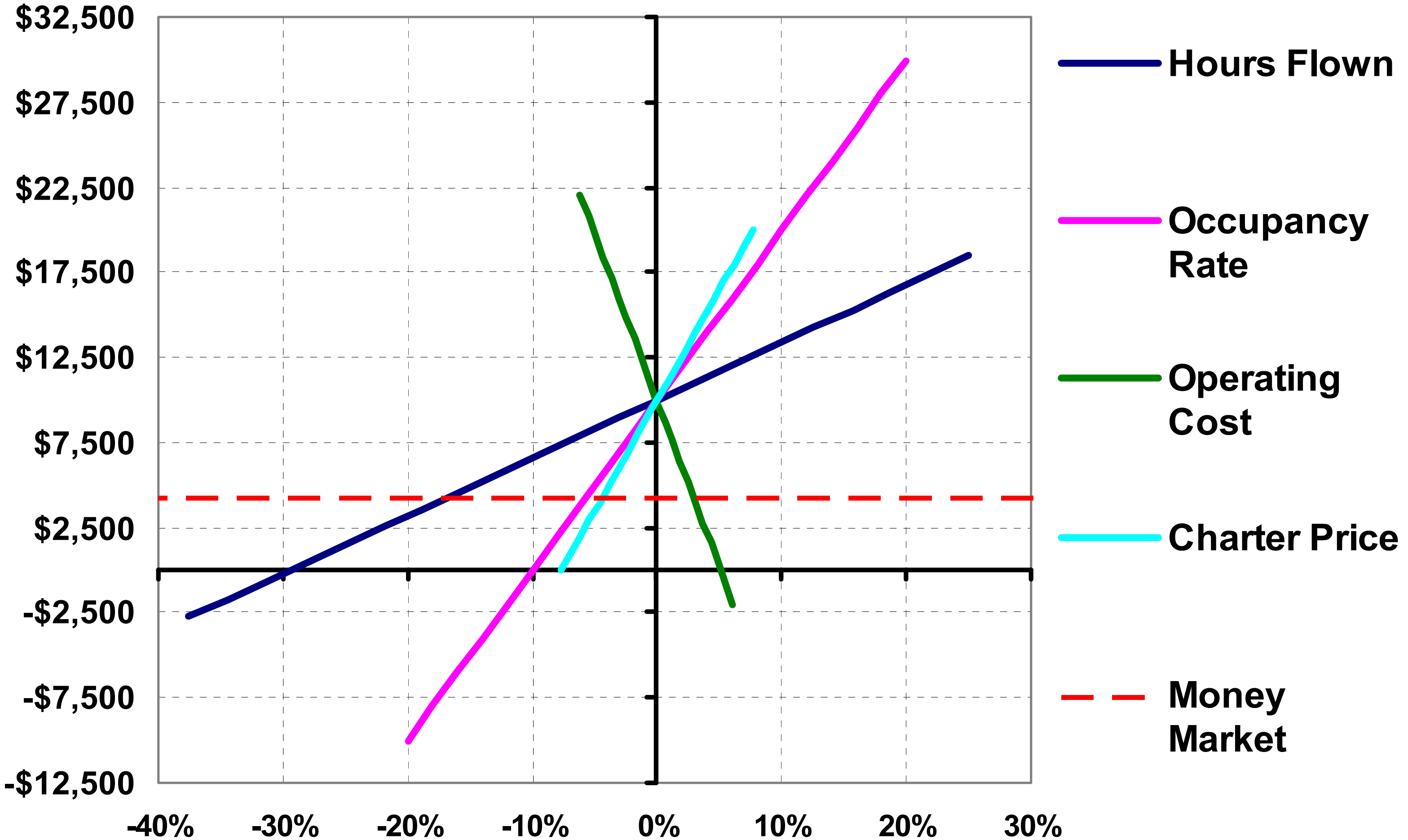
One-Way Sensitivity Analysis



One-Way Sensitivity Analysis

STEP 3: Perform a one-way sensitivity analysis for all variables and plot results in a **Spider Diagram**.



