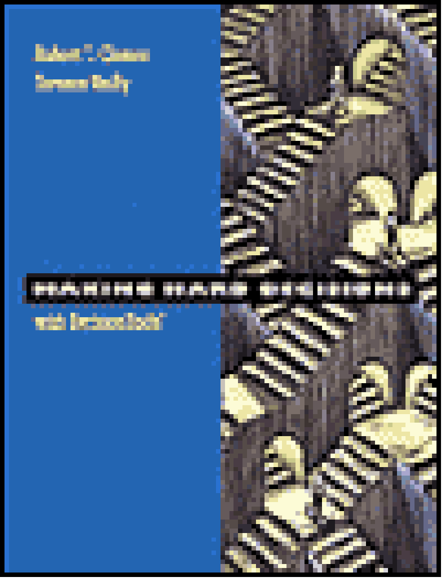


Making Hard Decisions

R. T. Clemen, T. Reilly

Chapter 4

Making Choices

Draft: Version 1

Texaco Versus Pennzoil

In early 1984, **Pennzoil and Getty Oil agreed to the terms of a merger.** But before any formal documents could be signed, **Texaco offered Getty a substantially better price,** and Gordon Getty, who controlled mos of the Getty Stock, reneged on the Pennzoil deal and sold to Texaco. Naturally, **Pennzoil felt as if it had been dealt with unfairly and immediately files a lawsuit against Texaco** alleging that Texaco had interfered illegally in the Pennzoil-Getty negotiations. **Pennzoil won the case**: in late 1985, it was awarded **$11.1 billion**, the largest judgment ever in the United States. A Texas appeal court reduced the judgement to **$2 billion**, but interest and penalties drove the total back up to **$10.3 billion**. James Kinnear, Texaco's Chief executive officer, had said that **Texaco would file for bankruptcy** if Pennzoil obtained court permission to secure the judgment by filing liens against Texaco's assets.

Texaco Versus Pennzoil - Continued

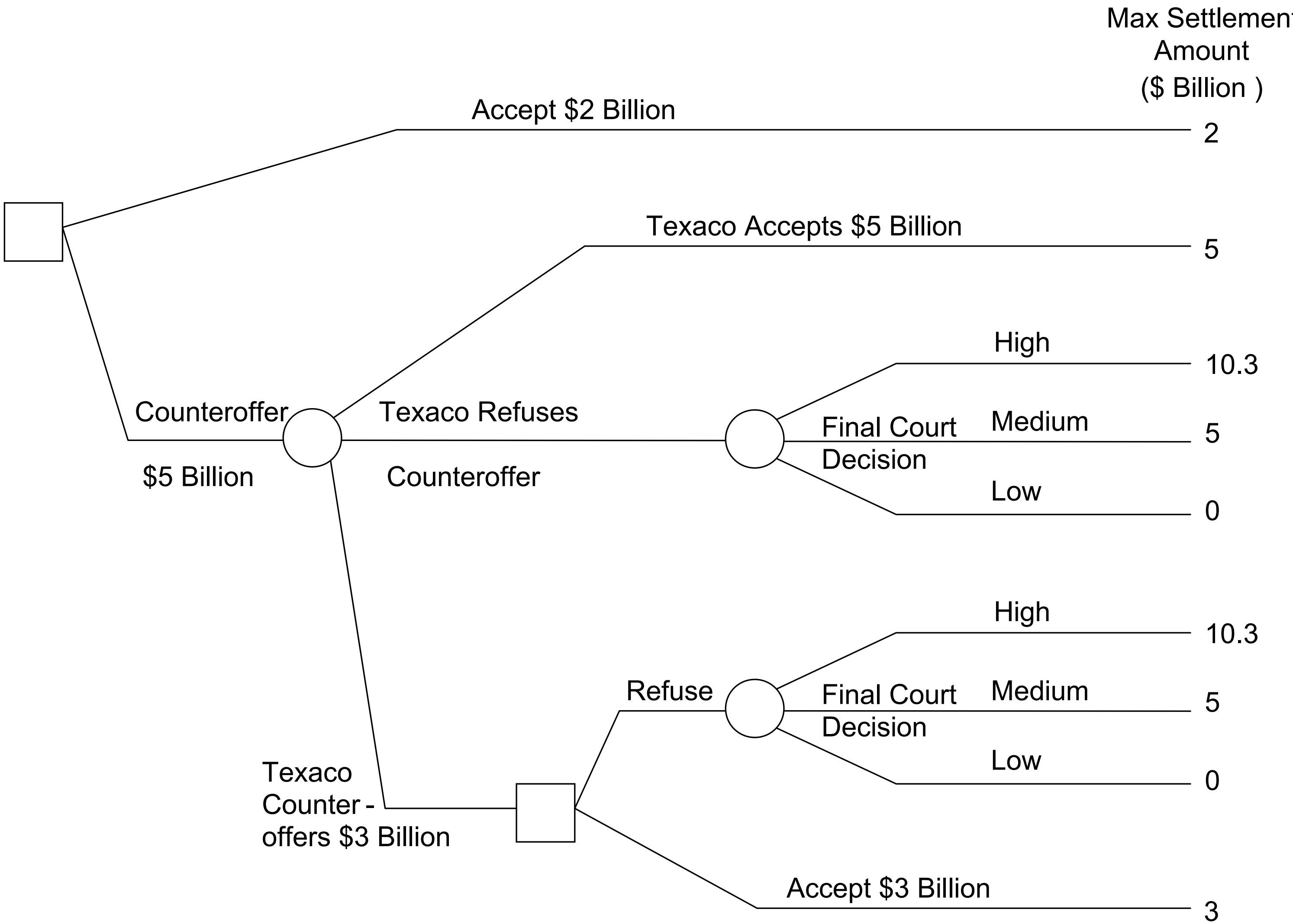
Furthermore, Kinnear had promised to fight the case all the way to the **U.S. Supreme Court** if necessary, arguing in part that Pennzoil had not followed Security and Exchange Commission regulations in its negotiations with Getty. In April 1987, just before Pennzoil began to file liens, **Texaco offered to Penzoil $2 billion dollars** to settle the entire case. Hugh Liedtke, chairman of Pennzoil, indicated that his advisors were telling him that a settlement **between $3 billion and $5 billion would be fair.**

What should Hugh Liedtke do?

1. Accept $2 Billion
2. Refuse $2 Billion and counter offer $5 Billion

Draft: Version 1

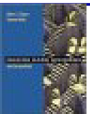
Texaco Versus Pennzoil – Decision Tree



Draft: Version 1

Texaco Versus Pennzoil - Continued

- Given tough negotiation positions of the two executives, their could be **an even chance (50%)** that Texaco will refuse to negotiate further.
- Liedtke and advisor figure that it is **twice as likely** that Texaco would counter offer \$3 billion than accepting the \$5 billion. Hence, because there is a **50% of refusal**, there must be a **33% chance** of a Texaco counter offer and a **17% chance** of Texaco accepting \$5 billion.
- What are the probabilities of the final court decision?
  - Liedtke **admitted** that Pennzoil could lose the case. Thus there is a significant possibility the outcome would be zero. It's probability is assessed at **30%.**
  - Given the strength of the Pennzoil case it is also possible that the court will **upheld the judgment** as it stands. It's probability is assessed at **20%.**
  - Finally, the possibility exists that the judgment could be **reduced somewhat to \$5 billion.** Thus there must be a chance of **50%** of this happening.



Texaco Versus Pennzoil - Continued

- Given tough negotiation positions of the two executives, it could be **an even chance (50%)** that Texaco will refuse to negotiate further.
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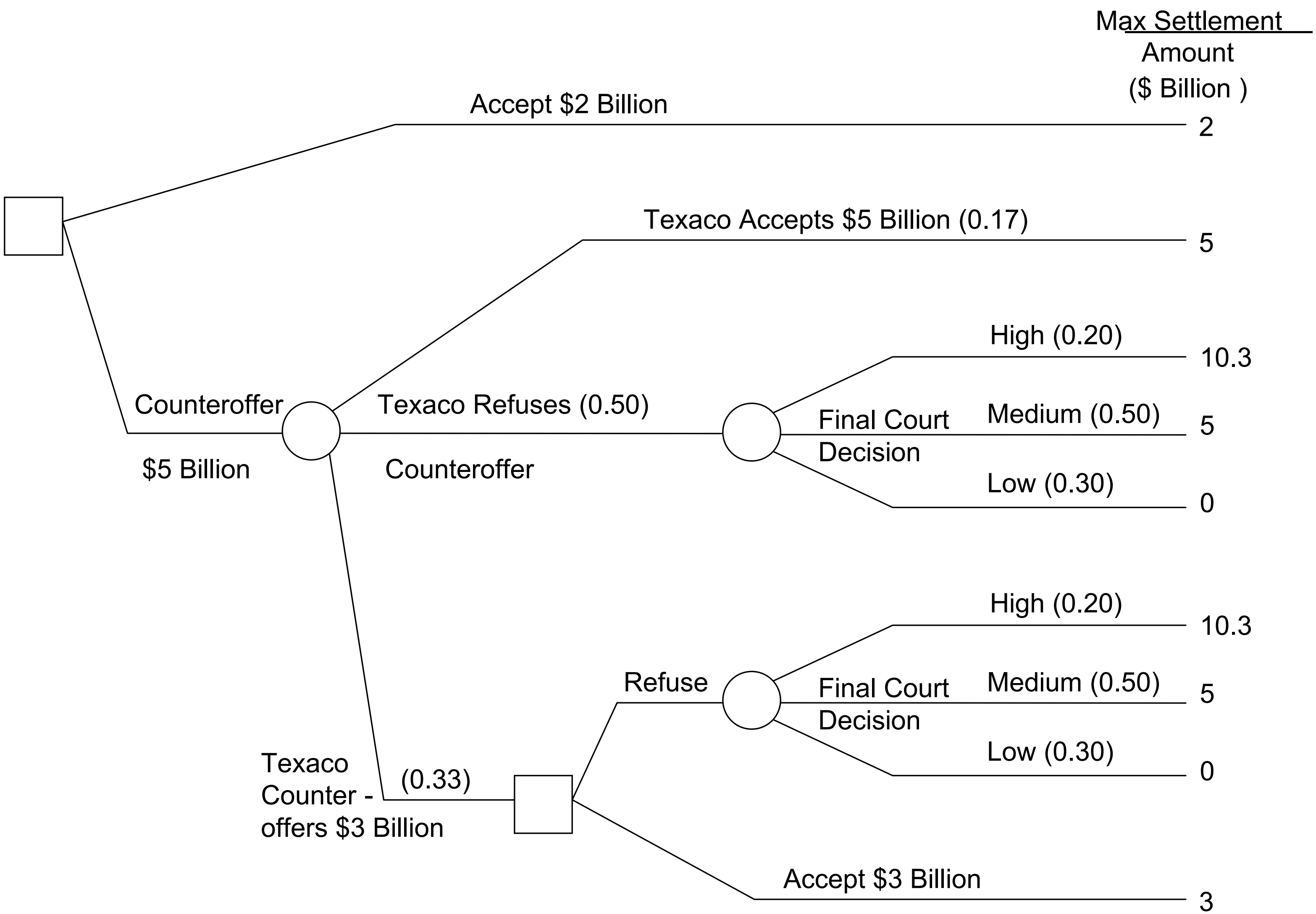
Draft: Version 1

Texaco Versus Pennzoil – Decision Tree



Draft: Version 1

Decision Tree and Expected Monetary Value (EMV)

When objective is measured in **dollars**

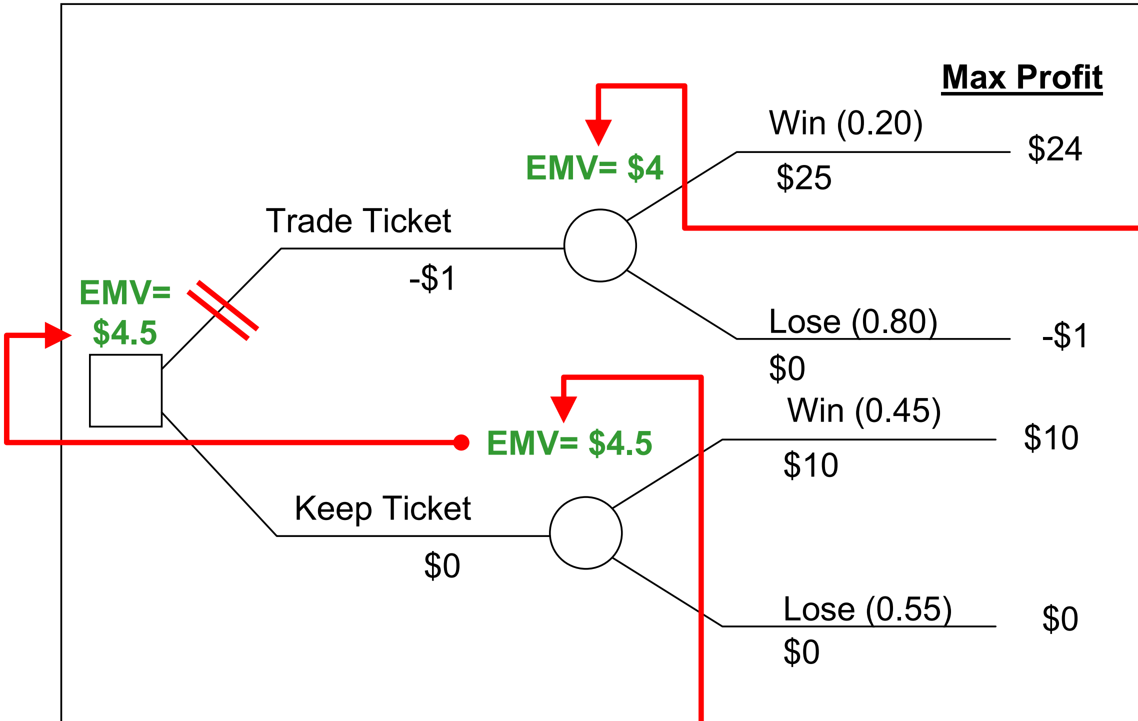
First Suggestion:
Solve decision problem by choosing that alternative that maximizes the EMV

Expected value of discrete random variable Y:

$$E_Y[Y] = \sum_{i=1}^{n} y_i * \Pr(Y = y_i) = \sum_{i=1}^{n} y_i * p_i$$

Draft: Version 1

A double-risk dillema



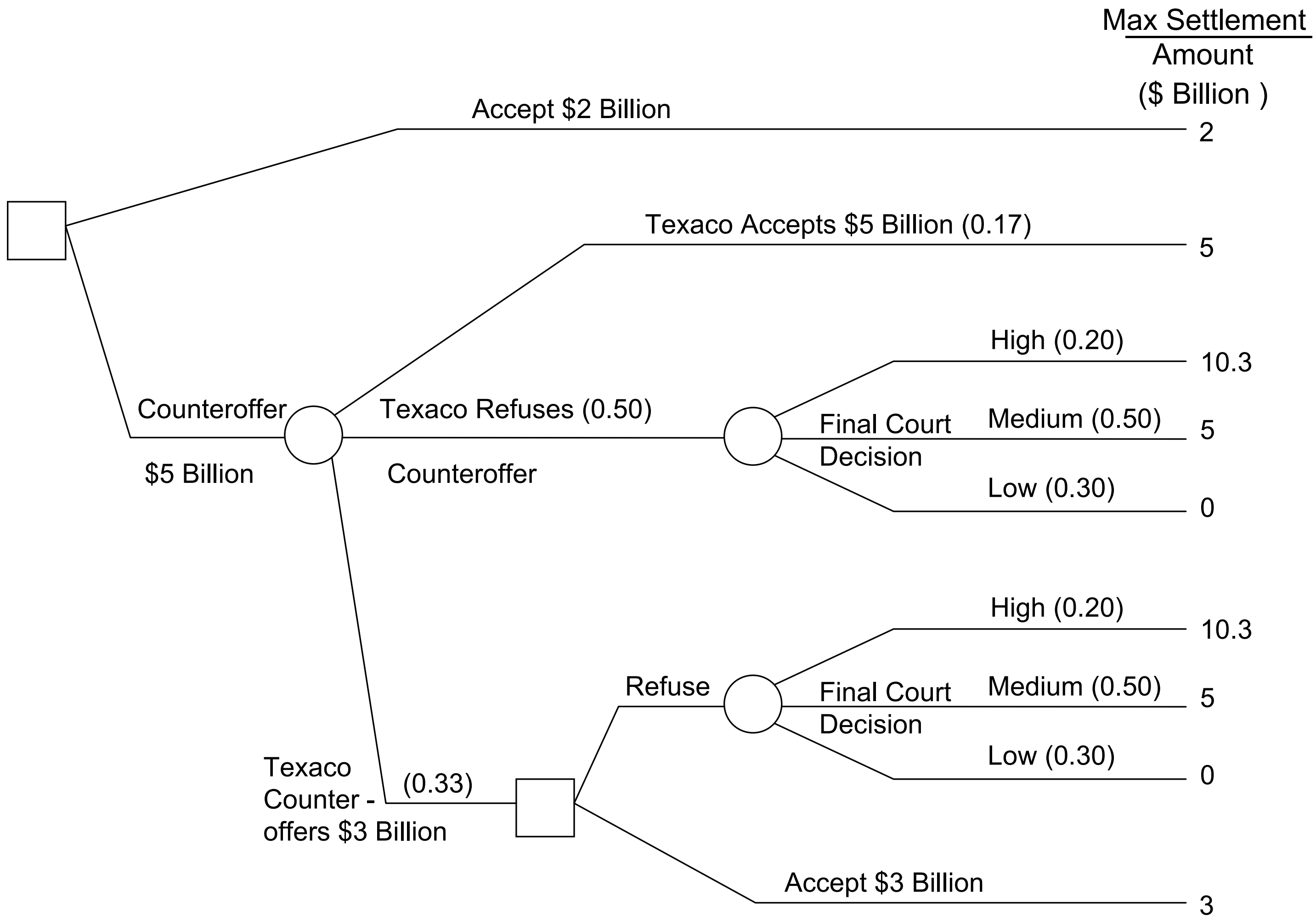
y	Pr(Y=y)	y*Pr(Y=y)	
$24.00	0.2	$4.80	
-$1.00	0.8	-$0.80	
		$4.00	= EMV

y	Pr(Y=y)	y*Pr(Y=y)	
$10.00	0.45	$4.50	
$0.00	0.55	$0.00	
		$4.50	=EMV

Draft: Version 1

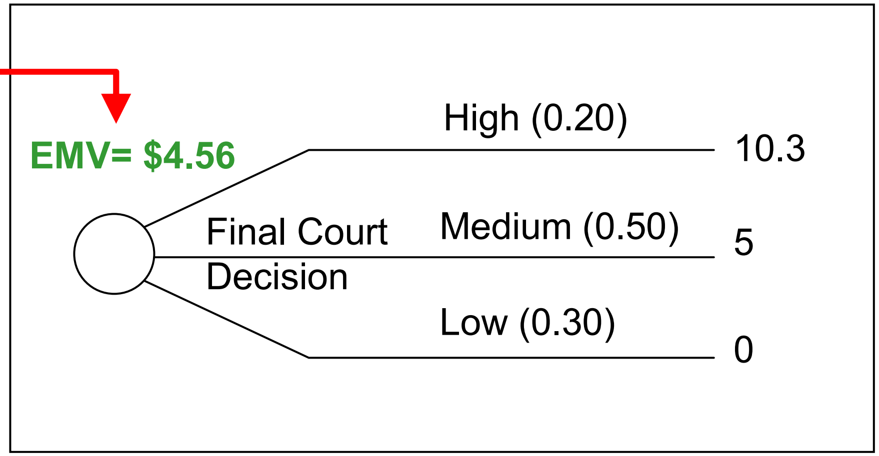
Interpretation EMV: Playing the same lottery a lot of times will result over time in an **average pay-off** equal to the EMV

