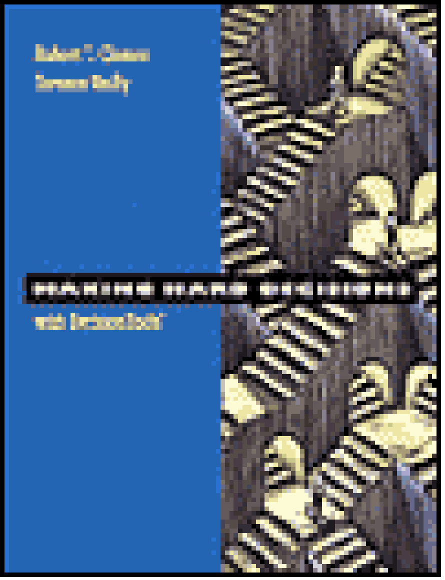

Making Hard Decisions

R. T. Clemen, T. Reilly

Chapter 12

Value of Information

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Introduction

Often you **pay for information** you are asking for:

- Investment Advice
- Management Consultants
- Market Investigation
- Palm Reading

You need this information to make **a decision in the future:**

- To invest in a particular stock or not
- To restructure the organization of your company or not
- To introduce a product or not
- Should I marry this person or not

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Introduction

Problem at hand:

Given your decision problem, how much should you be willing to pay for this information?

- To answer this questions you have to determine the value (in dollars) of information.

- We will first discuss a method for determining the value of **perfect information** and next for **imperfect information**.

WHICH ONE DO YOU VALUE MORE?

Definition: Clairvoyant Expert on event A

If event A is about to occur, the expert says, it will. If event A is not to occur, the expert says, it will not. The expert is **NEVER** wrong. His information is **PERFECT**.

A = { Dow Jones index goes up}

"A" = {**Expert Says** Dow Jones index goes up}

You are considering investing in a company, but before you do you want to make sure that the **Dow Jones index will go up** as this increases your chances of making a good investment. Therefore, you decide to consult **a clairvoyant expert** on the event A.

Probability and Perfect Information

What does it mean to be clairvoyant in **probabilistic terms**?

Pr({ Expert Says Dow Jones ↑ } | { Dow Jones ↑ }) =
Pr("A" | A) = 1

Similarly:

$$\Pr("A" | A) = 1 \Leftrightarrow 1 - \Pr(\overline{"A"} | A) = 1 \Leftrightarrow \Pr(\overline{"A"} | A) = 0$$

$$\Pr("A" | \overline{A}) = 0 \Leftrightarrow 1 - \Pr(\overline{"A"} | \overline{A}) = 0 \Leftrightarrow \Pr(\overline{"A"} | \overline{A}) = 1$$

Perhaps more importantly, what about?

Pr({ Dow Jones ↑ } | { Expert Says Dow Jones ↑ }) =
Pr("A" | A)

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Probability and Perfect Information

$$\Pr(A\,|\,"A") = \frac{\Pr("A"\,|\,A)\Pr(A)}{\Pr("A")} =$$

$$\frac{\Pr("A"\,|\,A)\Pr(A)}{\Pr("A"\,|\,A)\Pr(A) + \Pr("A"\,|\,\overline{A})\Pr(\overline{A})} =$$

$$\frac{1\cdot\Pr(A)}{1\cdot\Pr(A) + 0\cdot\Pr(\overline{A})} = 1$$

Conclusion:

Pr(A|"A") equals 1 no matter what the value of Pr(A) is.

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Probability and Perfect Information

What about the probability Pr({Expert Says Dow Jones ↑})?

$$\Pr(\text{"A"}) = \Pr(\{\text{Expert Says Dow Jones} \uparrow\}) =$$

$$\Pr(\text{"}A\text{"} \mid A)\Pr(A) + \Pr(\text{"}A\text{"} \mid \overline{A})\Pr(\overline{A}) =$$

$$1 \cdot \Pr(A) + 0 \cdot \Pr(\overline{A}) = \Pr(A) = \Pr(\{\text{Dow Jones} \uparrow\})$$

This is true in general: if we consult a **clairvoyant expert** about an event a with possible outcomes $\{A_1, \cdots, A_n\}$ then:

$$\Pr(\text{"}\boldsymbol{A_i}\text{"}) = \Pr(\boldsymbol{A_i}),\ \text{for all } \boldsymbol{i} = 1, \cdots, \boldsymbol{n}$$

After consulting the **clairvoyant expert** about event a, **no uncertainty** remains about event a.