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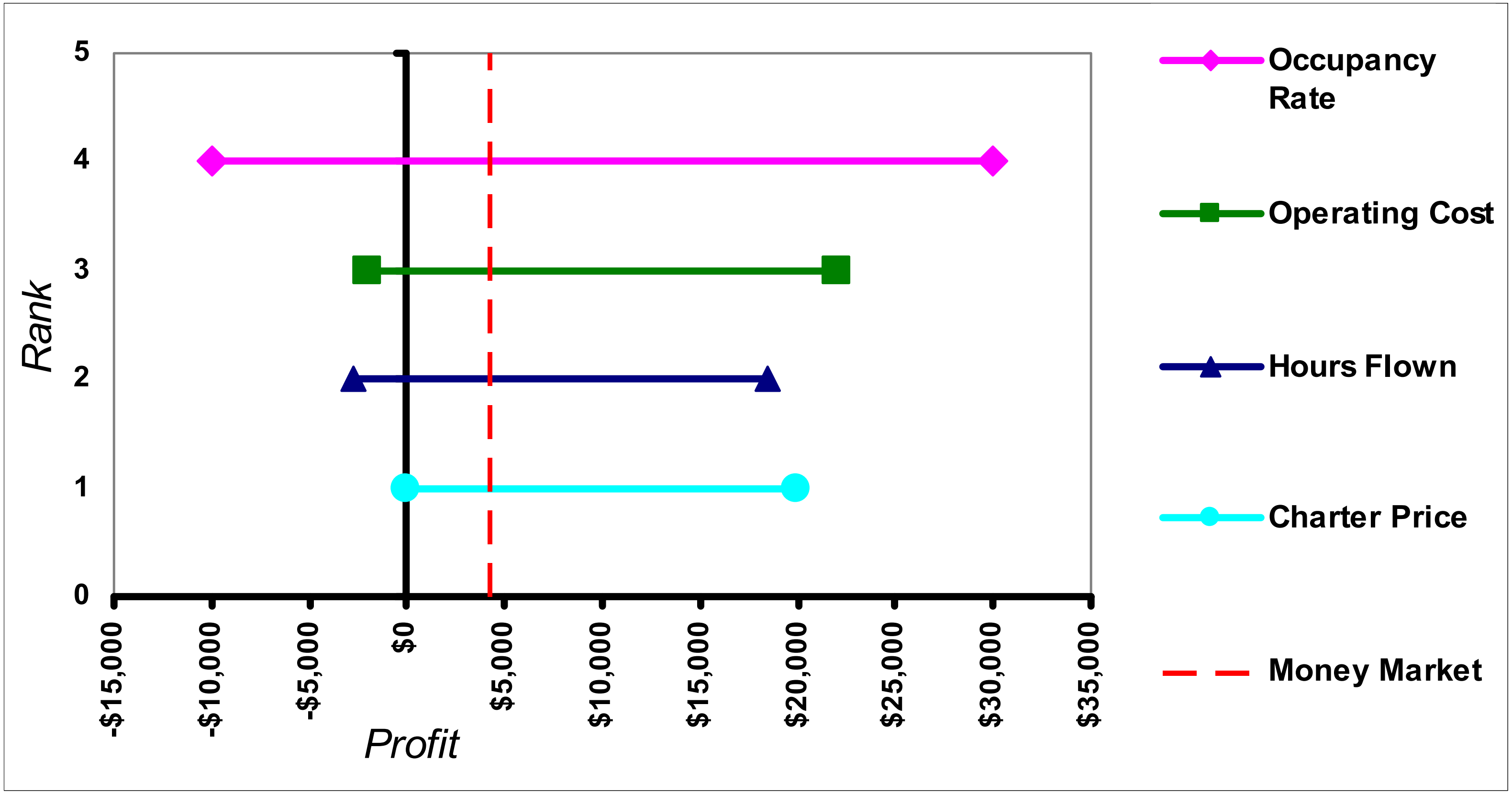
One-Way Sensitivity Analysis

STEP 4: Calculated payoff range is a measure of uncertainty in payoff due to uncertainty in the free variable. Plot the payoff ranges in a **Tornado Diagram** and visually determine the important variables.



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Two-Way Sensitivity Analysis

One-way sensitivity analysis ignores the effect of changing **multiple** variables at the same time

"One Way" under estimates sensitivity due to:

- **Additive effects** of varying more than one variable
- **Multiplicative effects** of varying more than one variable

Eagle Airlines Case:
Tornado Diagram indicates that **Occupancy Rate (OR)** and **Operating Cost (OC)** on scheduled flight are critical (suggesting a two-way sensitivity analysis of these variables)

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Eagle Airlines Case:

1. Determine **Annual Profit (AP)** as a function of OR and OC:

AP = R*H*CP + (1-R)*H*TP*NPS*OR-H*OC-I-PP*F*IR

2. Set all other parameters at their **base values**, yielding

AP(OR,OC) = $130000 + $200000*OR-800*OC-$24025

3. For what values of OR and OC is "buying the plane" worse than "putting money in the savings account". Hence for what values of OR and OC is the following true?