Texaco Versus Pennzoil – Decision Tree



Solve tree using EMV by folding back the tree

Step 1: Calculate EMV of court decision uncertainty node

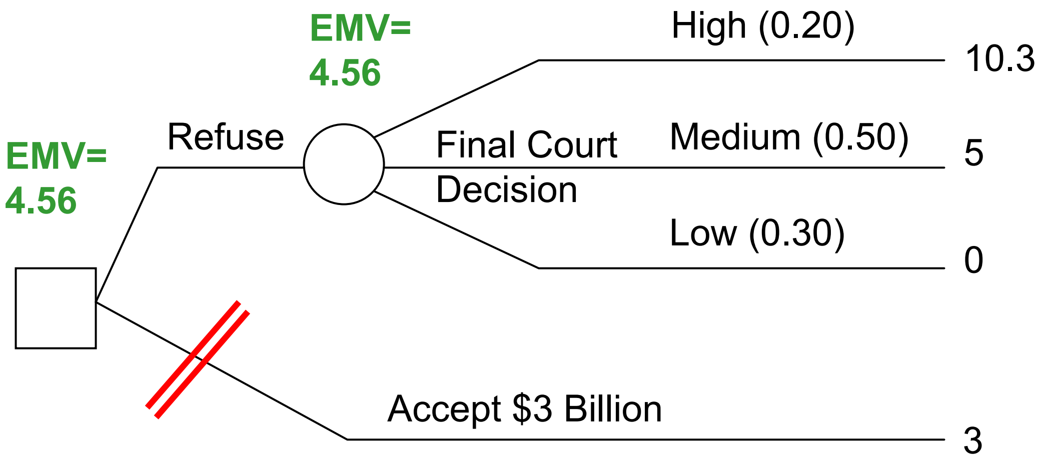


	Step 1		
y	Pr(Y=y)	y*Pr(Y=y)	
10.300	0.2	$2.06	
5.000	0.5	$2.50	
0.000	0.3	$0.00	
		$4.56	=EMV

Draft: Version 1

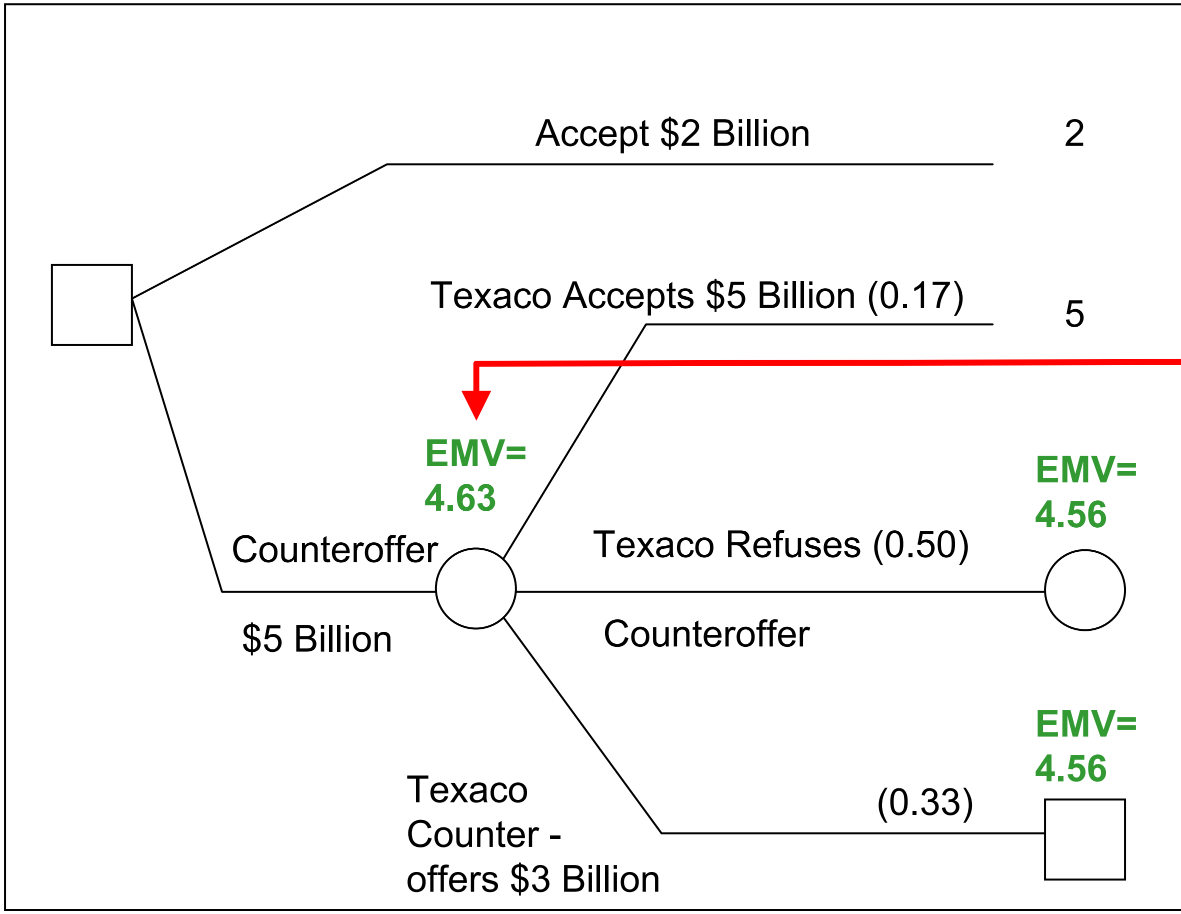
Decision Tree and Expected Monetary Value (EMV)

Step 2: Evaluate decision regarding Texaco's counter offer



Draft: Version 1

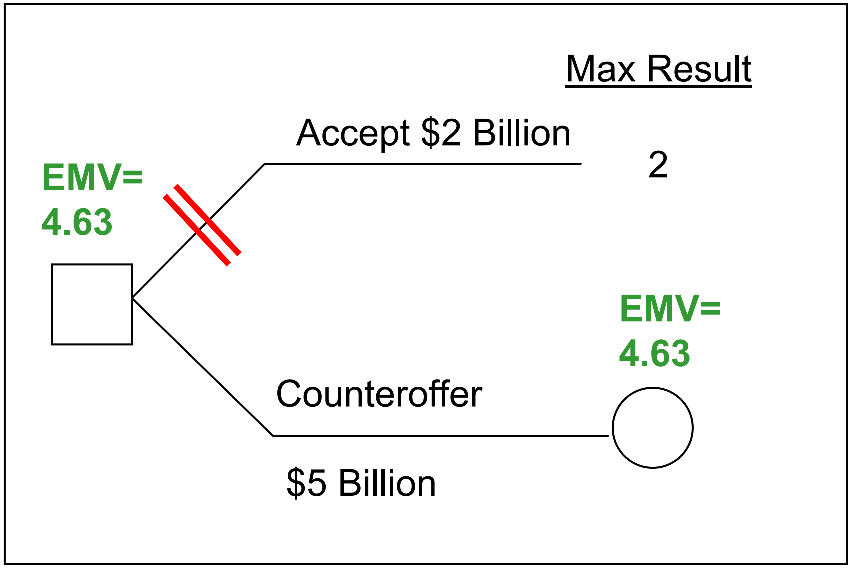
Step 3: Calculate EMV Texaco's reaction uncertainty node



y	Pr(Y=y)	y*Pr(Y=y)	
5.000	0.17	$0.85	
4.560	0.5	$2.28	
4.560	0.33	$1.50	
		$4.63	= EMV

Draft: Version 1

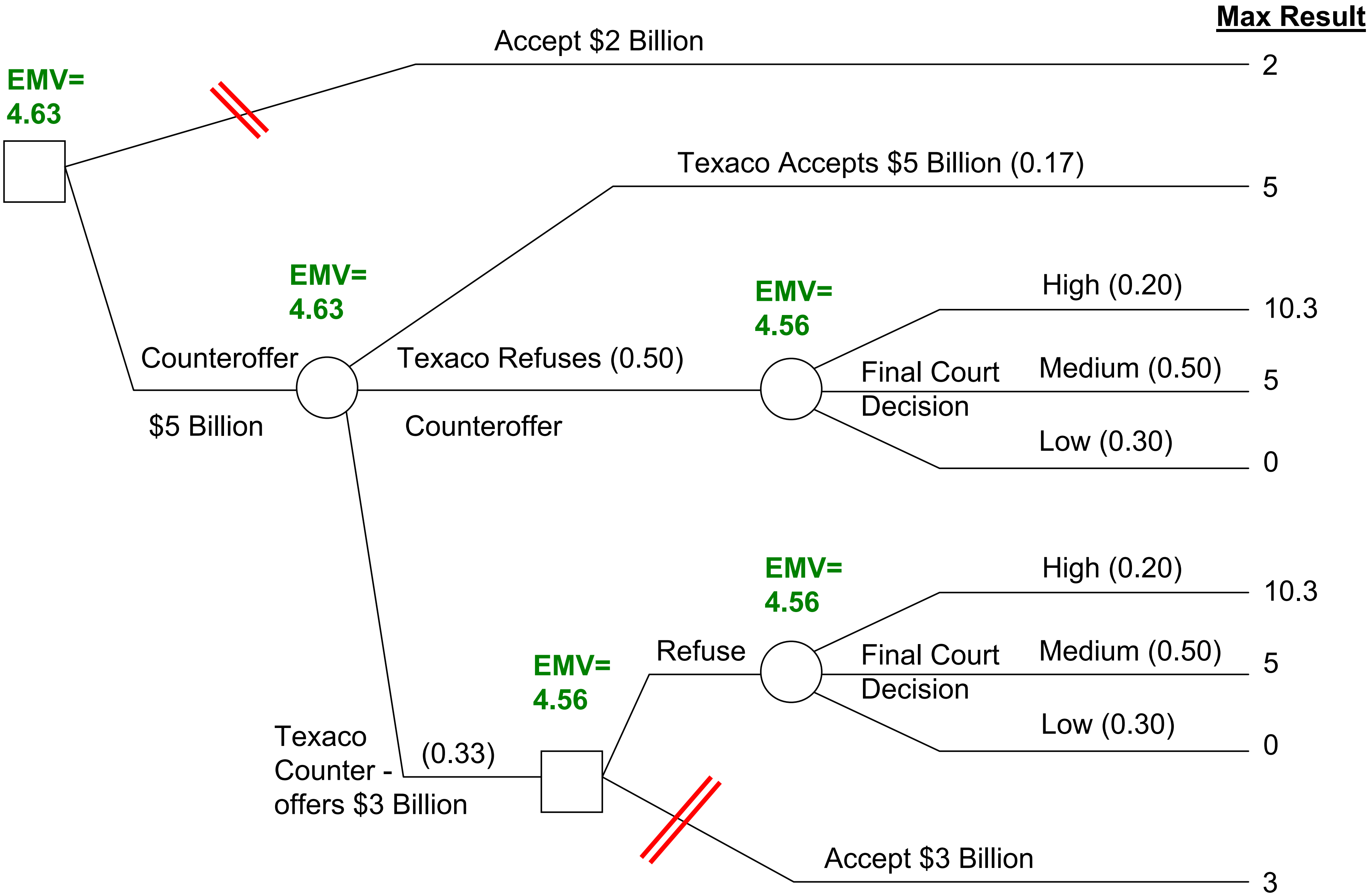
Step 4: Evaluate the immediate decision



Optimal decision: Counteroffer $5 Billion

Optimal decision strategy: Counteroffer $5 Billion and **if Texaco counteroffers $3 Billion**, then refuse this counteroffer.

Folding back the Decision Tree from right to left using EMV



Definitions Decision Path and Strategy

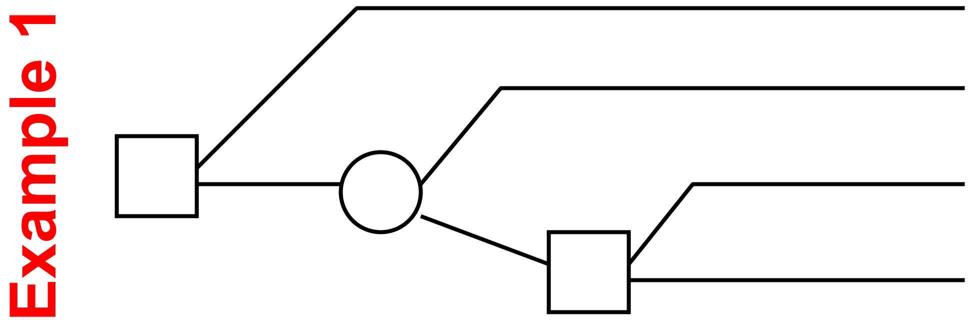
Definition decision path:
A path starting at the left most node up to the values at the end of a branch by selecting **one alternative** from decision nodes or by following **one outcome** from uncertainty nodes. **Represents a possible future scenario.**

Definition decision strategy:
The collection of decision paths connected to one branch of **the immediate decision** by selecting **one alternative** from each decision node along these paths. Represents specifying at every decision in the decision problem **what we would do, if we get to that decision** (we may not get there due to outcome of previous uncertainty nodes).

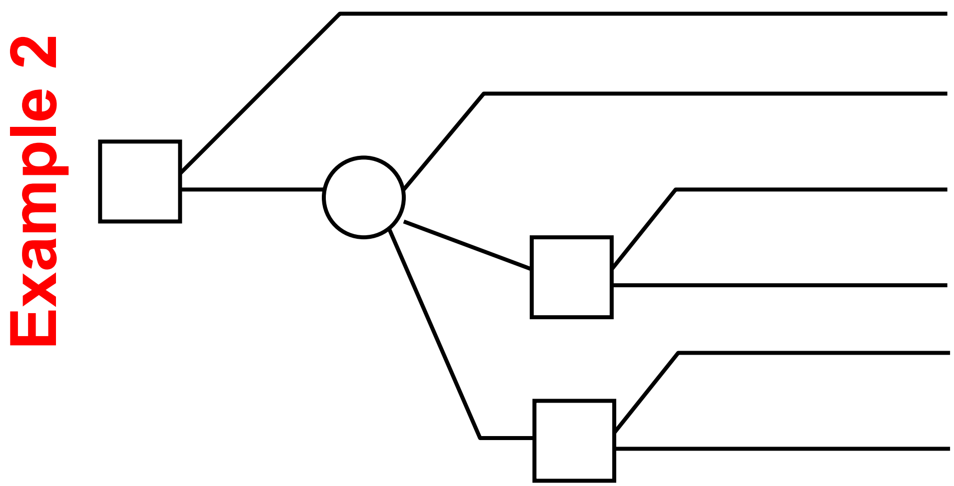
Optimal decision strategy:
That decision strategy which results in **the highest EMV** if we **maximize profit** and **the lowest EMV** if we **minimize cost**.

Counting Strategies

How many decision strategies in Example 1?

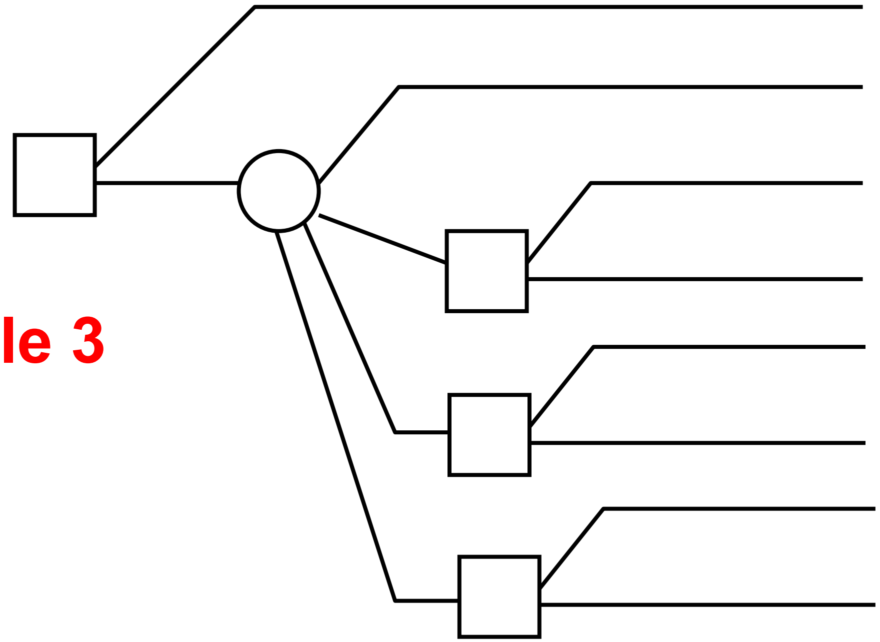


How many decision strategies in Example 2?