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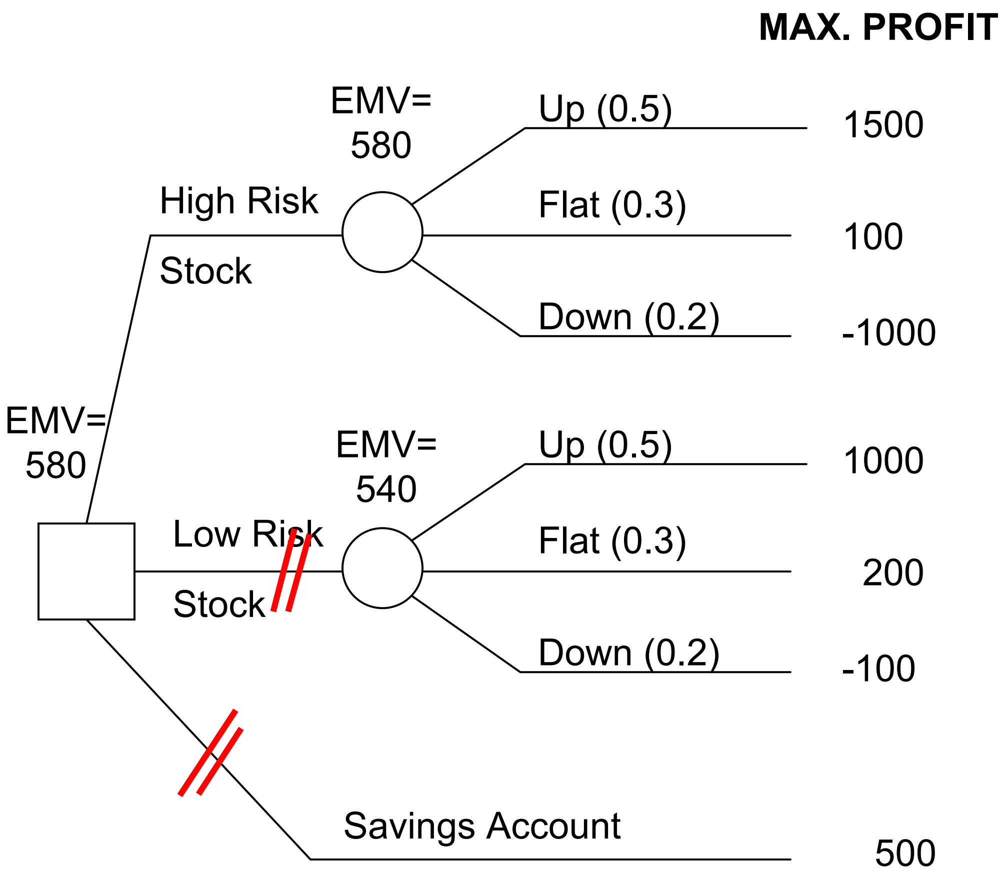
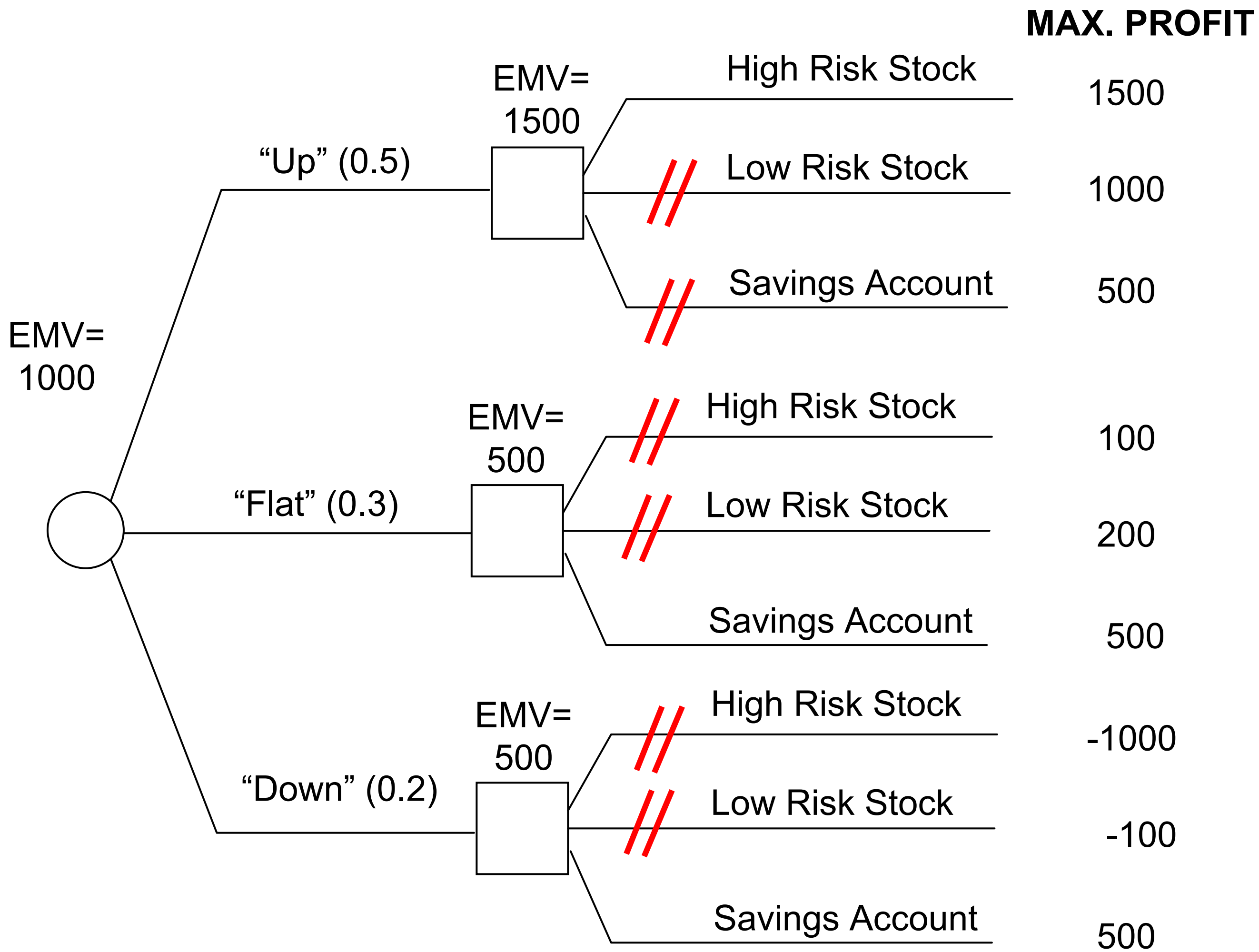
STOCK MARKET EXAMPLE:



Expected Value of Perfect Information

Consider first talking to **a clairvoyant expert** and then making the investment decision:



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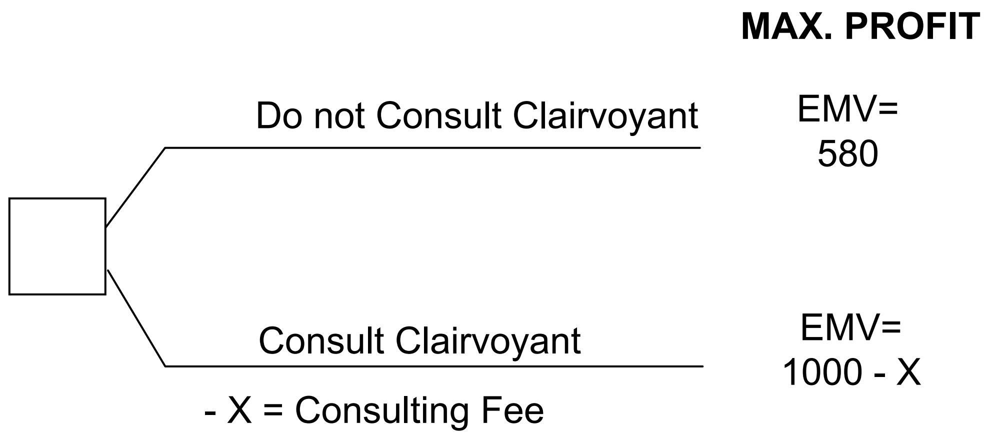
Expected Value of Perfect Information

Of course, the **clairvoyant expert** will charge a fee and you would like to know how much you would be willing to pay before using his services.



Conclusion:

You would be willing to consult the **clairvoyant expert** if:

$$1000 - X \geq 580 \Leftrightarrow X \leq 1000 - 580 = 420 \text{ (=EVPI)}$$

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Expected Value of Perfect Information

EVPI = Expected Value of Perfect Information

Interpretation:

EVPI is the **maximum amount of money** you would be willing to pay for the services of the **clairvoyant expert.** If he charges more than $420 you would not consult the expert.

A = { Dow Jones index goes up}

"A" = {Expert Says Dow Jones index goes up}

Consider an Expert about event A, who is **not clairvoyant,** but is considered to be an expert. What does it mean in for an expert **not to be perfect** in his assessment about event A?

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Expected Value of Imperfect Information

- Pr({Expert Says Dow Jones ↑} | {Dow Jones ↑}) =

$$\Pr("A" \mid A) < 1$$

Hopefully, the probability above is close to 1
(otherwise why consider him/her and Expert?)