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Two-Way Sensitivity Analysis

$$AP < \$4200 \Leftrightarrow$$

$$\$130000 + \$200000*OR-800*OC-\$24025 < \$4200 \Leftrightarrow$$

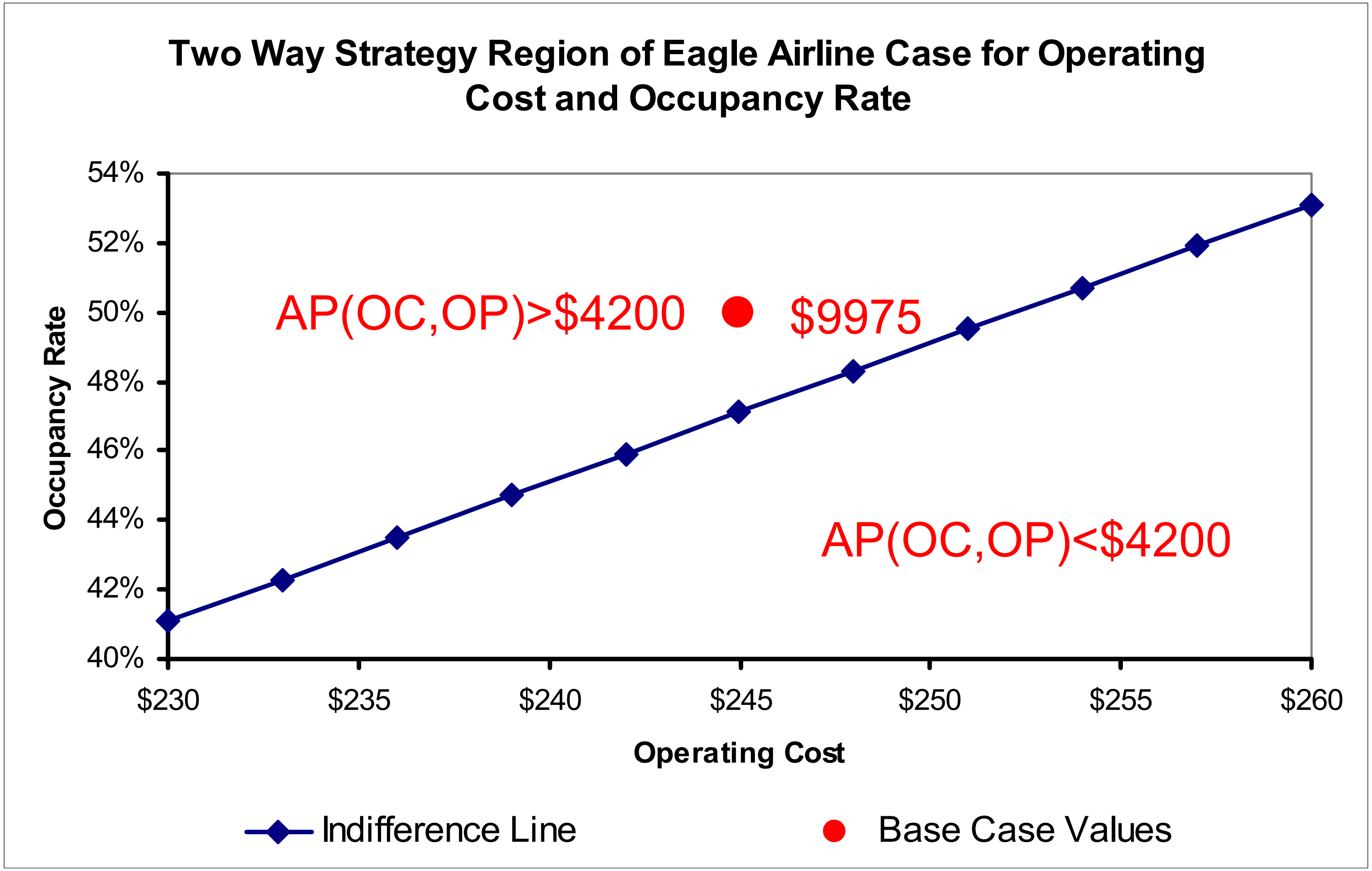
$$\$200000*OR < \$800*OC - \$101775 \Leftrightarrow$$

- First, draw graph of values of OR and OC such that one is **indifferent** between "buying the plane" and "the saving account".

$$OR = 0.004*OC - 0.509$$

- Second, determine which alternative is preferred above and below the **indifference curve**.