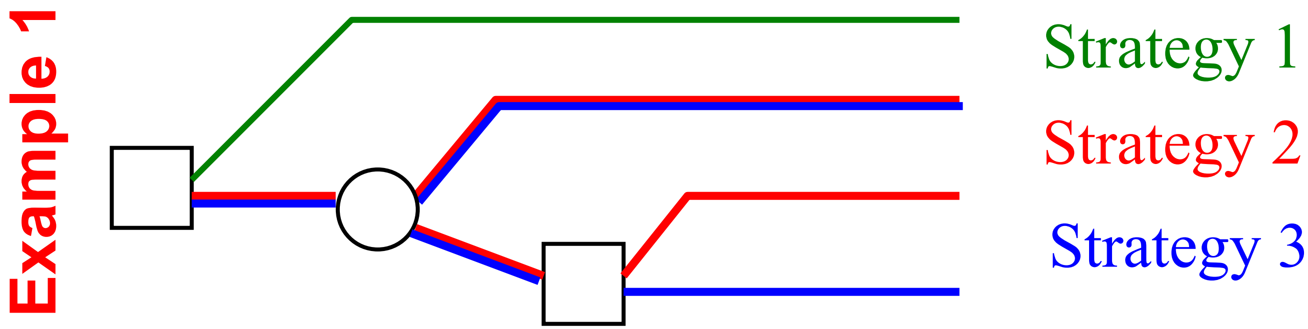
How many decision strategies in Example 3?

Example 3

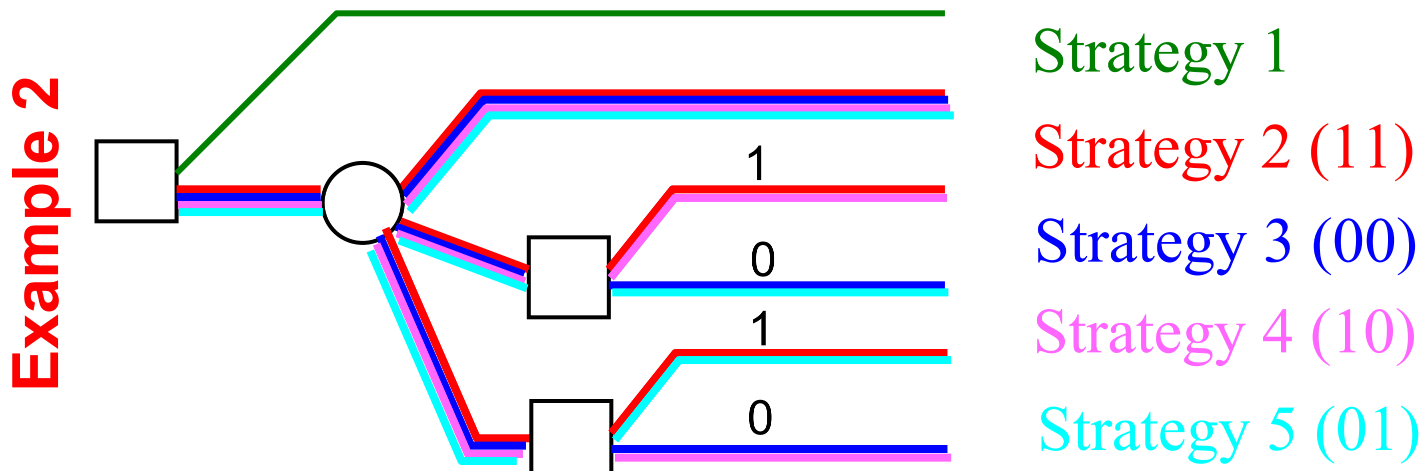


Counting Strategies

How many decision strategies in Example 1?



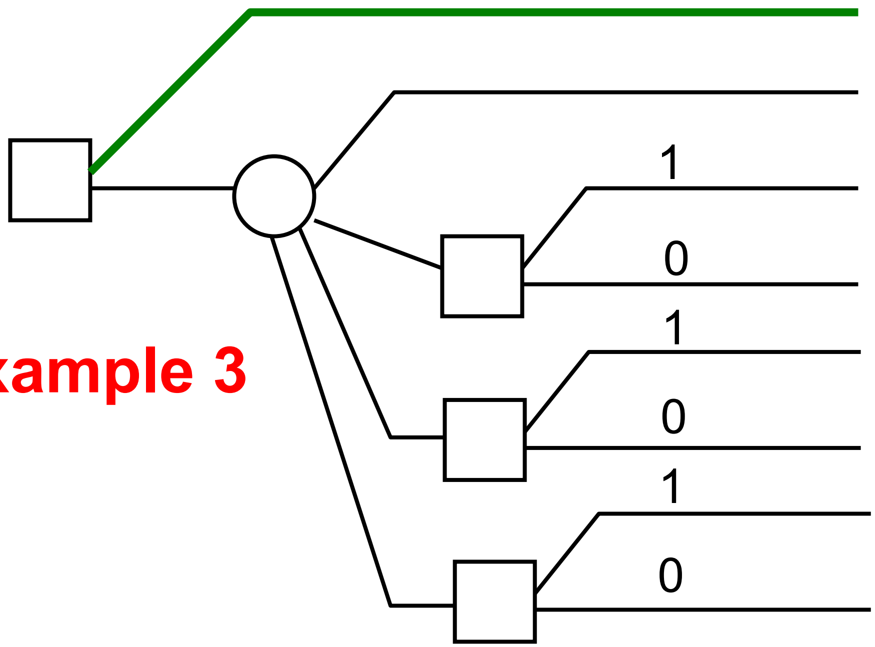
How many decision strategies in Example 2?



Draft: Version 1

Counting Strategies

How many decision strategies in Example 3?



Example 3

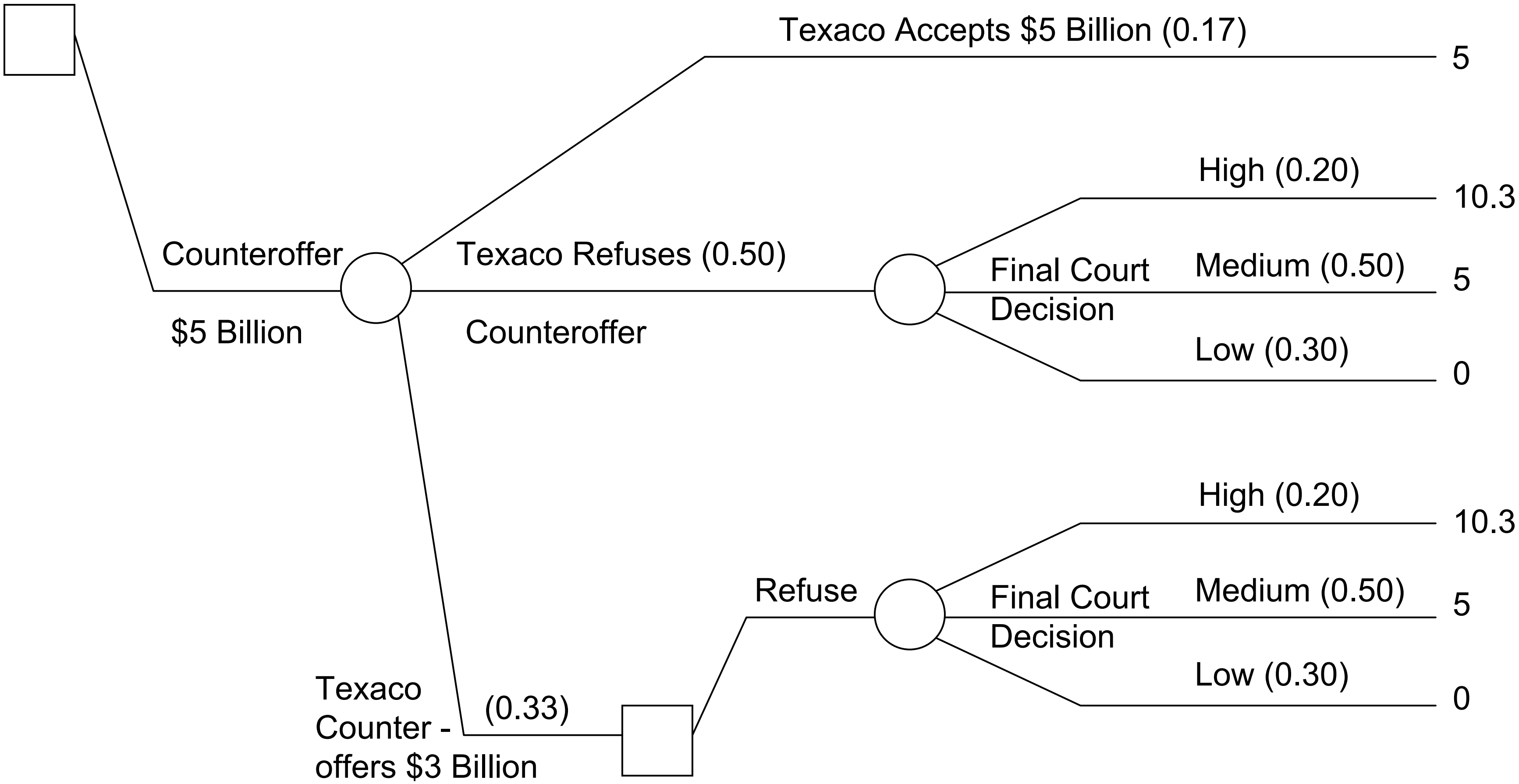
Strategy 1
Strategy 2 (111)
Strategy 3 (001)
Strategy 4 (101)
Strategy 5 (011)
Strategy 6 (110)
Strategy 7 (000)
Strategy 8 (100)
Strategy 9 (010)

How many decision strategies do we have in the Texaco – Penzoil decision tree?

First strategy: "Accept $2 billion"

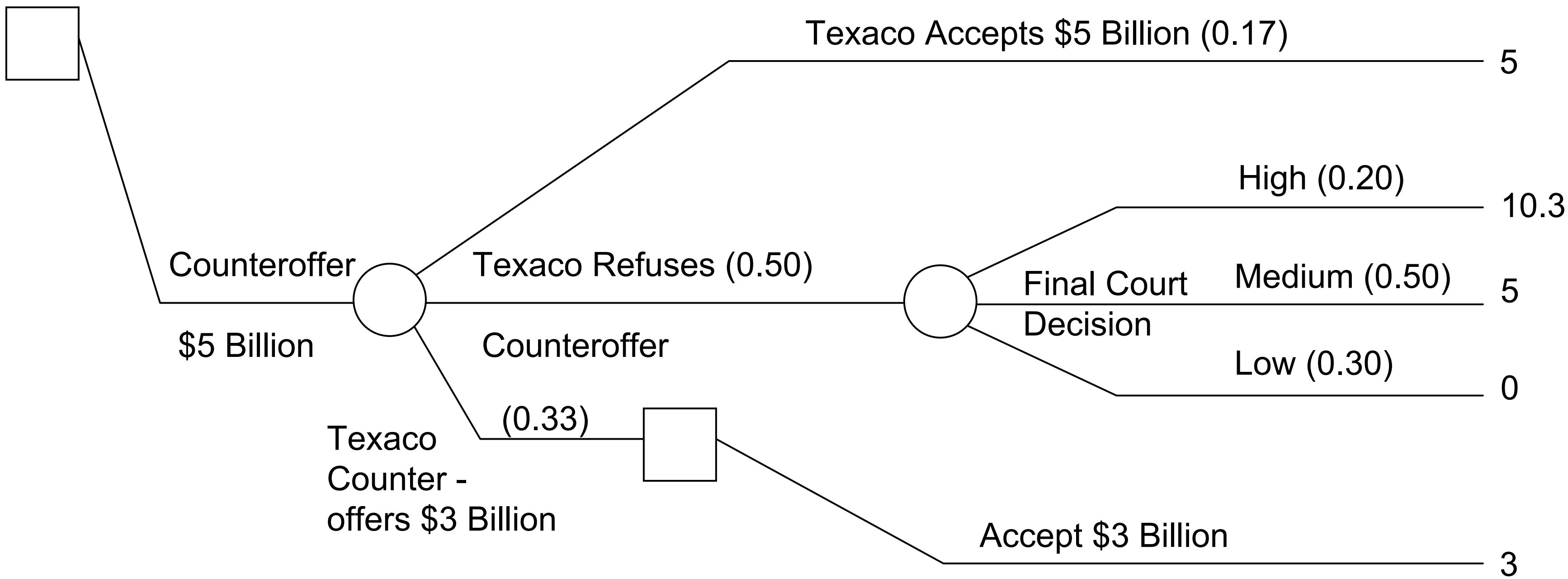
Decision Strategies Texaco-Pennzoil Case

Second strategy: "Counter $5 billion and if Texaco counter offers $3 billion refuse this counteroffer of $3 Billion"



Draft: Version 1

Third strategy: "Counter $5 billion and if Texaco counter offers $3 billion accept this counteroffer of $3 Billion"



Risk Profiles and Cumulative Risk Profiles

RISK PROFILES = Graph that shows probabilities for each of the possible outcomes **given a particular decision strategy.**

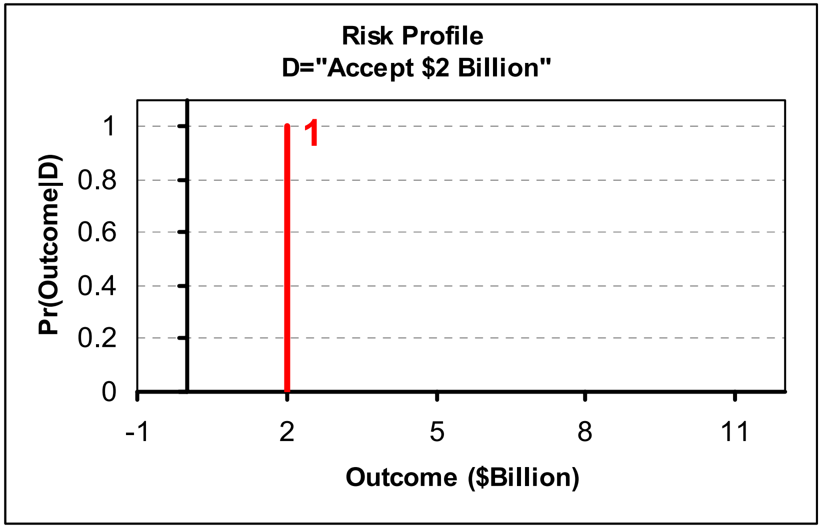
Note: Risk Profile is **a probability mass function** for the discrete random variable Y representing the outcomes for the given decision strategy.

CUMMULATIVE RISK PROFILES = Graphs that shows **cumulative probabilities** associated with a risk profile

Note: Cumulative risk profile is **a cumulative distribution function** for the discrete random variable Y representing the outcomes for **the given decision strategy**.

First strategy: "Accept $2 billion"

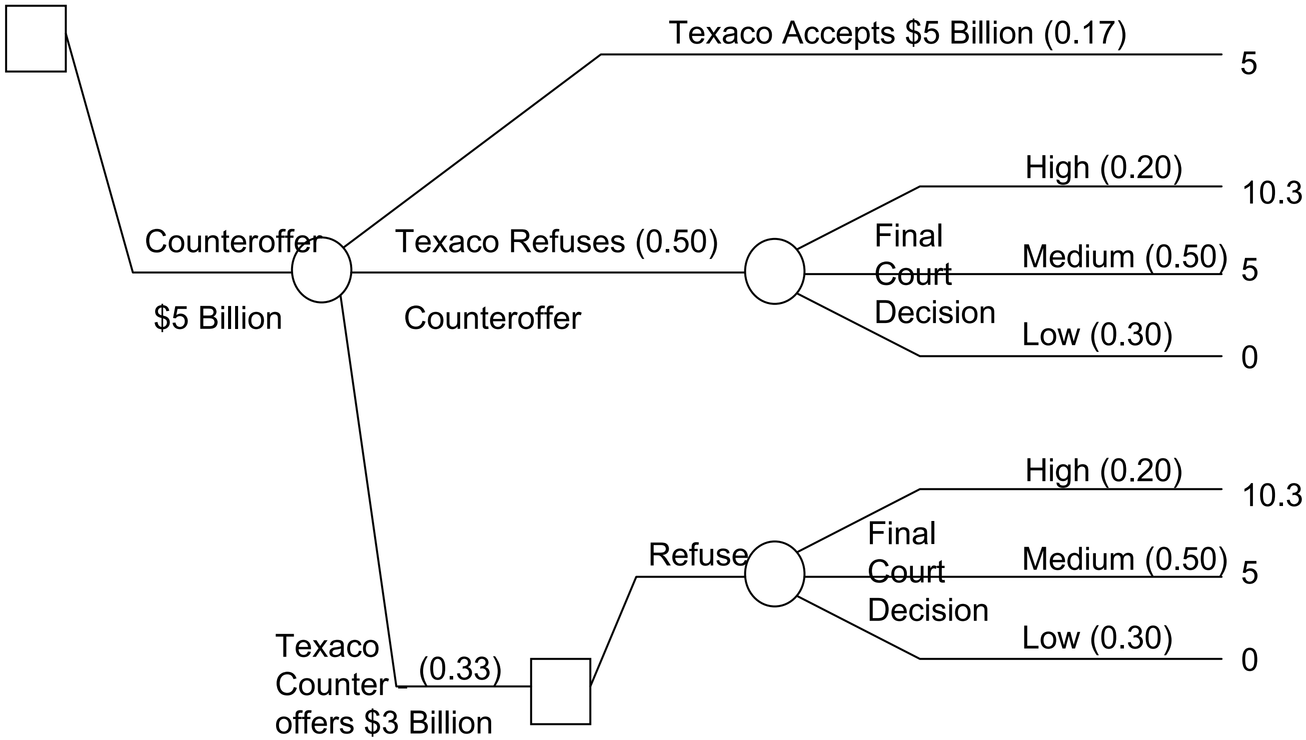
Outcome x ($Billion)	Pr(Outcome|D)
2	1



Draft: Version 1

Risk Profiles

Second strategy: "Counter $5 billion and if Texaco counter offers $3 billion refuse this counteroffer of $3 Billion"