- Pr({Expert Says Dow Jones ↑ } | {Dow Jones ↓}) =

$$\Pr("A" \mid \overline{A}) > 0$$

Hopefully, the probability above is close to 0
(otherwise why consider him/her and Expert?)

When an expert about an event is **not clairvoyant** you need to express **your trust** in his assessment by for example, checking his past performances and inter-viewing references.

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Expected Value of Imperfect Information

Based on **your background-check** of the expert you assess your trust in terms of subjective probabilities.

	True Market State		
Expert Prediction	UP	FLAT	DOWN
"UP"	Pr("UP"|UP)	Pr("UP"|FLAT)	Pr("UP"|DOWN)
"FLAT"	Pr("FLAT"|UP)	Pr("FLAT"| FLAT)	Pr("FLAT"|"|DOWN)
"DOWN"	Pr("DOWN"|UP)	Pr("DOWN"| FLAT)	Pr("DOWN"|"|DOWN)
Total	1	1	1

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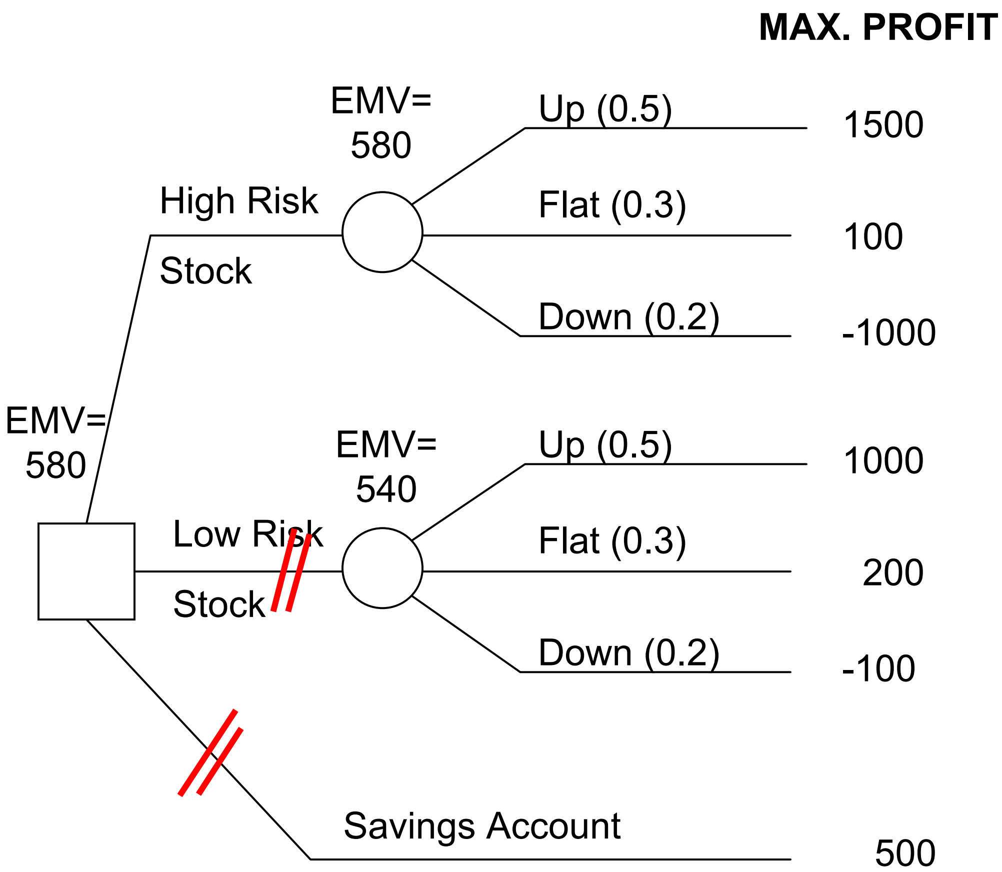
Actual Assessment of the Expert:

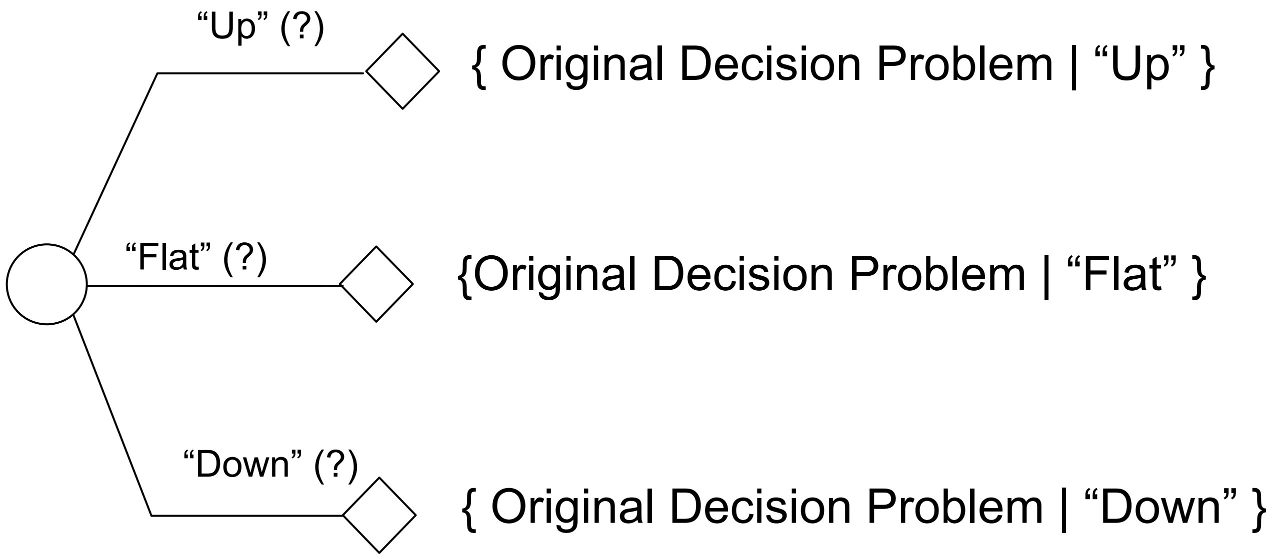
	True Market State		
Expert Prediction	UP	FLAT	DOWN
"UP"	80%	15%	20%
"FLAT"	10%	70%	20%
"DOWN"	10%	15%	60%
Total	1	1	1

STOCK MARKET EXAMPLE:



Expected Value of Imperfect Information

Consider first talking to an **"Imperfect expert"** and then making the investment decision:



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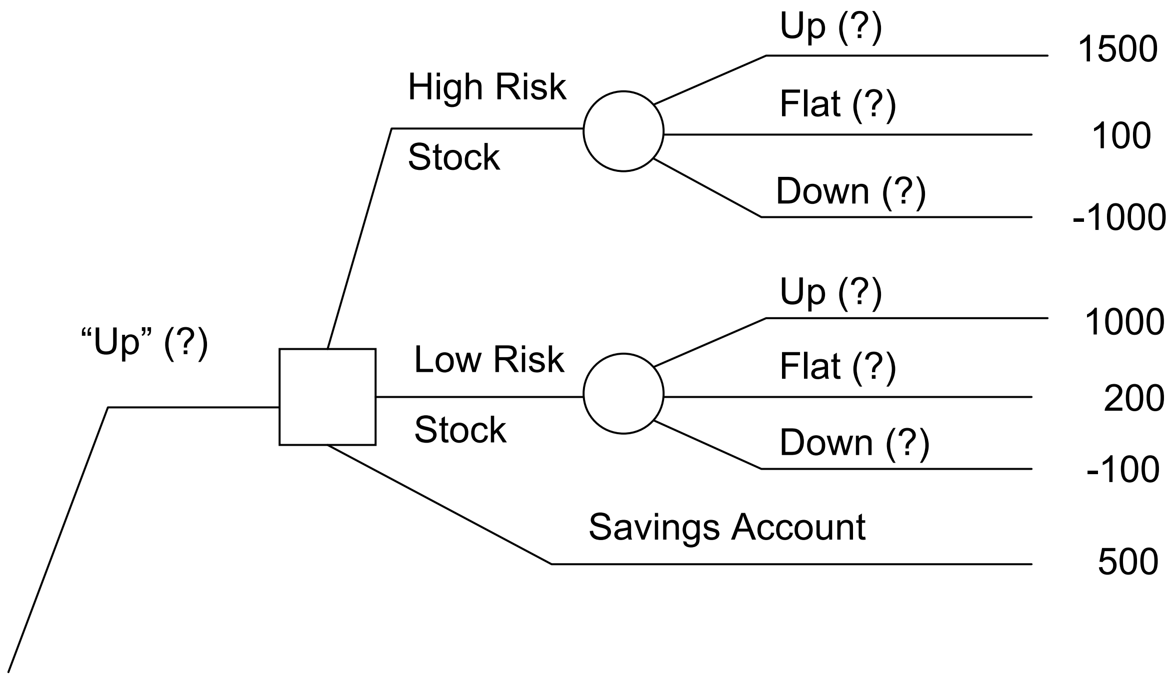
Expected Value of Imperfect Information