Two-Way Sensitivity Analysis



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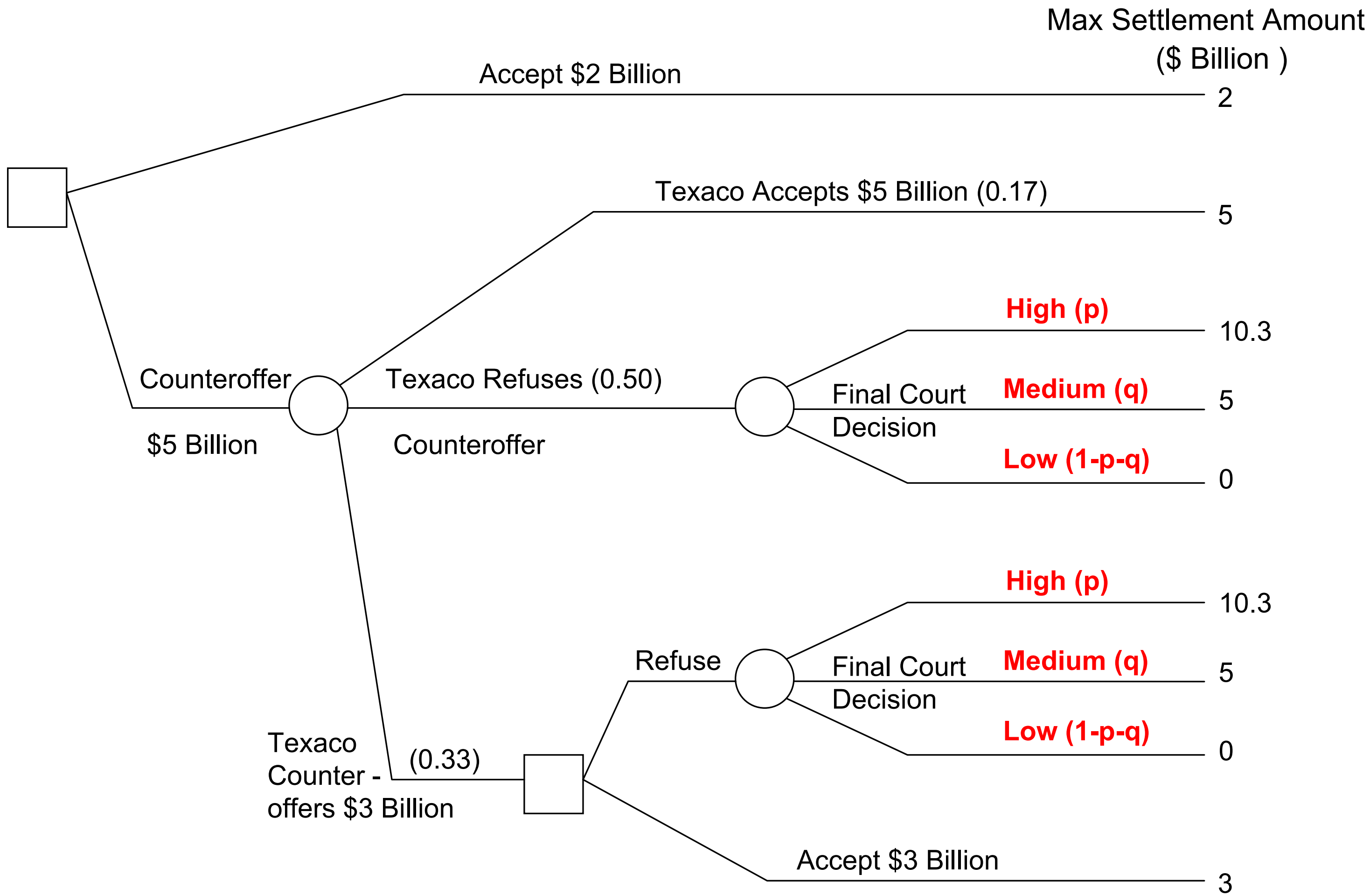
Two-Way Sensitivity Analysis: Texaco - Pennzoil



Liedtke is **unsure of court probabilities**. If Liedtke thinks that **p must be more than 0.15 and q must be more than 0.35** can he make the decision without further probability assessment?

Two-Way Sensitivity Analysis: Texaco - Pennzoil

Step 1: Create a two-way sensitivity graph that shows optimal strategies for Liedtke for all possible values of p and q.

Strategy A = Accept $2 billion.

Strategy B = Counteroffer $5 billion, then refuse if Texaco offers $3 billion.

Strategy C = Counteroffer $5 billion, then accept if Texaco offers $3 billion.