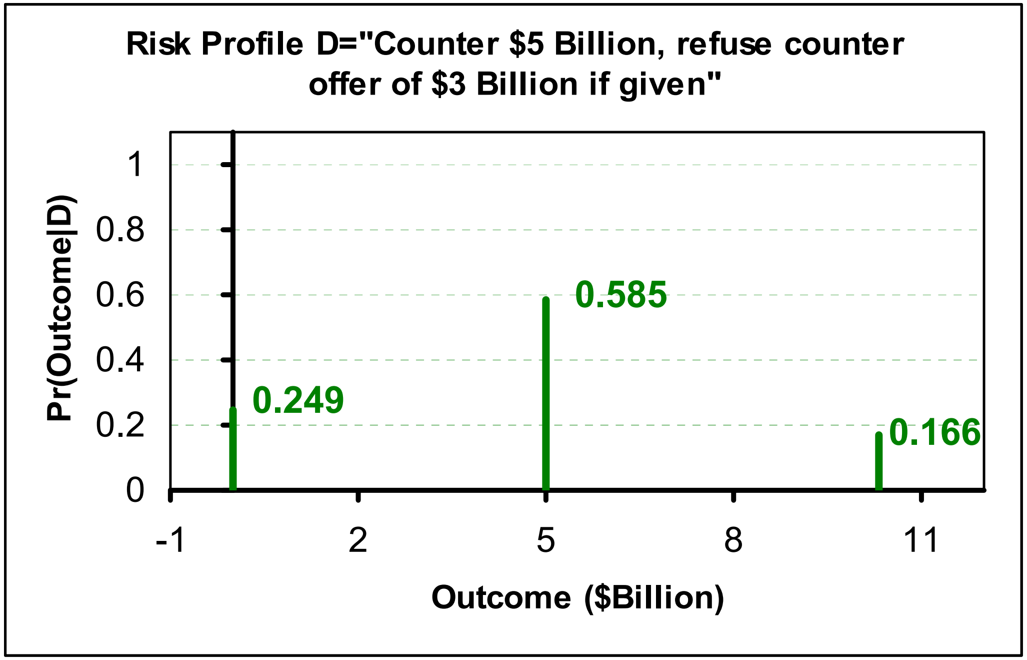


Calculation	Prob
0.17	0.170
0.50*0.20	0.100
0.50*0.50	0.250
0.50*0.30	0.150
0.33*0.20	0.066
0.33*0.50	0.165
0.33*0.30	0.099
Total	1.000

Draft: Version 1

Second strategy: "Counter $5 billion and if Texaco counter offers $3 billion refuse this counteroffer of $3 Billion"

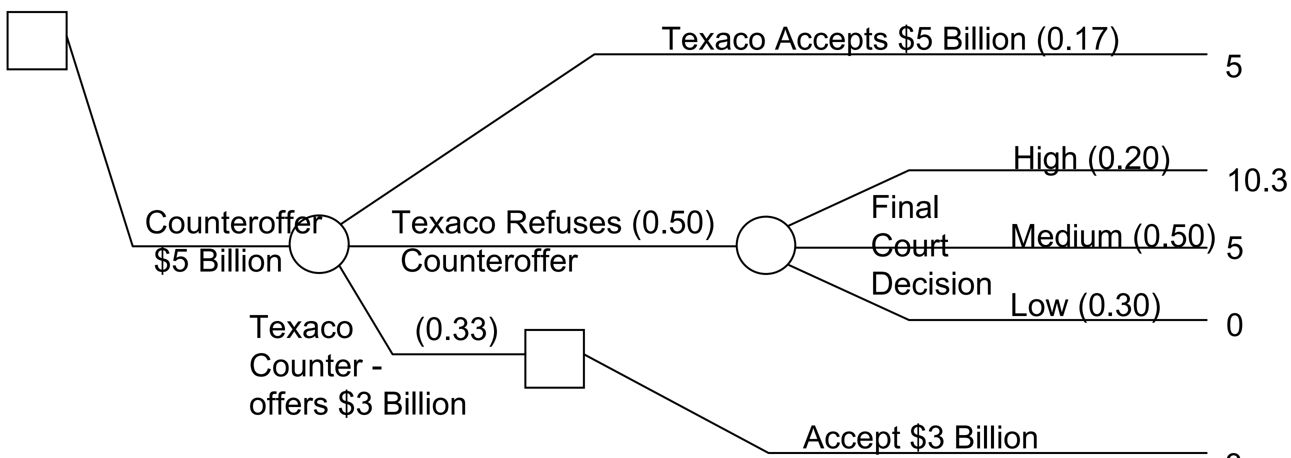
Outcome x ($Billion)	Calculation	Pr(Outcome| D)
0	0.150+0.099	0.249
5	0.170+0.250+0.165	0.585
10.3	0.100+0.066	0.166
		1.000



Draft: Version 1

Risk Profiles

Third strategy: "Counter $5 billion and if Texaco counter offers $3 billion accept this counteroffer of $3 Billion"

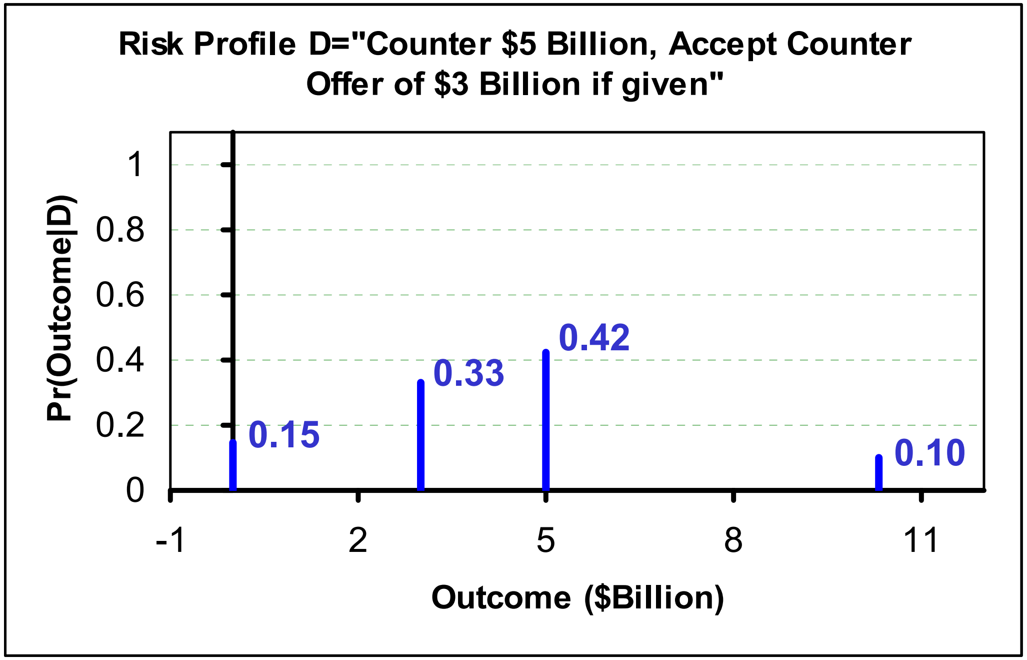


Calculation	**Prob**
0.17	0.170
0.50*0.20	0.100
0.50*0.50	0.250
0.50*0.30	0.150
0.33	0.330
Total	1.000

Risk Profiles

Third strategy: "Counter $5 billion and if Texaco counter offers $3 billion accept this counteroffer of $3 Billion"

Outcome x ($Billion)	Calculation	Pr(Outcome| D)
0	0.15	0.15
3	0.33	0.33
5	0.170+0.250	0.42
10.3	0.1	0.1
		1.000

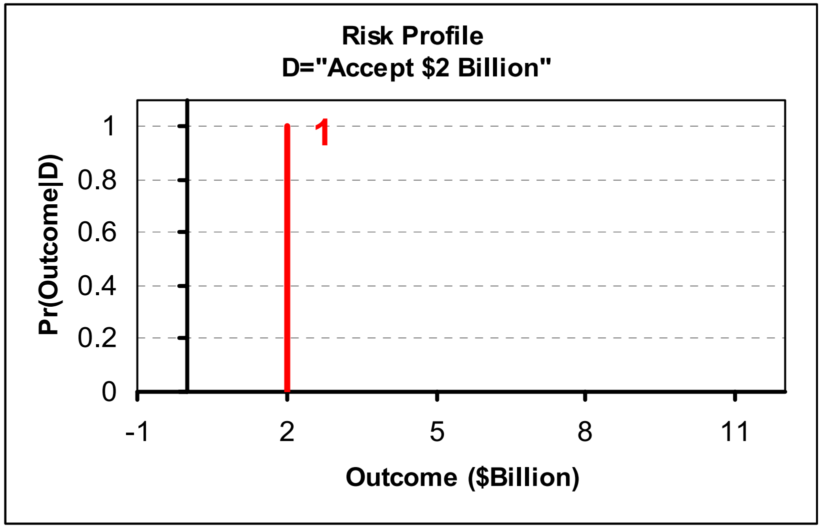


Draft: Version 1

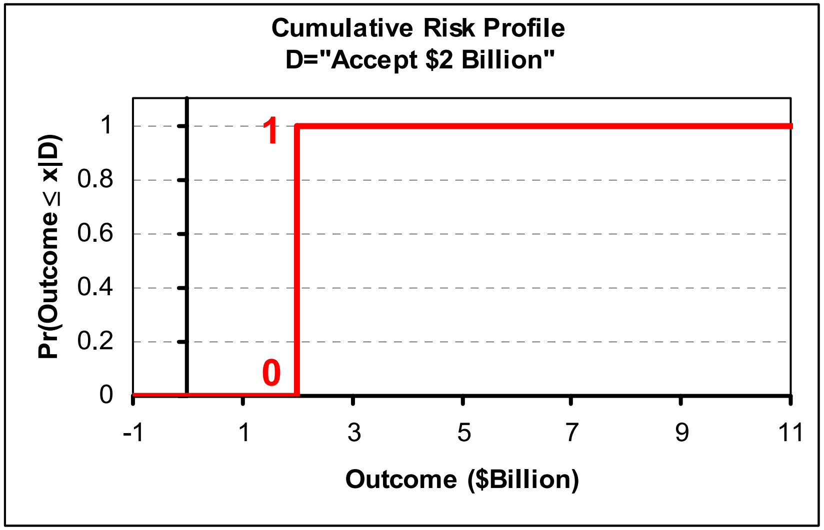
Cumulative Risk Profiles

First strategy: “Accept $2 billion”

Outcome x ($Billion)	Pr(Outcome|D)
2	1



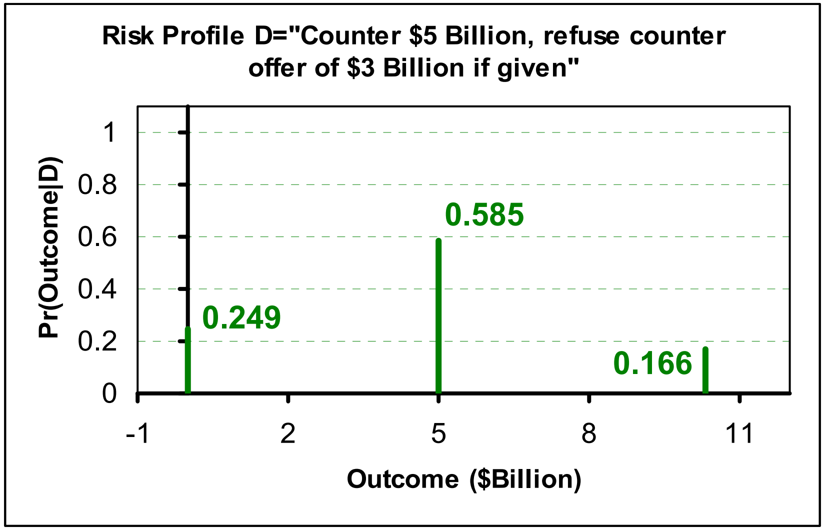
Outcome x ($Billion)	Pr(Outcome ≤ x|D)
2	1



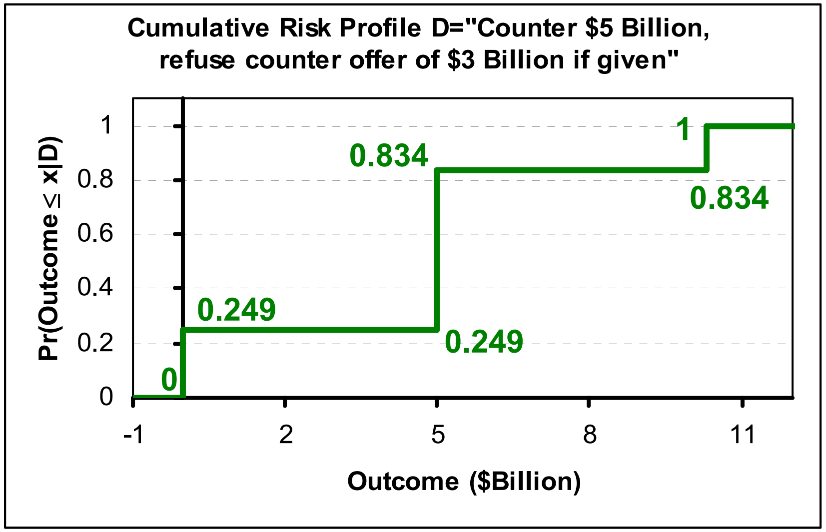
Draft: Version 1

Cumulative Risk Profiles

Second strategy: "Counter $5 billion and if Texaco counter offers $3 billion refuse this counteroffer of $3 Billion"



Outcome x ($Billion)	Pr(Outcome|D)
0	0.249
5	0.585
10.3	0.166



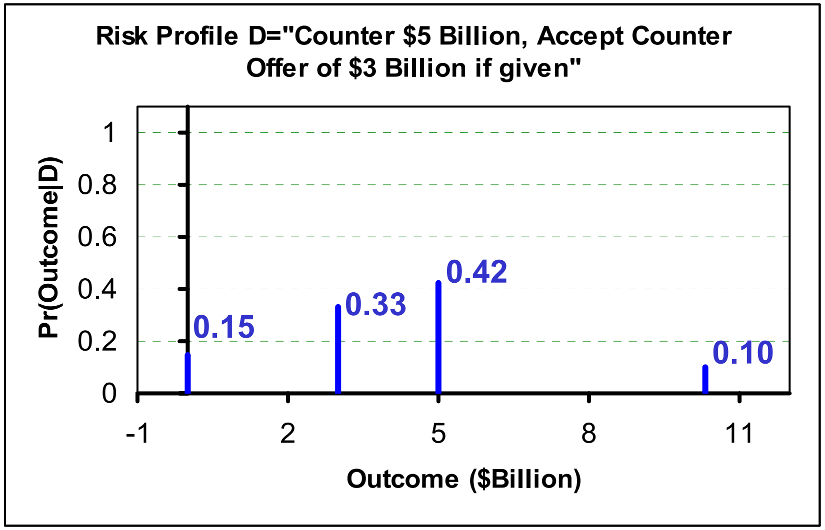
Outcome x ($Billion)	Pr(Outcome ≤ x|D)
0	0.249
5	0.249 + 0.585 = 0.834
10.3	0.834 + 0.166 = 1

Draft: Version 1

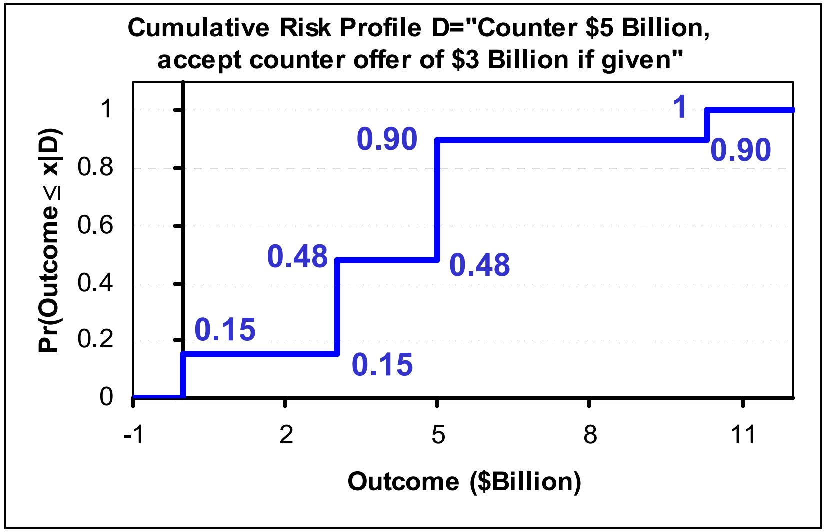
Cumulative Risk Profiles

Third strategy: "Counter \$5 billion and if Texaco counter offers \$3 billion accept this counteroffer of \$3 Billion"