Suppose the **Imperfect expert** said Dow Jones will go **UP**



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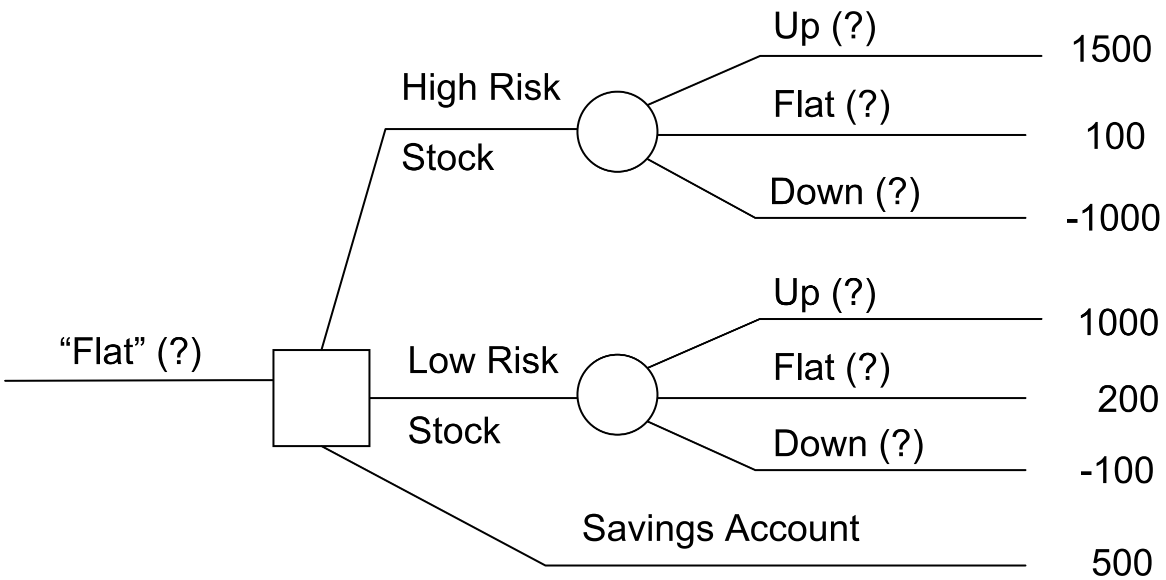
Expected Value of Imperfect Information

Suppose the **Imperfect expert** said Dow Jones will stay **FLAT**



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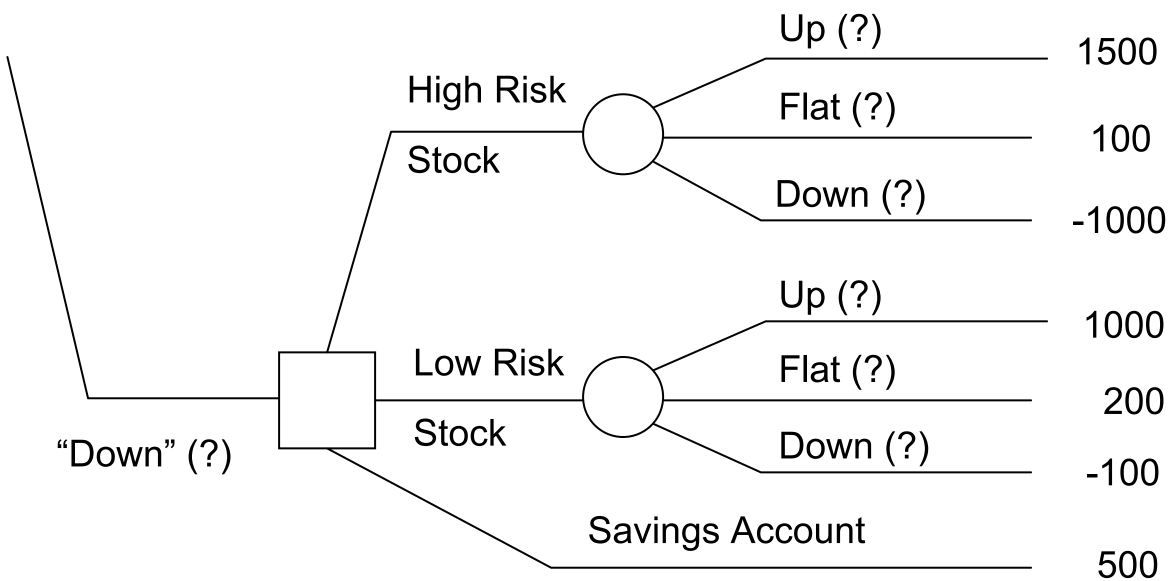
Expected Value of Imperfect Information

Suppose the **Imperfect expert** said Dow Jones will go **DOWN**



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Expected Value of Imperfect Information