$EMV(A) = 2$

$EMV(B) = 0.17\,(5) + 0.5\,[p\,10.3 + q\,5 + (1-p-q)\,0] + 0.33\,[p\,10.3 + q\,5 + (1-p-q)\,0]$

$= 0.85 + 8.549\,p + 4.15\,q.$

$EMV(C) = 0.17\,(5) + 0.5\,[p\,10.3 + q\,5 + (1-p-q)\,0] + 0.33\,(3)$

$= 1.85 + 5.15\,p + 2.5\,q.$

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Now construct three inequalities:

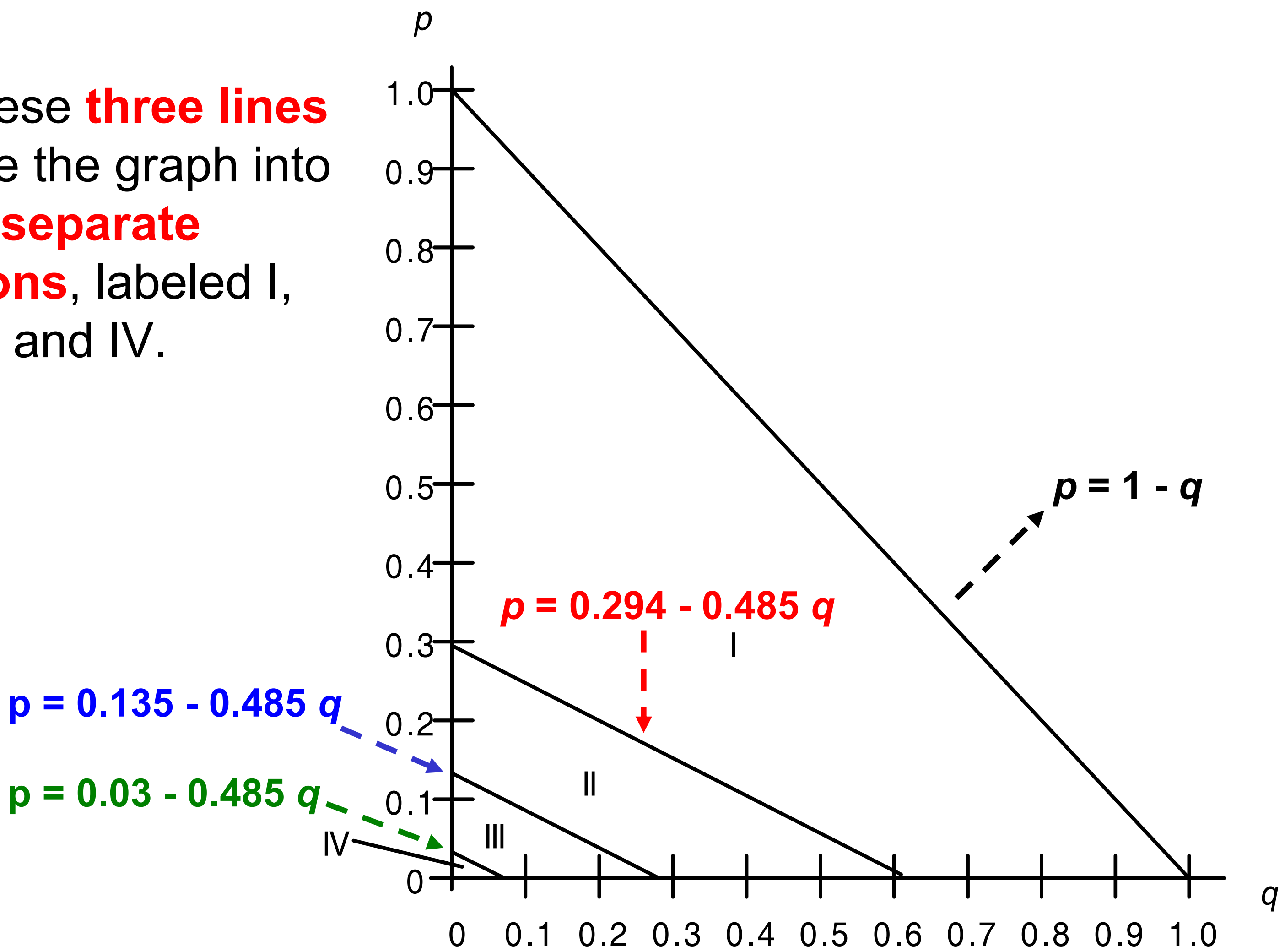
- **EMV(A) > EMV(B)** $\Leftrightarrow$
  $2 > 0.85 + 8.549\ p + 4.15\ q \Leftrightarrow$
  $0.135 - 0.485\ q > p$. **(1)**

- **EMV(A) > EMV(C)** $\Leftrightarrow$
  $2 > 1.85 + 5.15\ p + 2.5\ q \Leftrightarrow$
  $0.03 - 0.485\ q > p$. **(2)**