Outcome x (\$Billion)	Pr(Outcome|D)
0	0.15
3	0.33
5	0.42
10.3	0.1



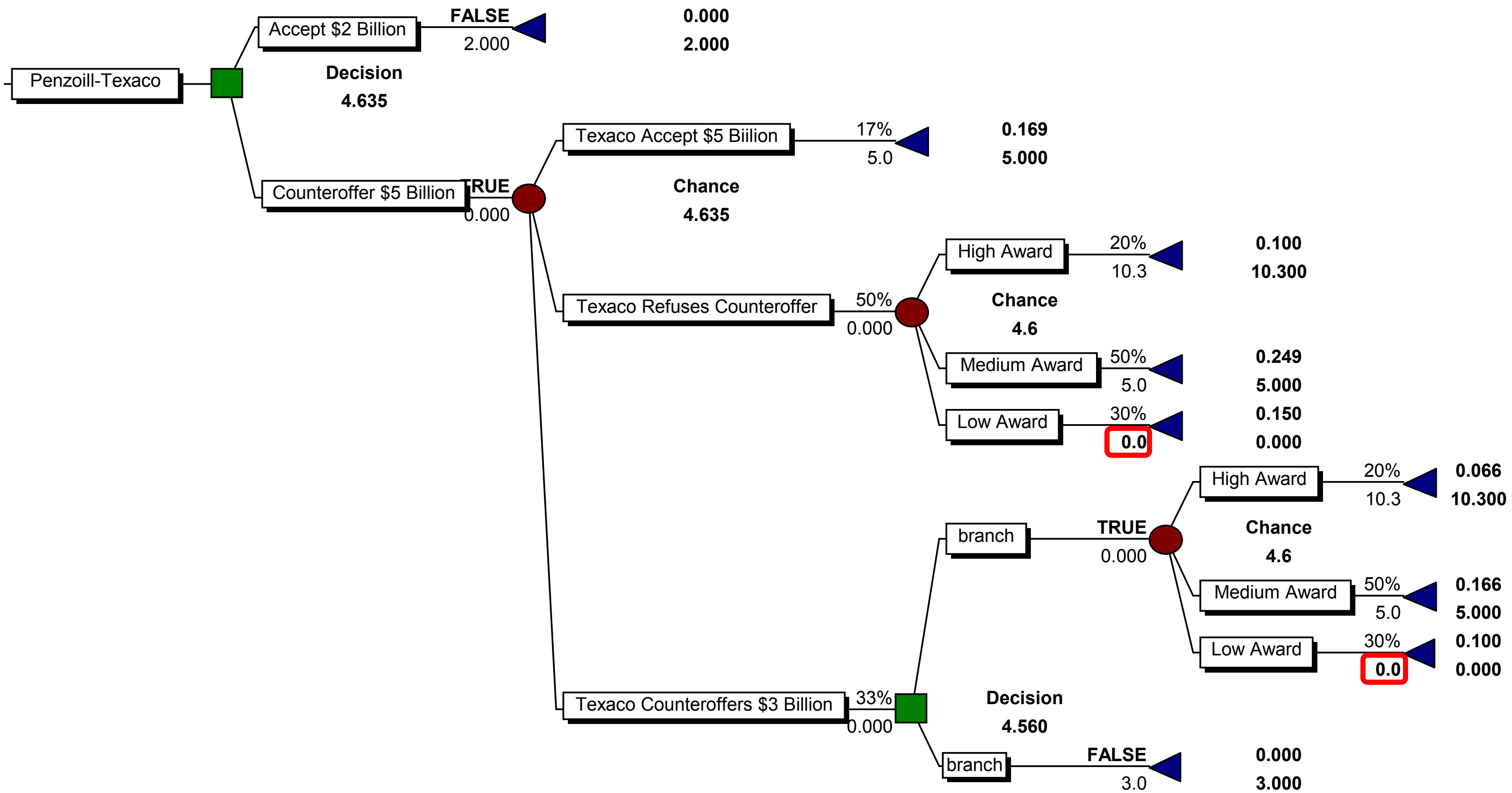
Outcome x (\$Billion)	Pr(Outcome ≤ x|D)
0	0.15
3	0.15 + 0.33 = 0.48
5	0.48 + 0.42 = 0.90
10.3	0.90 + 0.10 = 1



Draft: Version 1

Deterministic Dominance

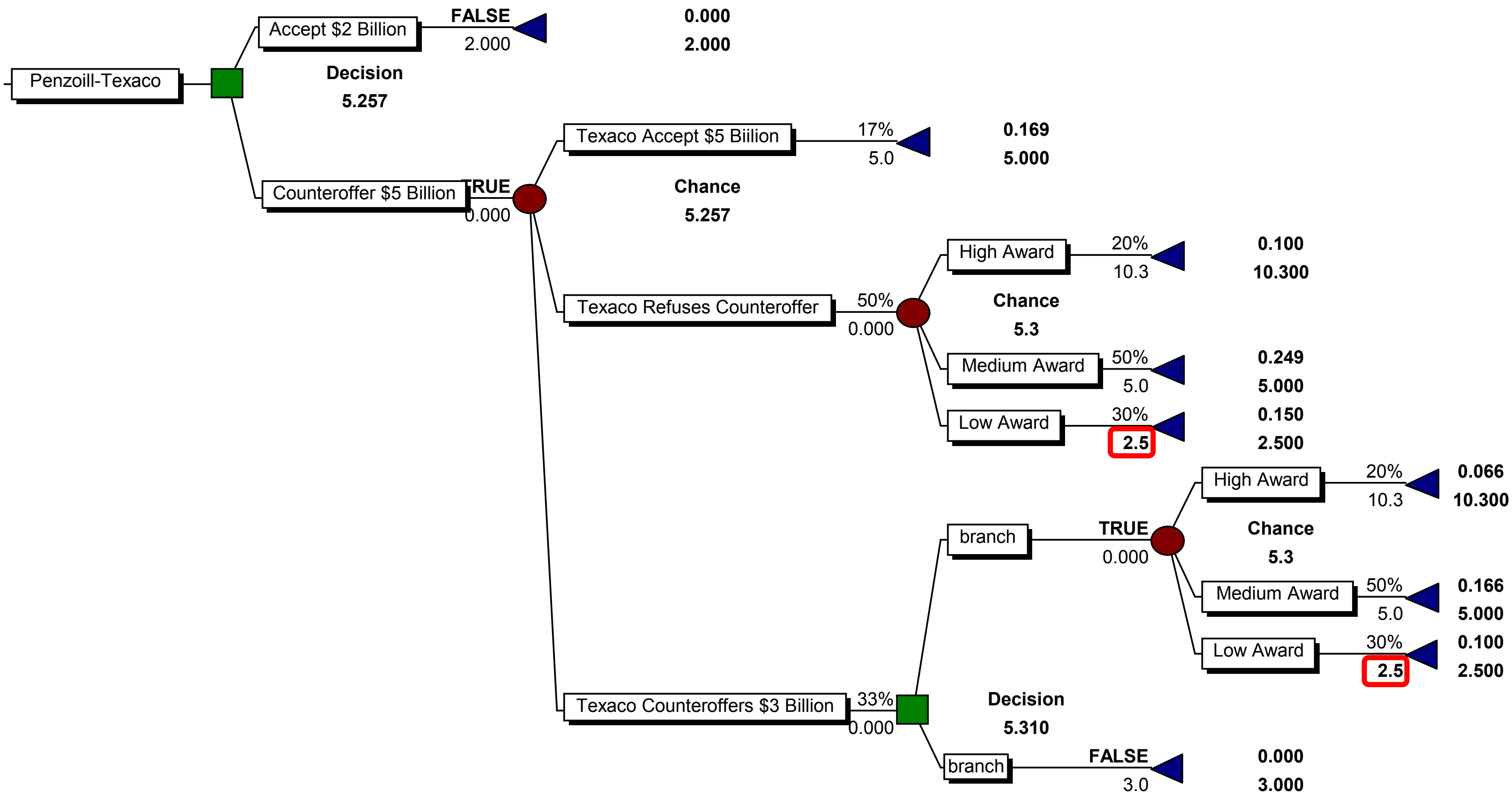
Original Tree



Draft: Version 1

Deterministic Dominance

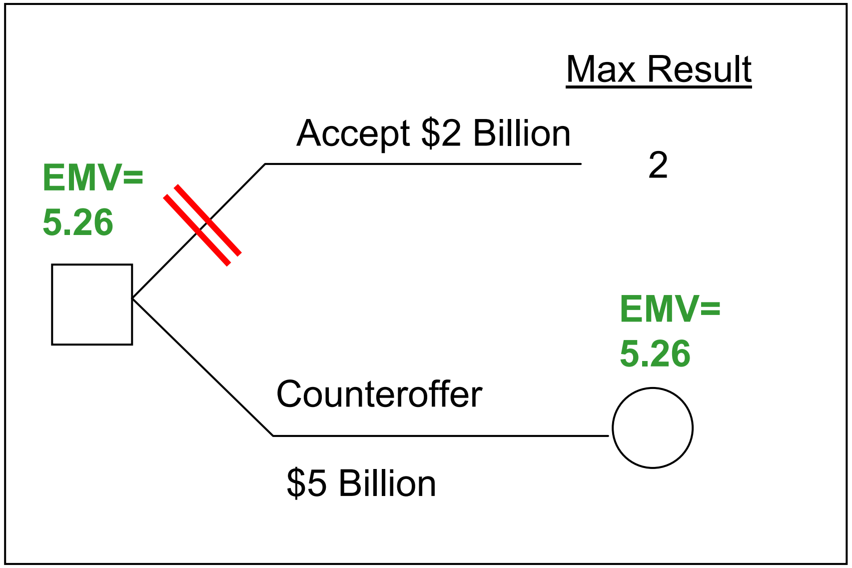
Modified Tree



Draft: Version 1

Deterministic Dominance

Based on EMV analysis we still choose the alternative “Counteroffer $5 Billion”



Could we have made a decision here without an EMV analysis ?

Deterministic Dominance

Formal Definition: Deterministic Dominance:

If the **worst outcome** of Alternative B is **at least as good** as that of the **best outcome** of Alternative A, then Alternative B **deterministically dominates** Alternative A.

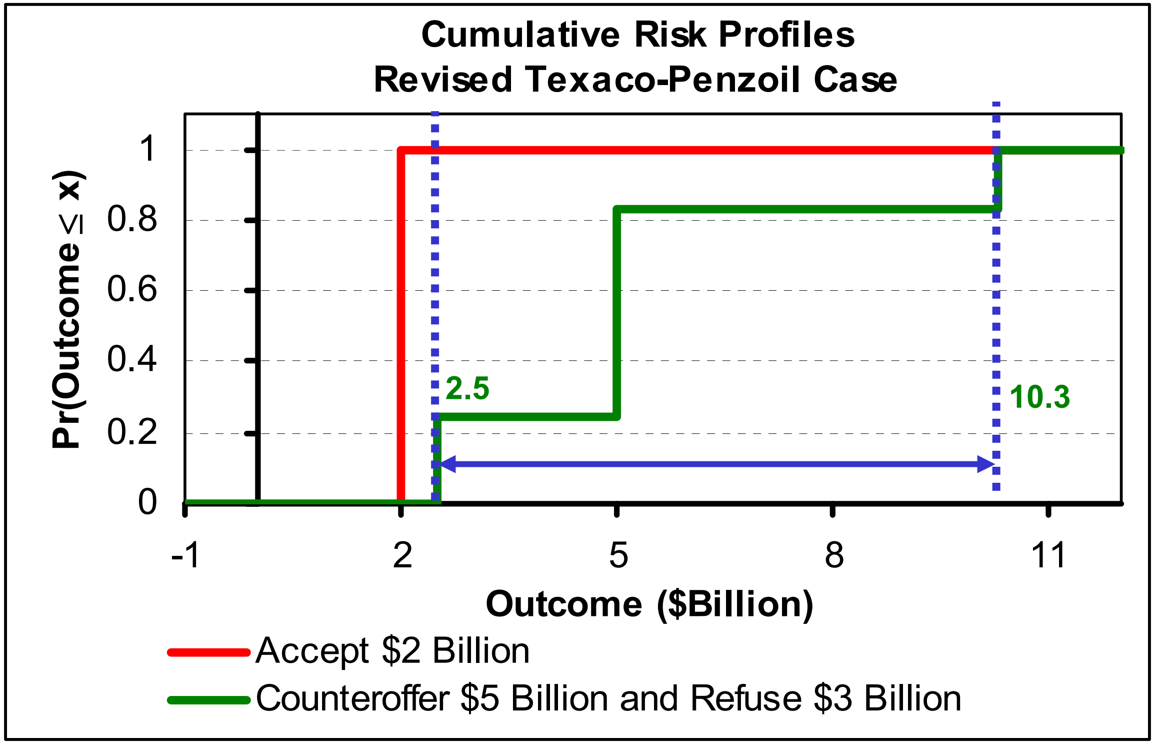
- Deterministic dominance may also be concluded by drawing cumulative risk profiles and using the definition:

Definition: Range of a Cumulative Risk Profile = [L,U], where L= Smallest 0% point in Cumulative Risk Profile and U= Largest 100% point in Cumulative Risk Profile

Draft: Version 1

Deterministic Dominance

- **Deterministic dominance via cumulative risk profiles:**
  - Step 1: Draw cumulative risk profiles in one graph
  - Step 2: Determine range for each risk profile
  - Step 3: If ranges are disjoint or their intersections contain a single point



Range 1: {2}

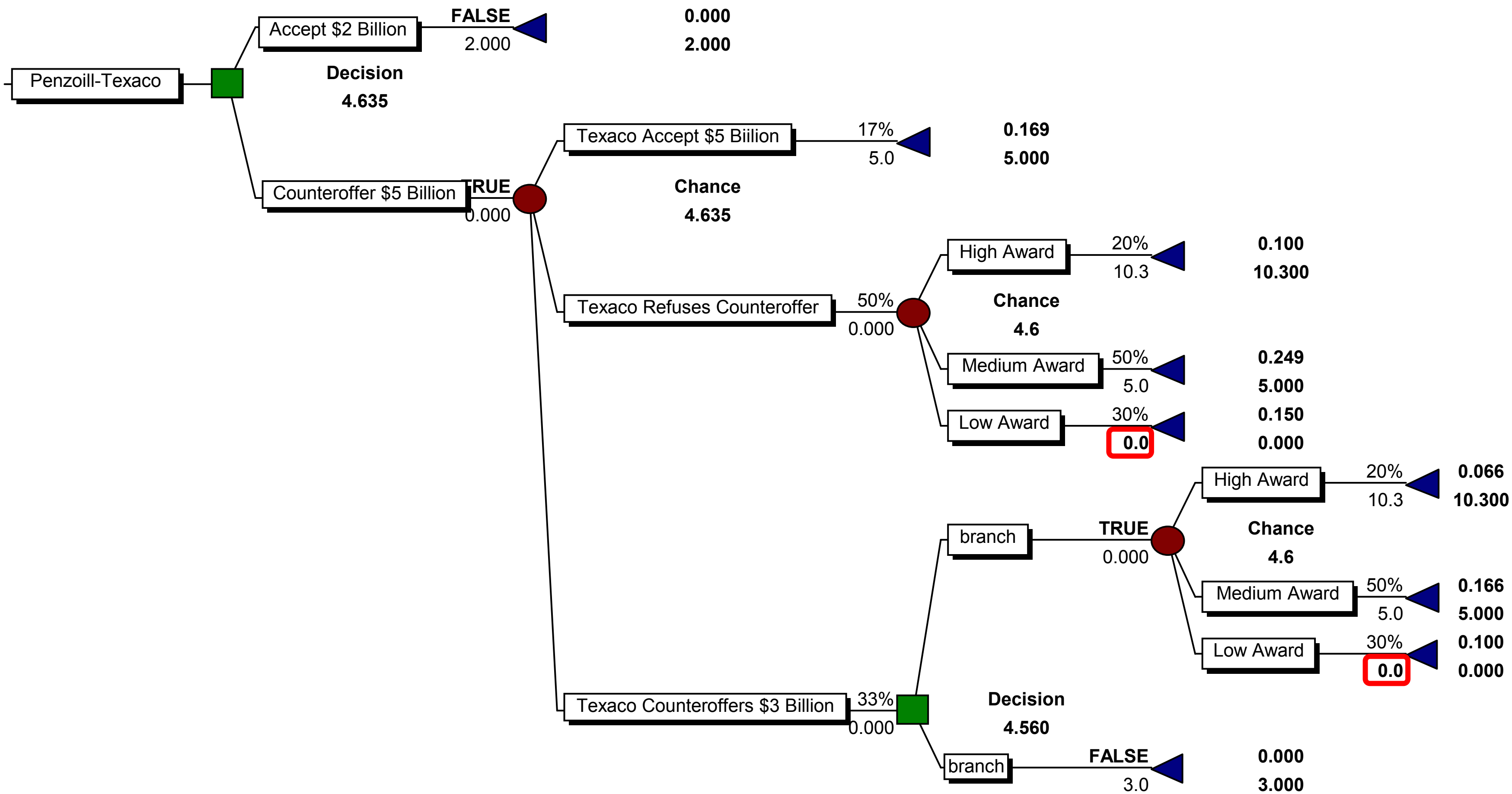
Range 2: [2.5,10.3]

Ranges 1 and 2 are disjoint. The Objective is **Max Result,** hence **Green CRP** deterministically dominates the **Red one.**