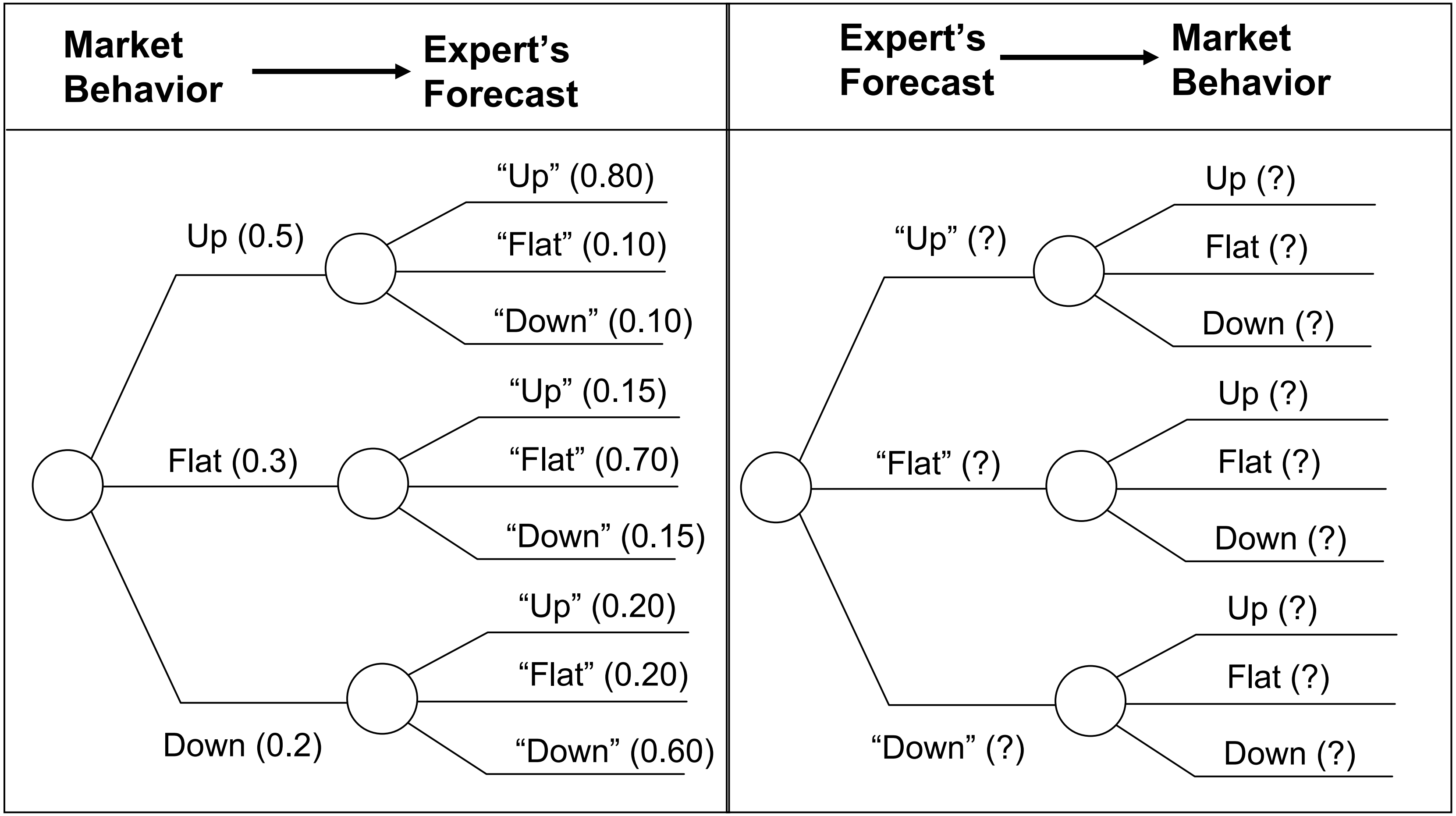
Note That:

- After consulting the expert the **uncertainty remains**

- After consulting an imperfect expert, **the original decision problem still remains**. The only difference is that probabilities of the original decision problem **have changed** to reflect the additional information, i.e the expert's advise.

To calculate the EMV of the decision problem **after consulting the imperfect expert** we have to solve for the probabilities in the decision tree above. Calculating these probabilities is equivalent with **FLIPPING** the order of the uncertainty nodes.

Expected Value of Imperfect Information



How can we solve for these probabilities?