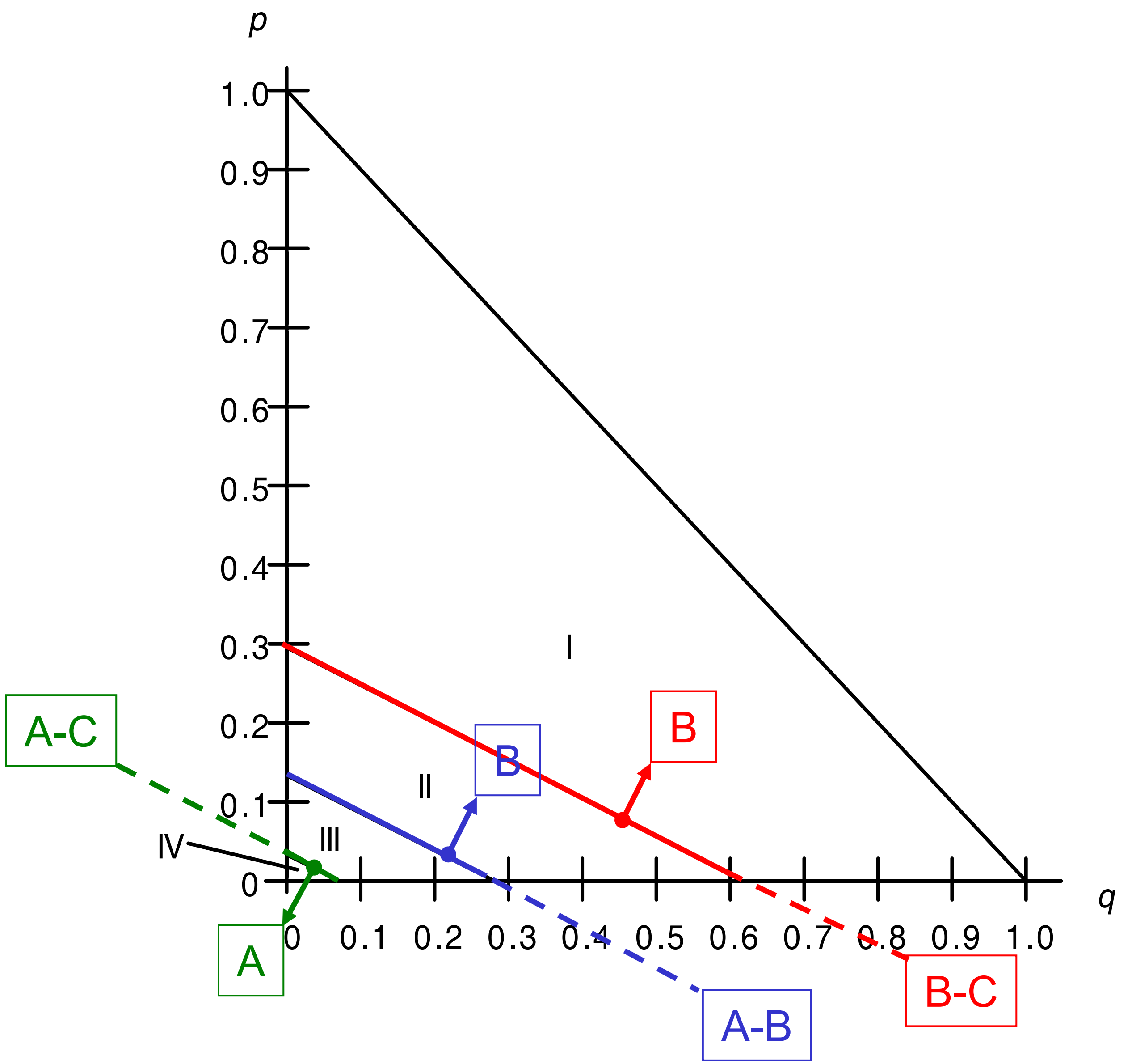
- **EMV(B) > EMV(C)** $\Leftrightarrow$
  $0.85 + 8.549\ p + 4.15\ q > 1.85 + 5.15\ p + 2.5\ q \Leftrightarrow$
  $0.294 - 0.485\ q < p$. **(3)**

Plot **three indifference lines** on a graph with p on the vertical axis and q on the horizontal axis. Note that only the region **below the line p + q = 1 is feasible** because p + q must be less than or equal to one.

- These **three lines** divide the graph into **four separate regions**, labeled I, II, III, and IV.



Two-Way Sensitivity Analysis: Texaco - Pennzoil



Two-Way Sensitivity Analysis: Texaco - Pennzoil

Inequality (3) divides regions I and II. For points above this line, $p > 0.294 - 0.485\ q$, and so EMV(B) > EMV (C).