Draft: Version 1

Stochastic Dominance: Example 1

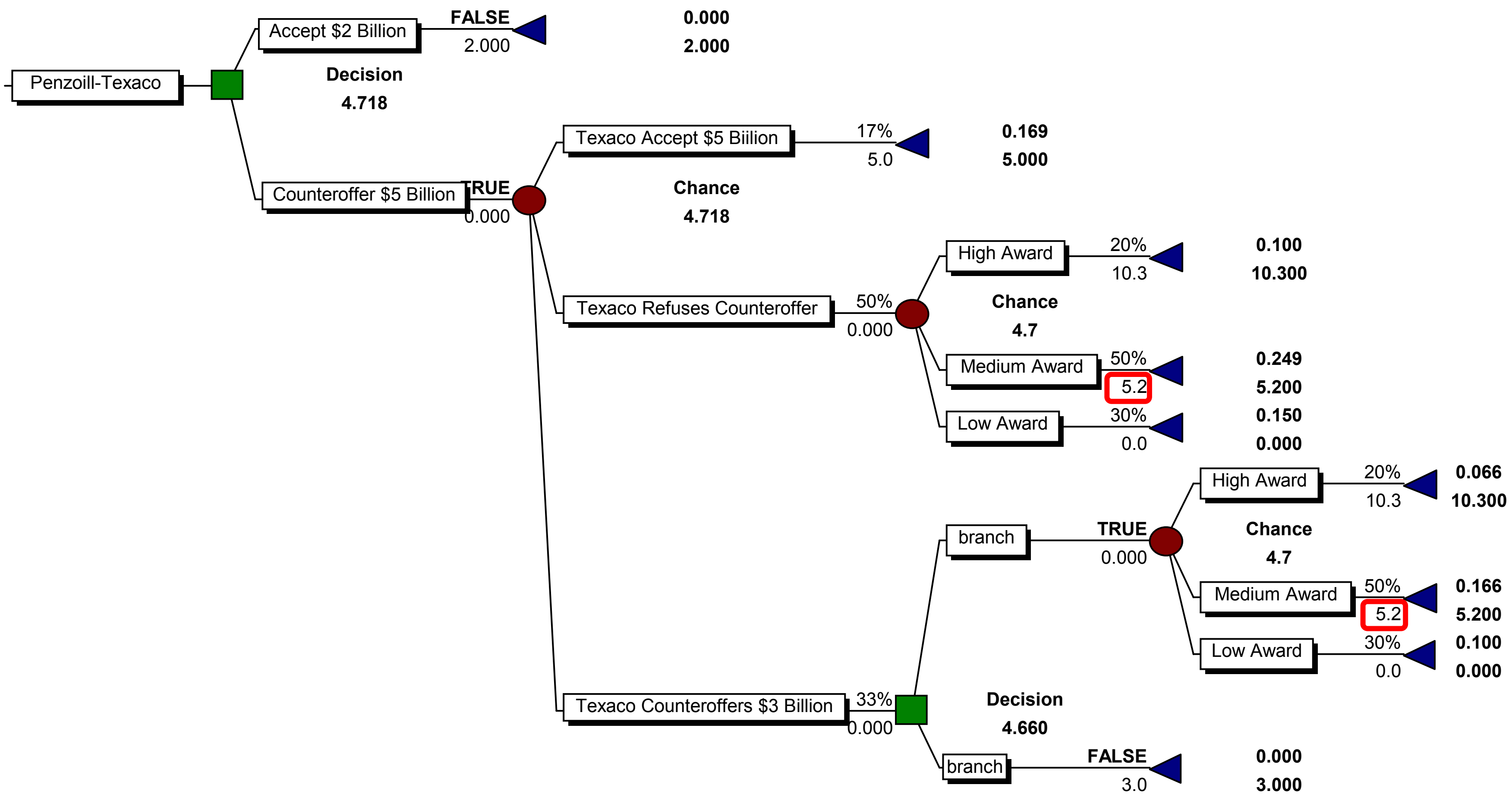
Firm A: Original Tree



Draft: Version 1

Stochastic Dominance: Example 1

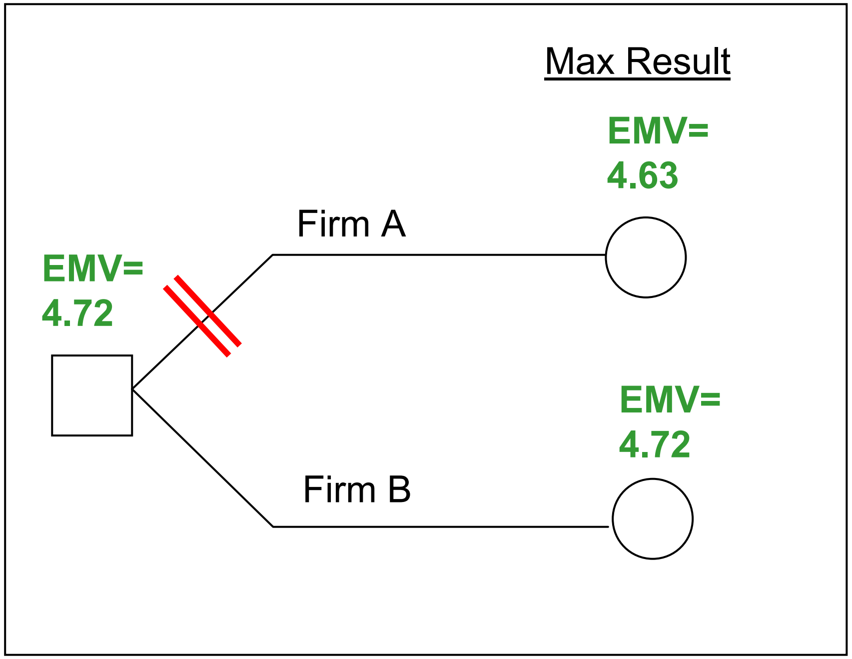
Firm B: Modified Tree



Draft: Version 1

Stochastic Dominance: Example 1

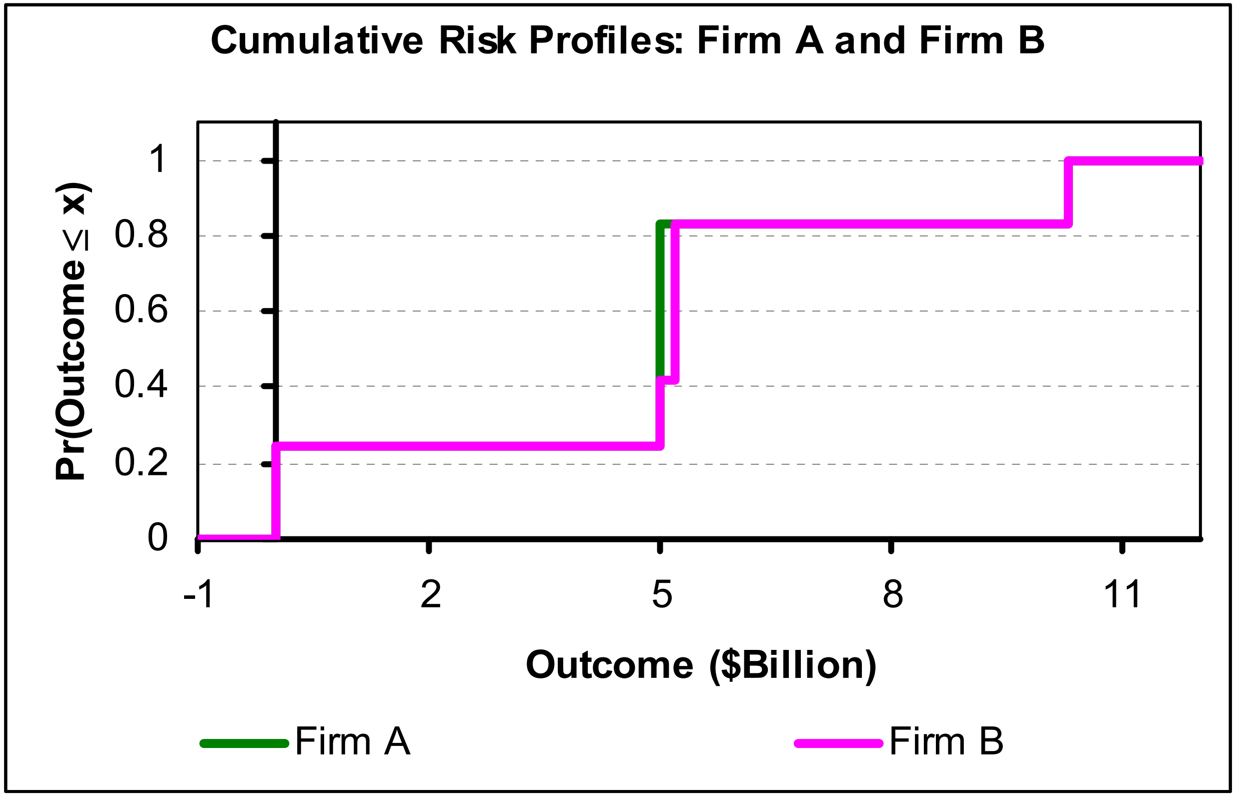
Based on EMV analysis we still choose the alternative "Firm B"



Could we have made a decision here without an EMV analysis ?

Stochastic Dominance: Example 1

Optimal Cumulative risk profiles in "Firm A" Tree and "Firm B" Tree



Draft: Version 1

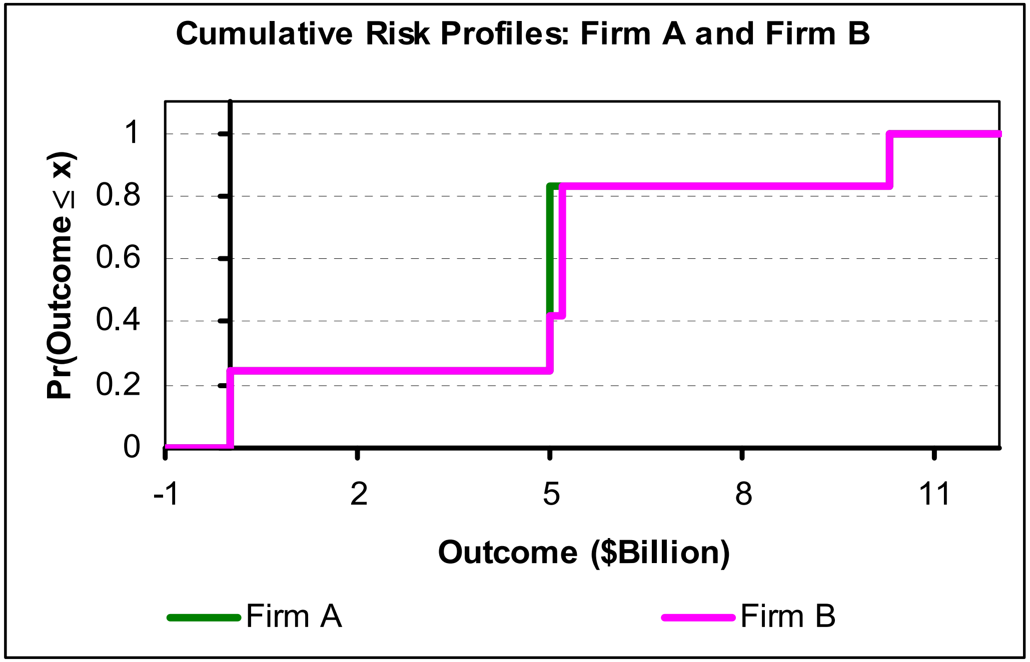
Stochastic Dominance: Example 1

Note that for all possible values of x:

Pr(Outcome ≤ x| Firm B) ≤ Pr(Outcome ≤ x| Firm A)

or equivalently:

Pr(Outcome ≥ x| Firm B) ≥ Pr(Outcome ≥ x| Firm A)



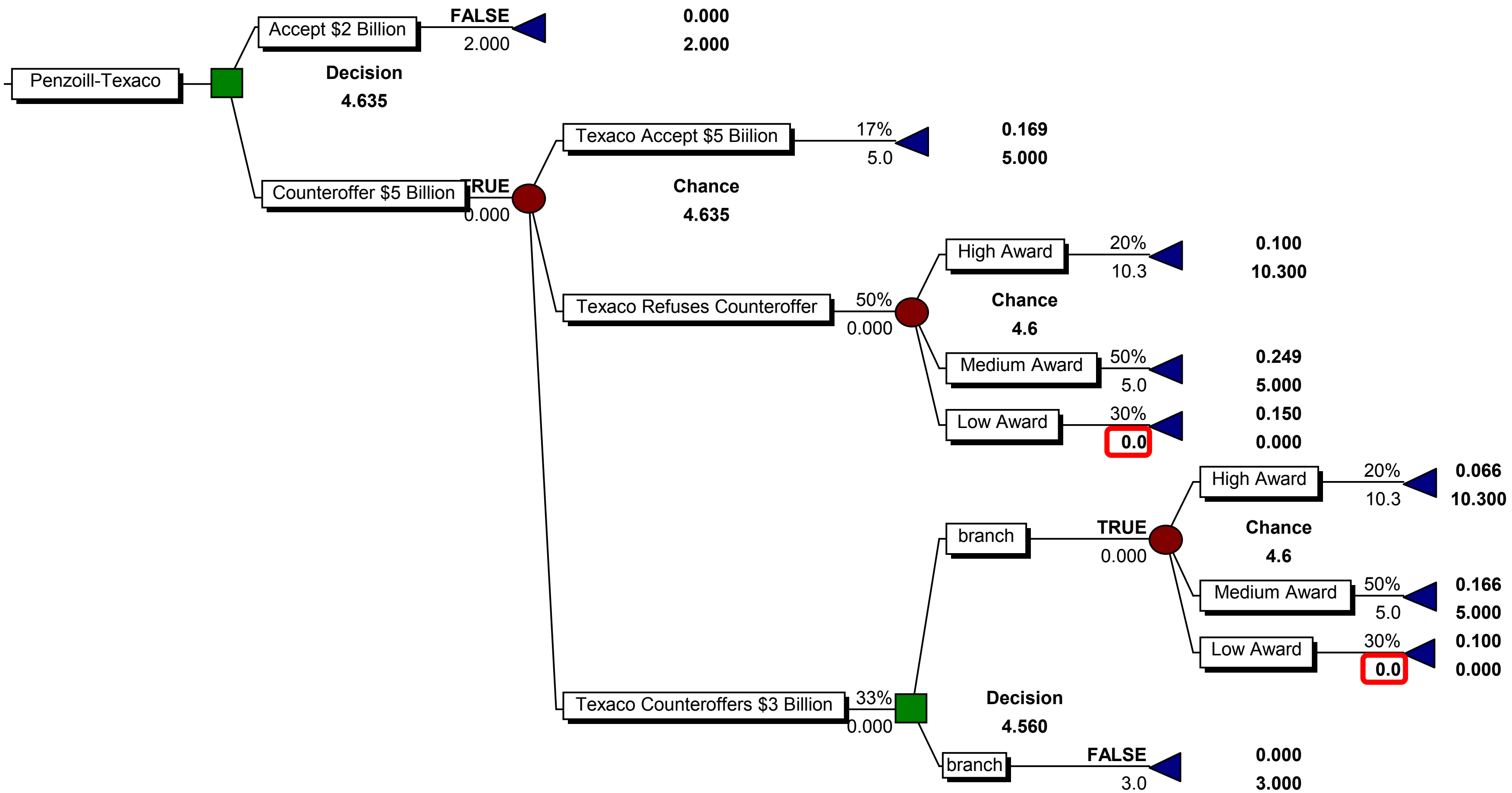
Hence the chances of winning with **Firm B** are always better than that of **Firm A.**

Conclusion: Firm B stochastically dominates Firm A

Draft: Version 1

Stochastic Dominance: Example 2

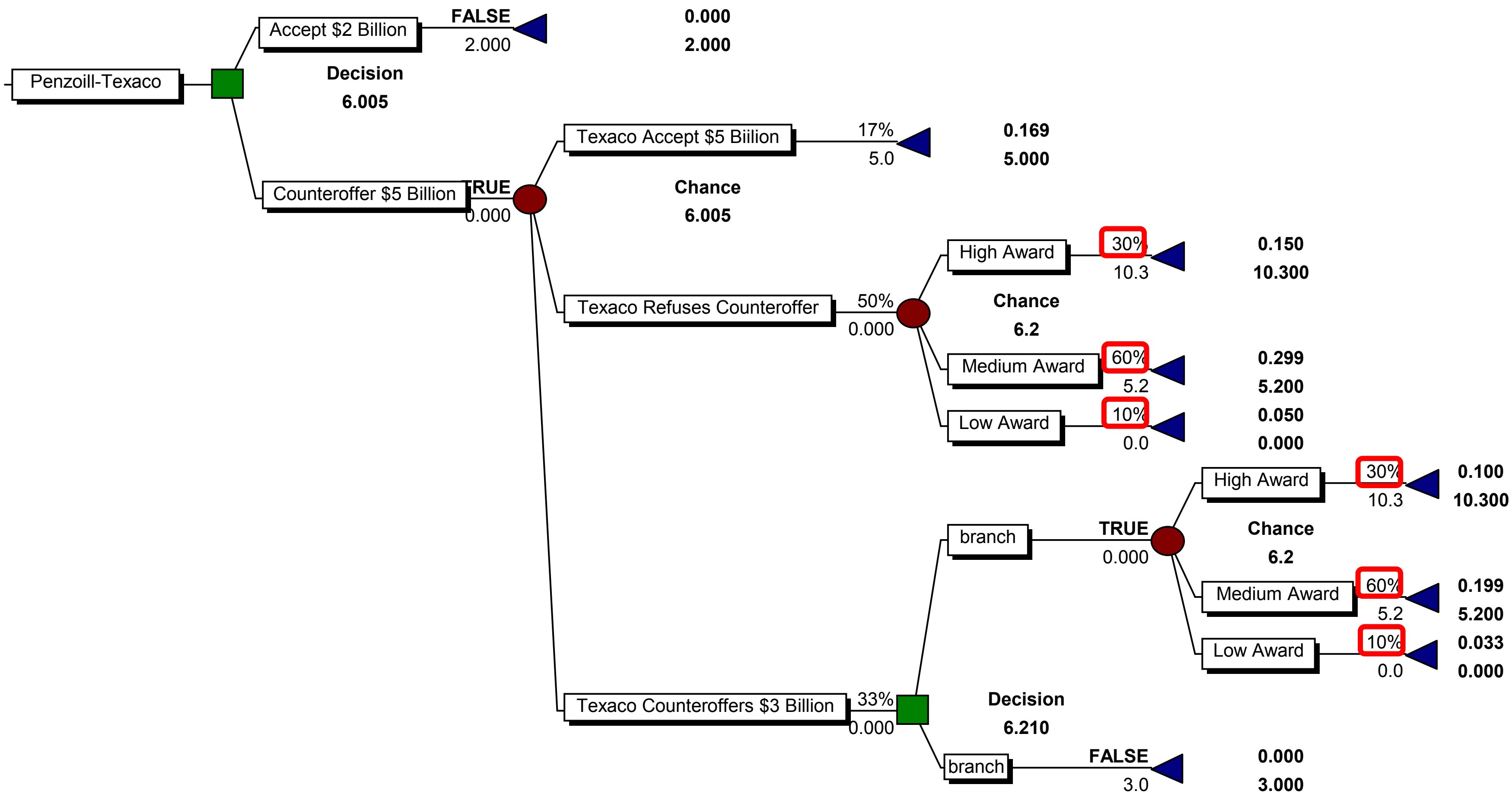
Firm A: Original Tree



Draft: Version 1

Stochastic Dominance: Example 2

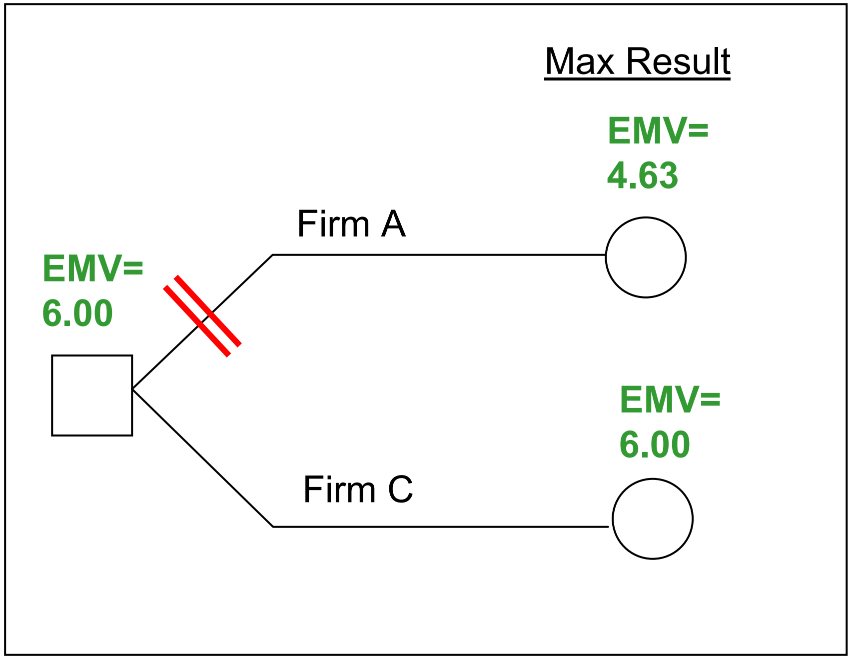
Firm C: Modified Tree



Draft: Version 1

Stochastic Dominance: Example 2

Based on EMV analysis we still choose the alternative “Firm C”