Inequality (1) divides regions II and III. For points above this line, $p > 0.135 - 0.485\ q$, and EMV(B) > EMV(A). As a result of this, we know that B is the preferred choice in region I and that C is the preferred choice in region II [where EMV(C) > EMV (B) > EMV(A)].

Inequality (2) divides regions III and IV. For points above this line, $p > 0.03 - 0.485\ q$, and EMV(C) > EMV (A). Thus, we now know that C is the preferred choice in region III [where EMV(C) > EMV(A) and EMV(C) > EMV(B)], and A is preferred in region IV.

Thus, we can redraw the graph, eliminating the line between regions II and III

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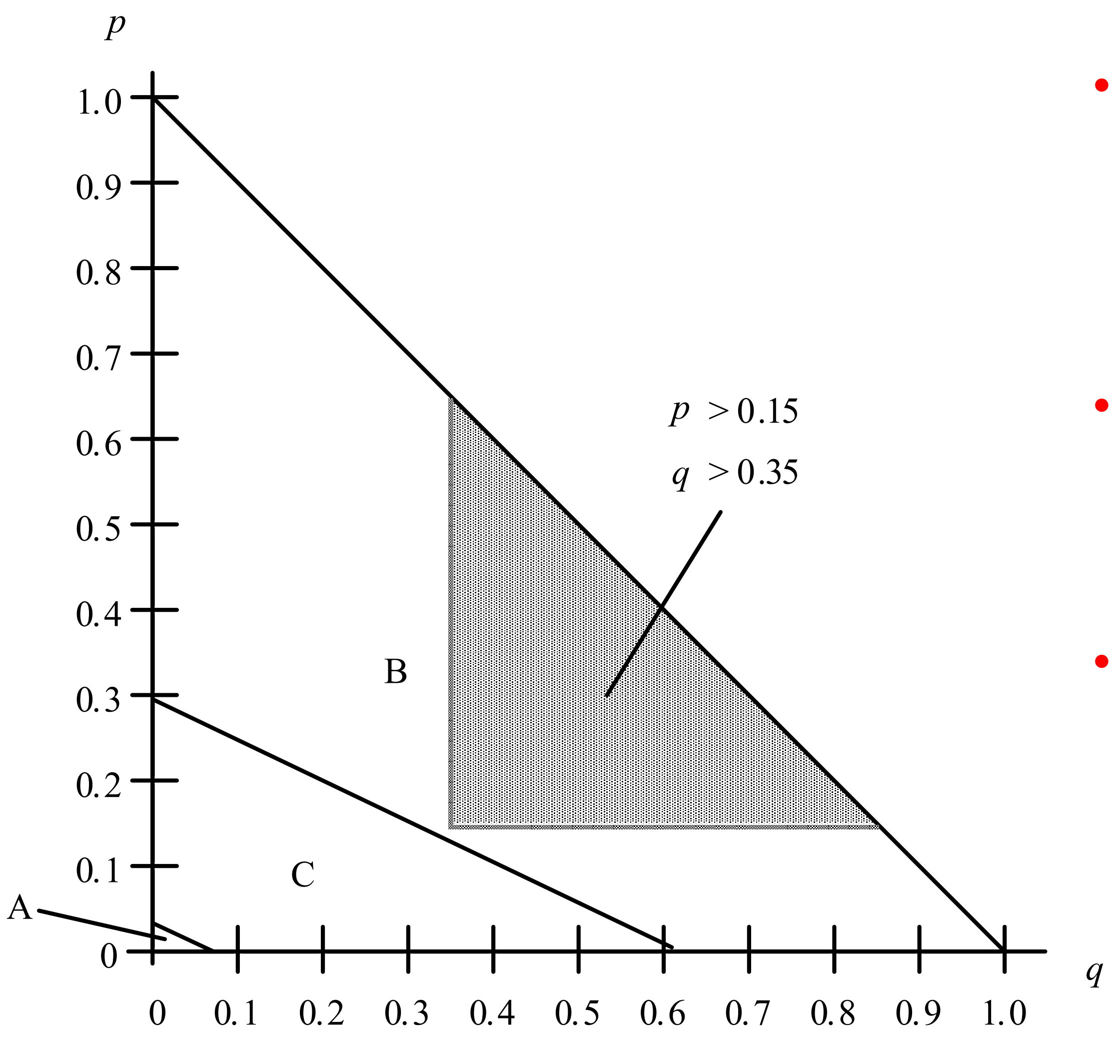
Two-Way Sensitivity Analysis: Texaco - Pennzoil



- The shaded area in the figure represents those points for which **p > 0.15 and q > 0.35.**
- Note that all of these points fall in the "Choose B" region.
- Thus, Liedtke should adopt strategy B:

Counteroffer \$5 billion, then refuse if Texaco offers \$3 billion.

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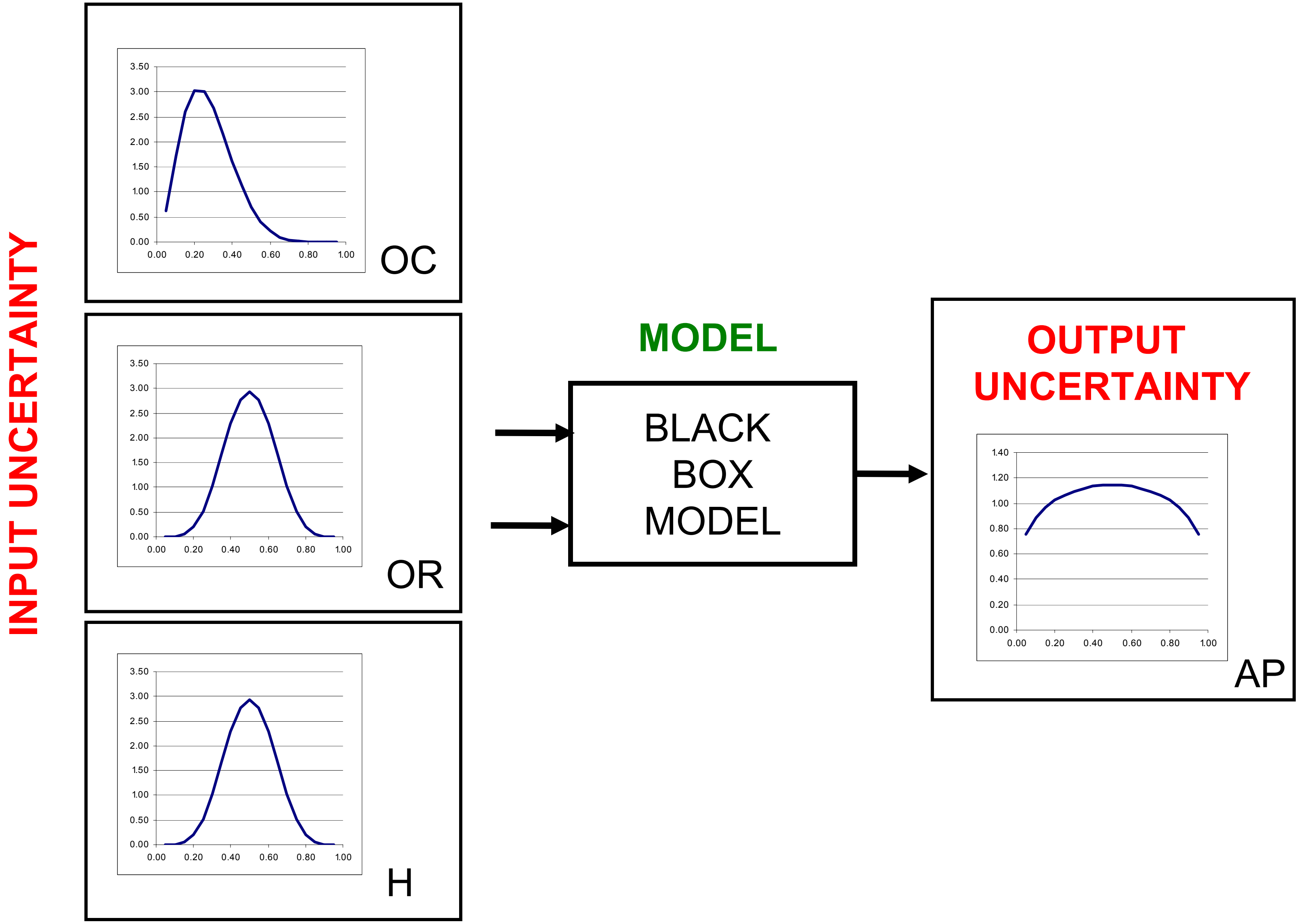
Uncertainty Analysis

- Thus far, **one-way** sensitivity analysis and **two-way** sensitivity analysis acknowledge that **output parameters** are uncertain by indicating a range for the different **input variables**.
- Of course, we can conduct a **three-way** sensitivity analysis, a **four- way** etc.?
- Perhaps we should vary all the parameters at the same time. The latter is called an: **Uncertainty Analysis (Chapter 11).**
- Parameters are uncertain as indicated by assessing a range. By specifying **probability distributions** for uncertainty of input parameters we assess **how uncertain** these parameters are.
- **Given the uncertainty distribution** of **the input parameters** and the calculation model **the uncertainty distribution of the output parameters is fixed.**

Uncertainty Analysis



Uncertainty Analysis