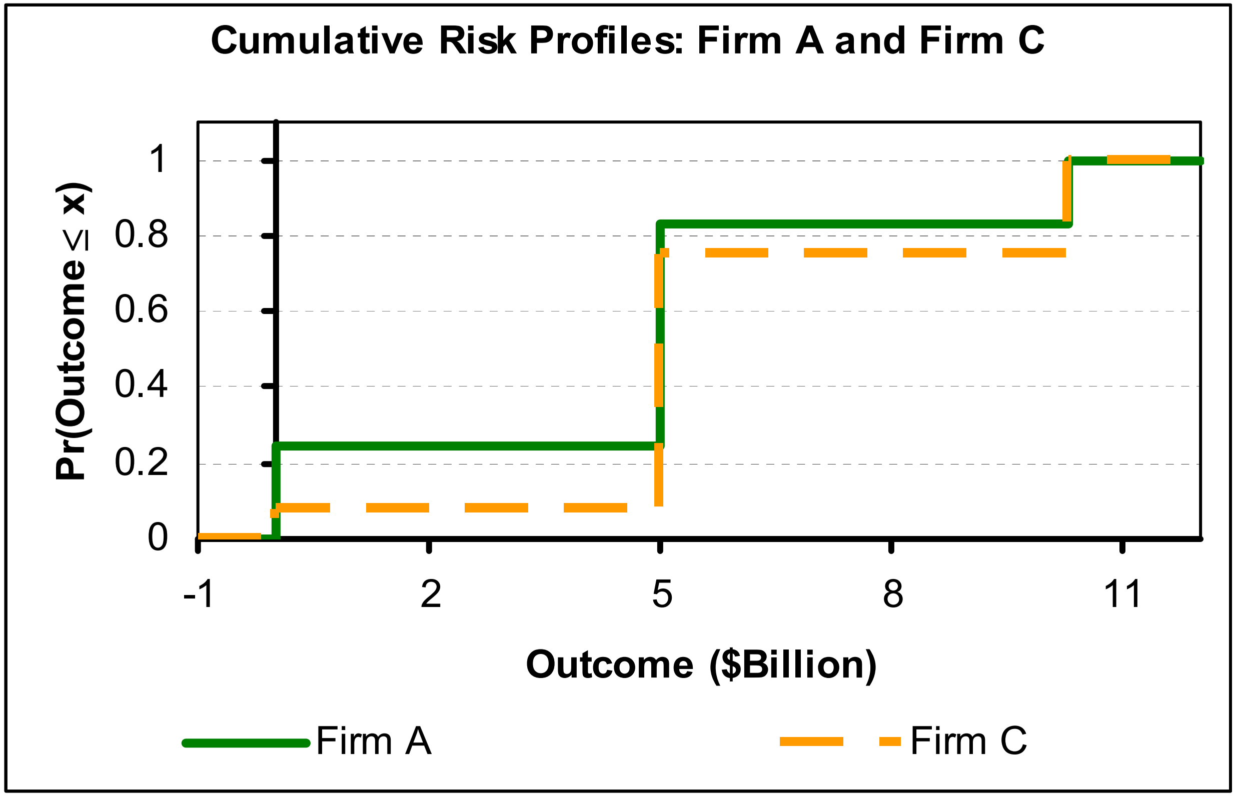


Could we have made a decision here without an EMV analysis ?

Draft: Version 1

Stochastic Dominance: Example 2

Optimal Cumulative risk profiles in "Firm A" Tree and "Firm C" Tree



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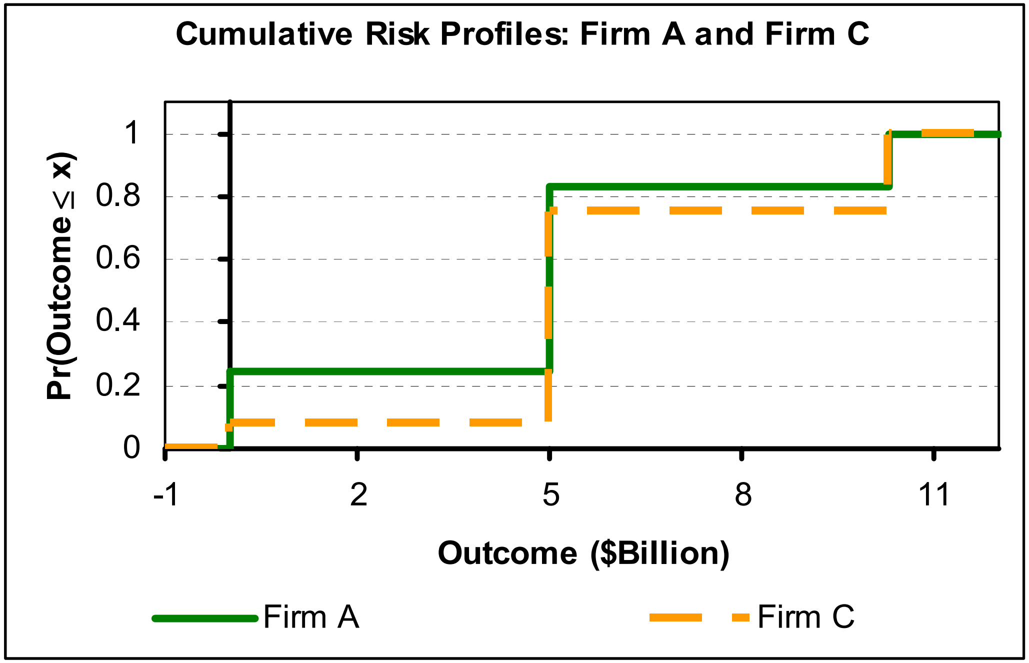
Stochastic Dominance: Example 2

Note that for all possible values of x:

Pr(Outcome $\leq$ x| Firm C) $\leq$
Pr(Outcome $\leq$ x| Firm A)

or equivalently:

Pr(Outcome $\geq$ x| Firm C) $\geq$
Pr(Outcome $\geq$ x| Firm A)



Hence the chances of winning with **Firm C** are always better than that of **Firm A.**

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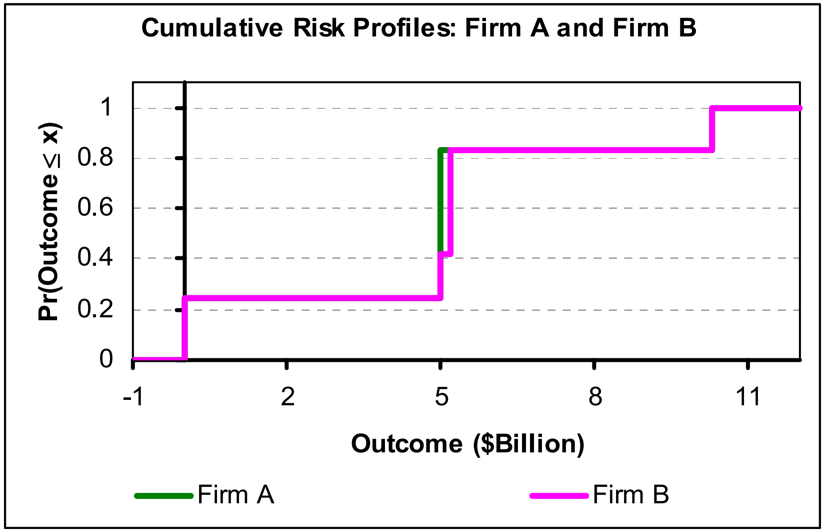
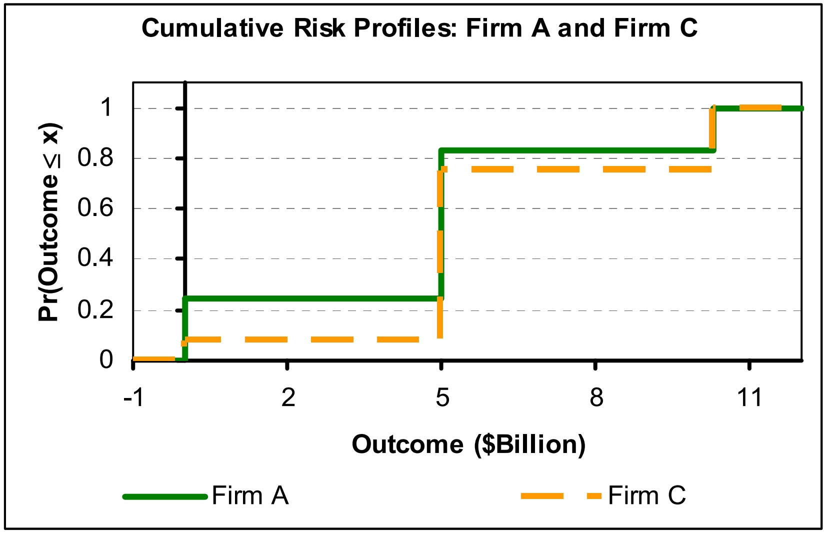
Conclusion: Firm C stochastically dominates Firm A

Stochastic Dominance: Examples 1 & 2

Commonality CRP plots:

- Cumulative risk profiles in both plots do not cross
- The CRP that is toward the **"right and below"** stochastically dominates
- The objective in both plots is to **Maximize the Result**
- What if the objective is **Minimize the Result?**





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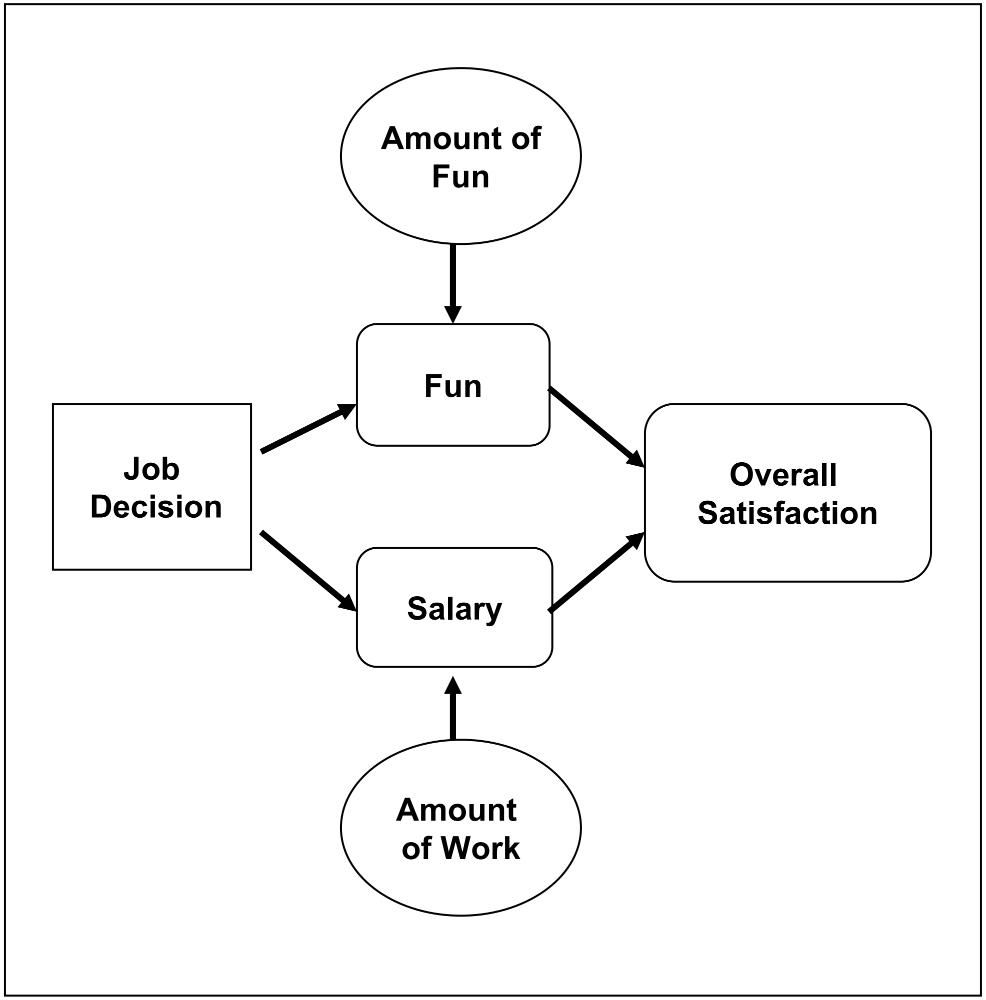
Making Decisions with Multiple Objectives

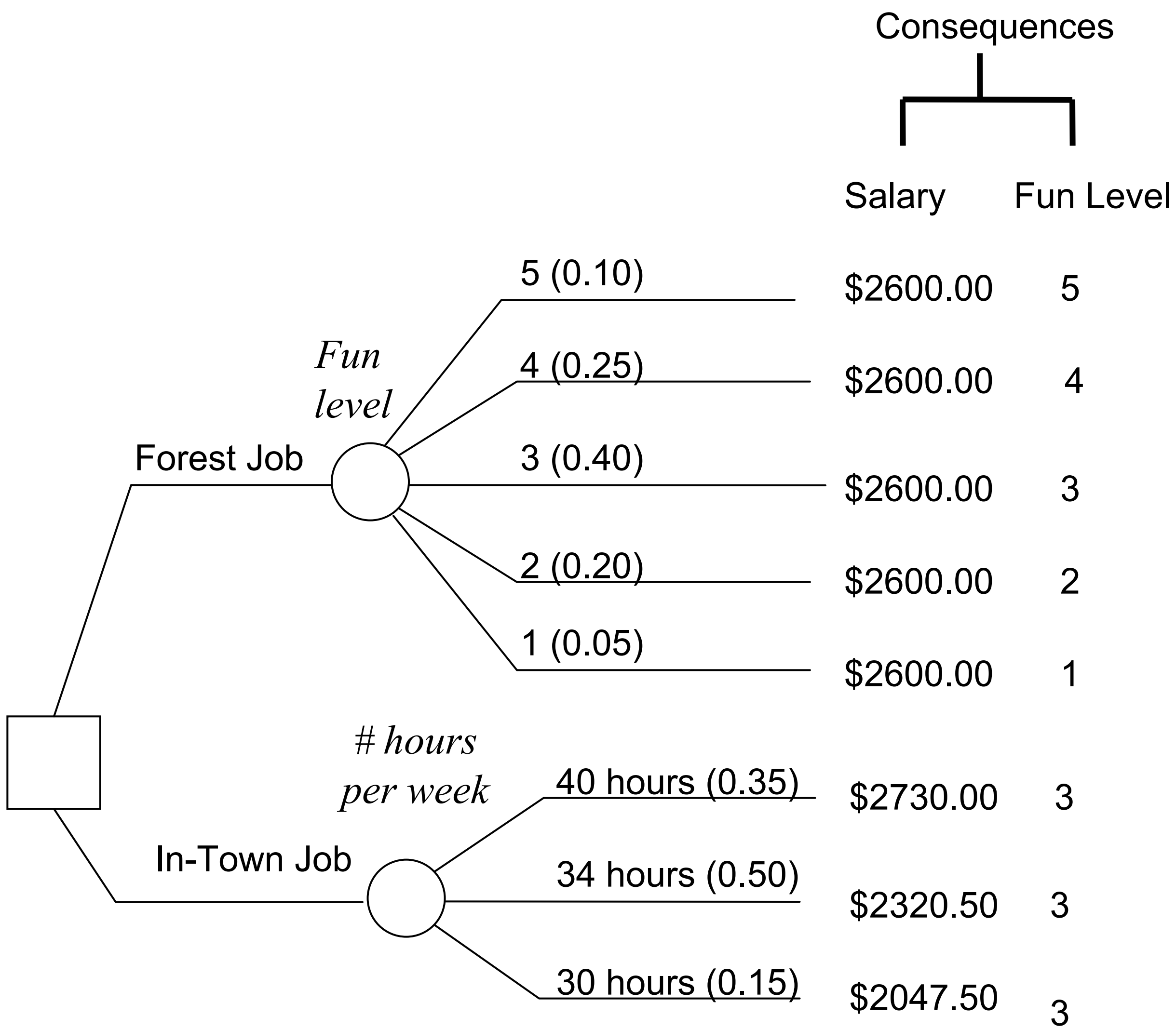
- **Two Objectives:**

  Making Money
  (Measured in $)

  Having Fun
  (Measured on Constructed attribute scale, see page 138):
  Best(5), Good(4), Middle(3), Bad(2), Worst (1)



Making Decisions with Multiple Objectives



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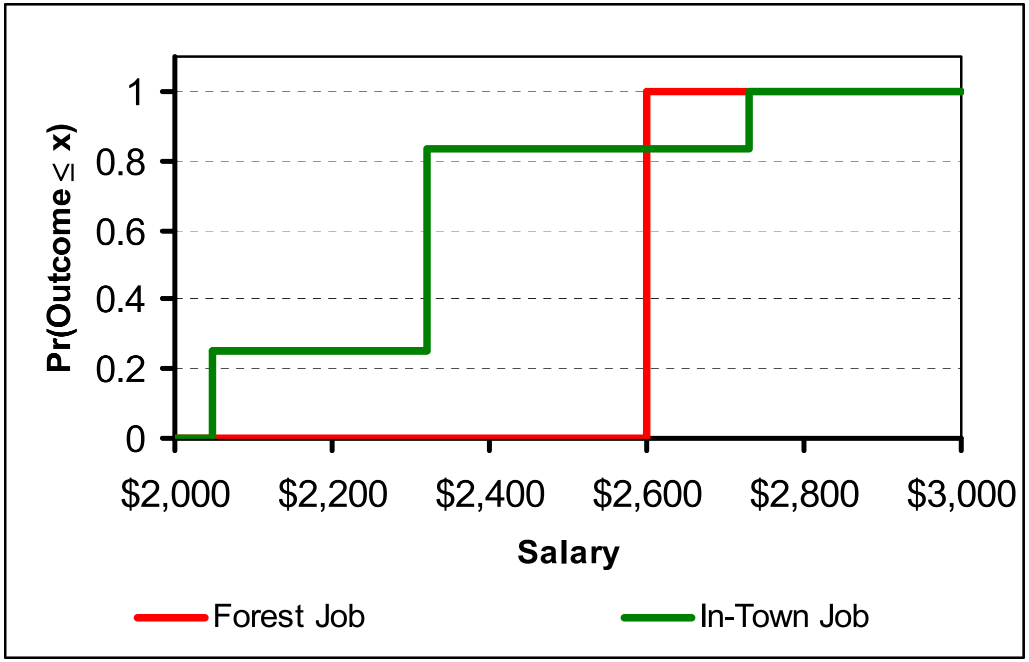
Analysis Salary Objective

	Forest Job		In-Town Job	
Salary	Prob	Salary*Prob	Prob	Salary*Prob
$2,047.50			0.15	$307.13
$2,320.50			0.50	$1,160.25
$2,600.00	1.00	$2,600.00		
$2,730.00			0.35	$955.50
	E[Salary]=	$2,600.00	E[Salary]=	$2,422.88

Conclusion:

- Forest Job preferred Over In-Town job
- CRP's cross. Hence, **No Stochastic Dominance**



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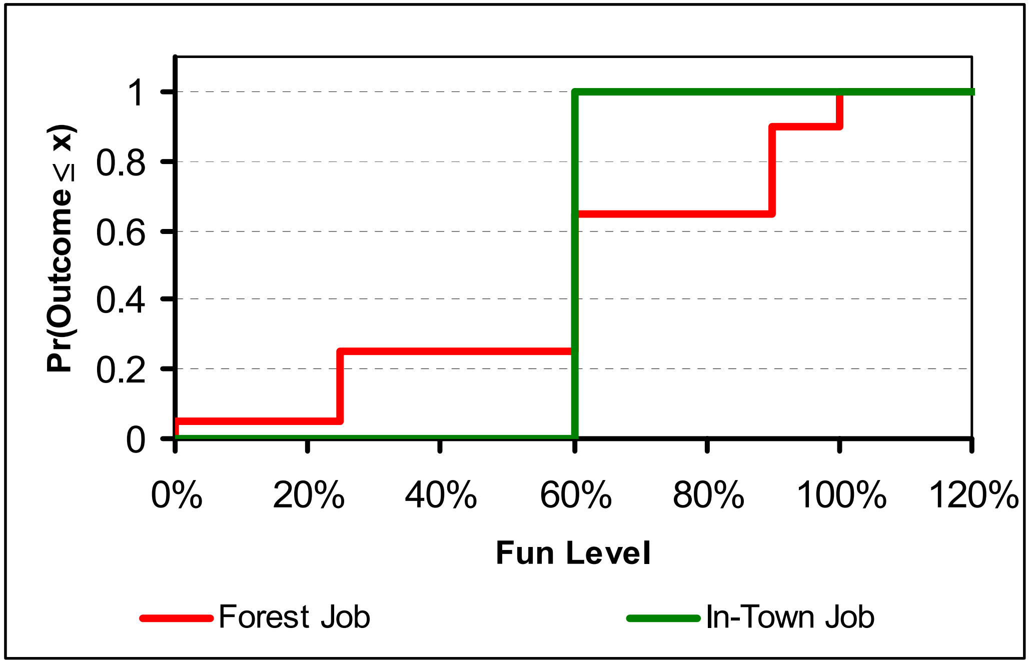
Fun Level Objective

		Forest Job		In-Town Job	
Outcome	**Fun Level**	**Prob**	**Fun Level*Prob**	**Prob**	**Fun Level*Prob**
5 -BEST	100.00%	0.10	10.0%		
4 -GOOD	90.00%	0.25	22.5%		
3 - MIDDLE	60.00%	0.40	24.0%	1.00	60.00%
2 - BAD	25.00%	0.20	5.0%		
1 - WORST	0.00%	0.05	0.0%		
		E[Fun Level]=	**61.5%**	**E[Fun Level]=**	**60.00%**

Conclusion:

- Forest Job preferred Over In-Town job
- CRP's cross. Hence, **No Stochastic Dominance**



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Multiple Objective Analysis

- It is clear from both objective analyses that the **Forest-Job is the strongly preferred,** although **neither Stochastic nor Deterministic Dominance** can be observed in them.

- Careful as you are in your decisions you decide to trade-off the salary objective and having fun objective in a multiple objective analysis.

- Before trade-off analysis can be conducted both objectives have to be measured on a “comparable” scale.

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Multiple Objective Analysis: Construct 0-1 Scale

- **Having Fun Objective** already has a 0-1 scale:

Transformed to 0-1 scale or 0%-100% scale

Set Best=100%, Worst=0%, Determine intermediate values

Having Fun objective:
Best(100%), Good(90%), Middle(60%), Bad(25%), Worst (0%)

- Construct 0-1 scale for **Salary Objective**:

\$2730.00=100%, \$2047.50=0%

Intermediate dollar amount X:

$$\frac{X - \$2047.50}{\$2730 - \$2047.50} \cdot 100\%$$

Multiple Objective Analysis: Assess Trade-Off

k_s = weight for salary $\qquad$ k_f = weight for fun

$$k_s + k_f = 1$$

Using Expert Judgment:

Going from **worst to best in salary objective** is 1.5 times more important than going from **worst to best in having fun objective**. Hence: $k_s = 1.5 \cdot k_f$

$$\begin{cases} k_s + k_f = 1 \\ k_s = 1.5 \cdot k_f \end{cases} \Leftrightarrow \begin{cases} 1.5 \cdot k_f + k_f = 1 \\ k_s = 1.5 \cdot k_f \end{cases} \Leftrightarrow \begin{cases} k_f = \dfrac{1}{2.5} = \dfrac{2}{5} \\ k_s = \dfrac{3}{2} \cdot \dfrac{3}{5} = \dfrac{3}{5} \end{cases}$$

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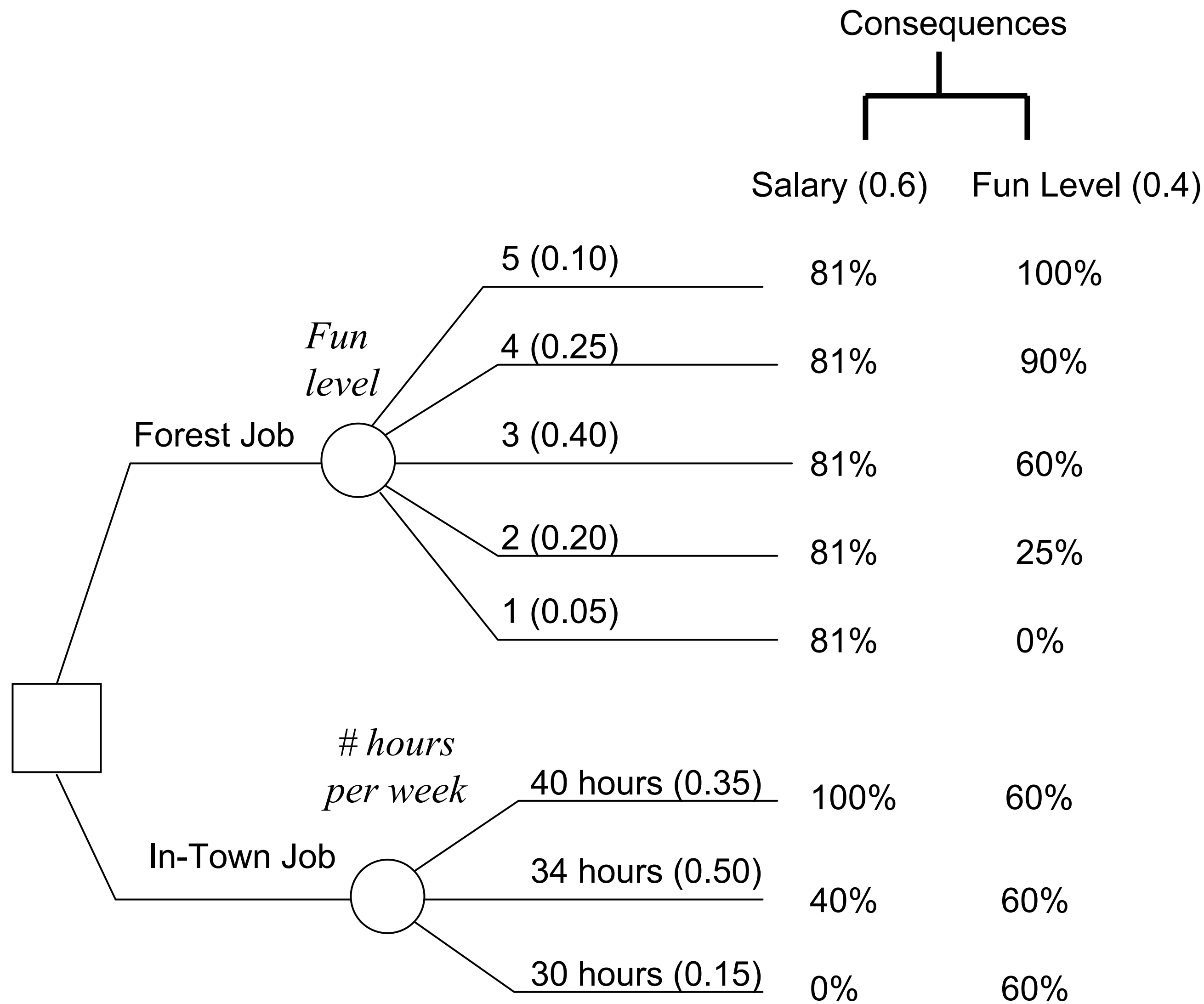
Multiple Objective Analysis: Convert Scales



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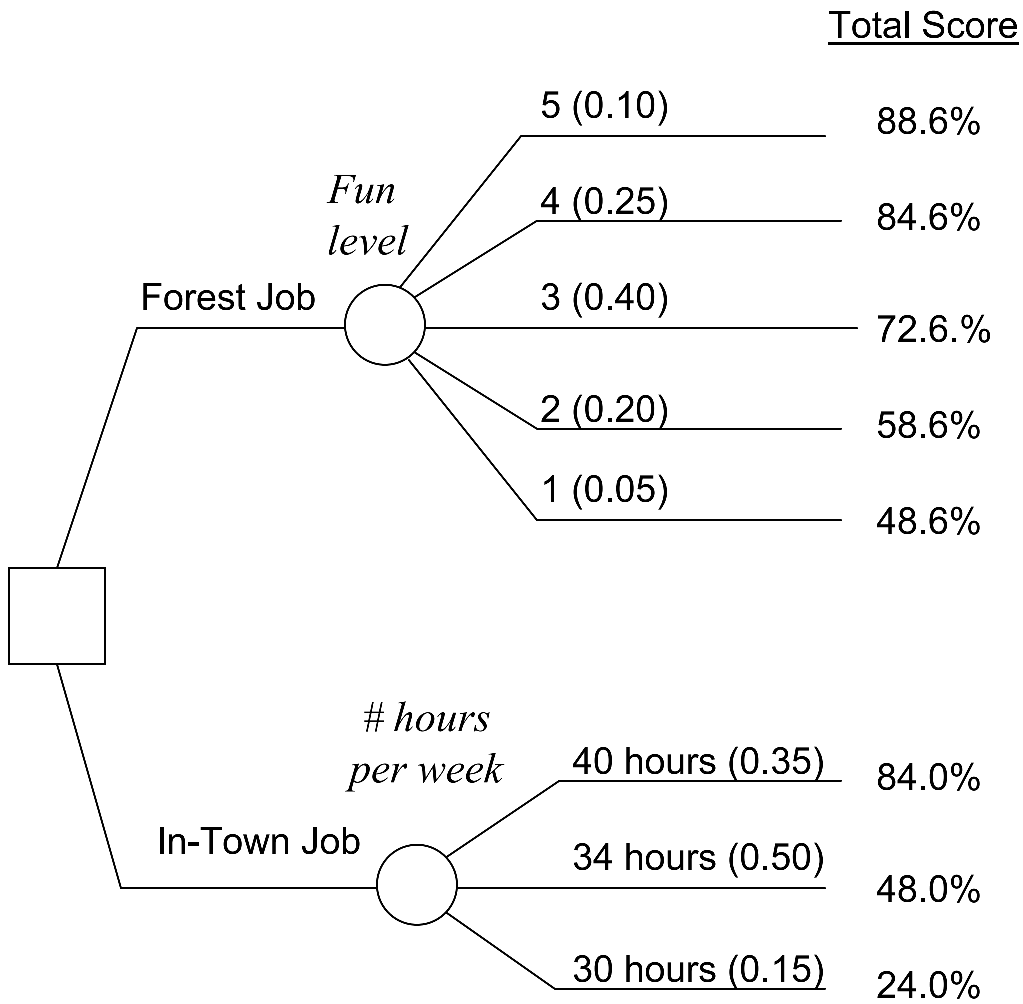
Multiple Objective Analysis: Combine Objectives



Analysis Overall Satisfaction

Forest Job

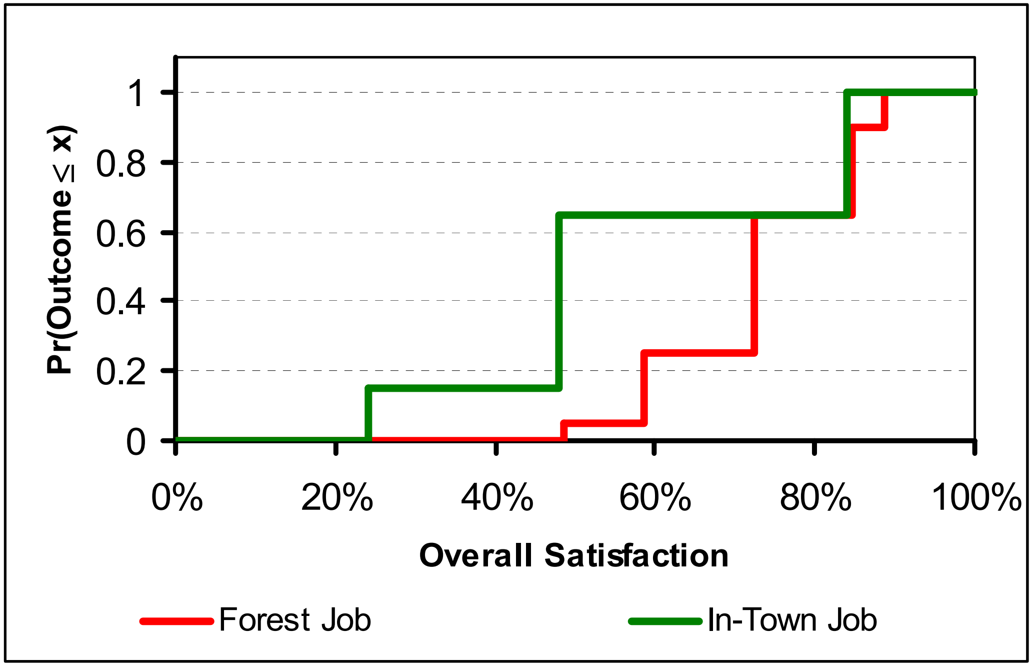
Overall Satisfaction	Prob	OS*Prob
88.57%	0.10	8.9%
84.57%	0.25	21.1%
72.57%	0.40	29.0%
58.57%	0.20	11.7%
48.57%	0.05	2.4%
	E[OS]=	**73.2%**

In-Town Job

Overall Satisfaction	Prob	OS*Prob
84.00%	0.35	29.40%
48.00%	0.50	24.00%
24.00%	0.15	3.60%
	E[OS]=	**57.00%**

Conclusion:

- Forest Job preferred Over In-Town job
- CRP's do not cross. Hence, **Stochastic Dominance present.**



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