Via **Bayes theorem** using a **probability table**

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Expected Value of Imperfect Information

STEP 1: Construct a probability table

	Pr(Up)	Pr(Flat)	Pr(Down)
	0.500	0.300	0.200
"A"	Pr("A"|Up)	Pr("A"|Flat)	Pr("A"|Down)
"Up"	0.800	0.150	0.200
"Flat"	0.100	0.700	0.200
"Down"	0.100	0.150	0.600
Check	1.000	1.000	1.000

	Pr("A" ∩ Up)	Pr("A" ∩ Flat)	Pr("A" ∩ Down)	Pr("A")	Pr(Up|"A")	Pr(Flat|"A")	Pr(Down|"A")	Check
	0.400	0.045	0.040	0.485	0.825	0.093	0.082	1.000
	0.050	0.210	0.040	0.300	0.167	0.700	0.133	1.000
	0.050	0.045	0.120	0.215	0.233	0.209	0.558	1.000
Check	0.500	0.300	0.200	1.000				

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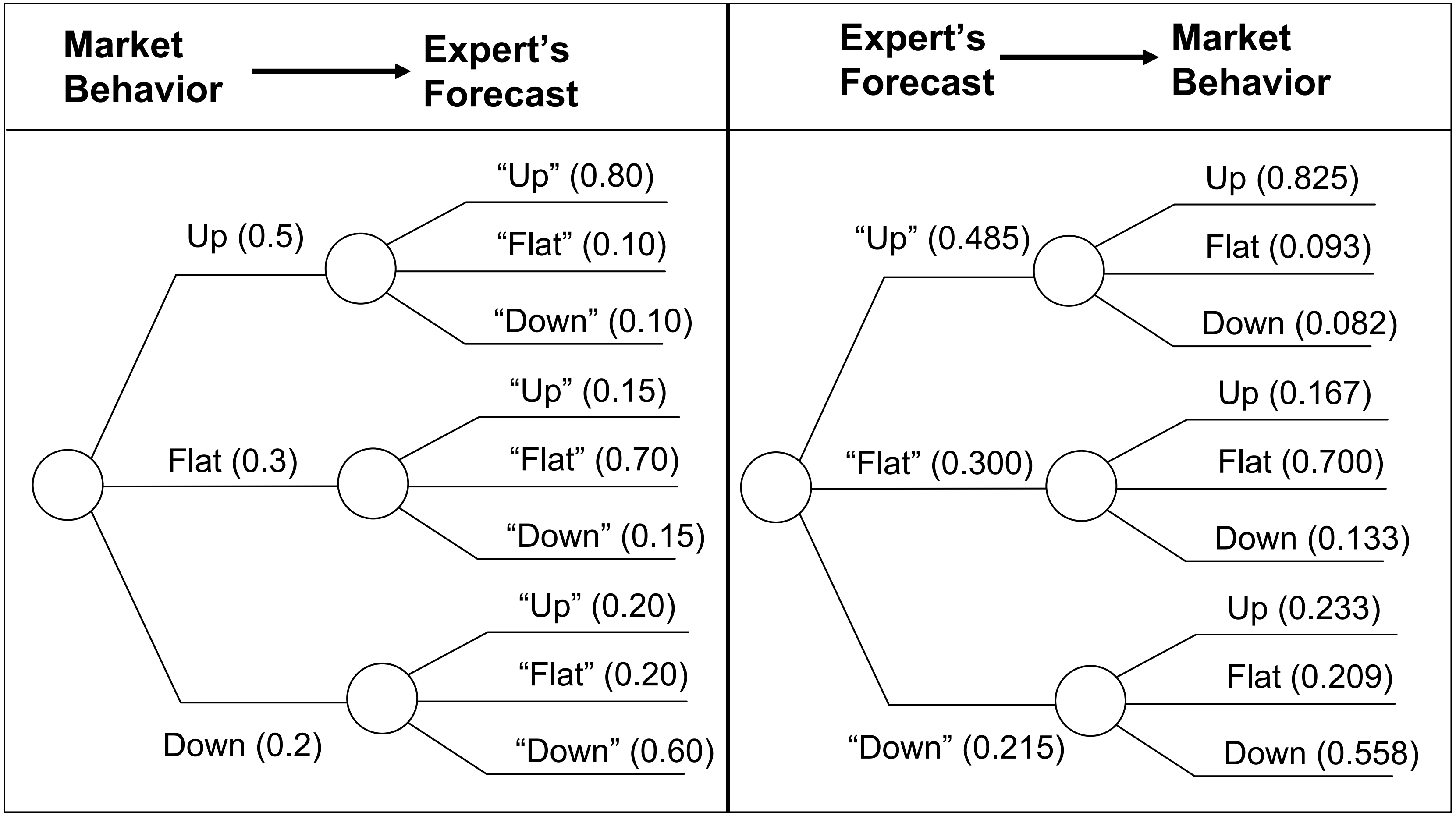
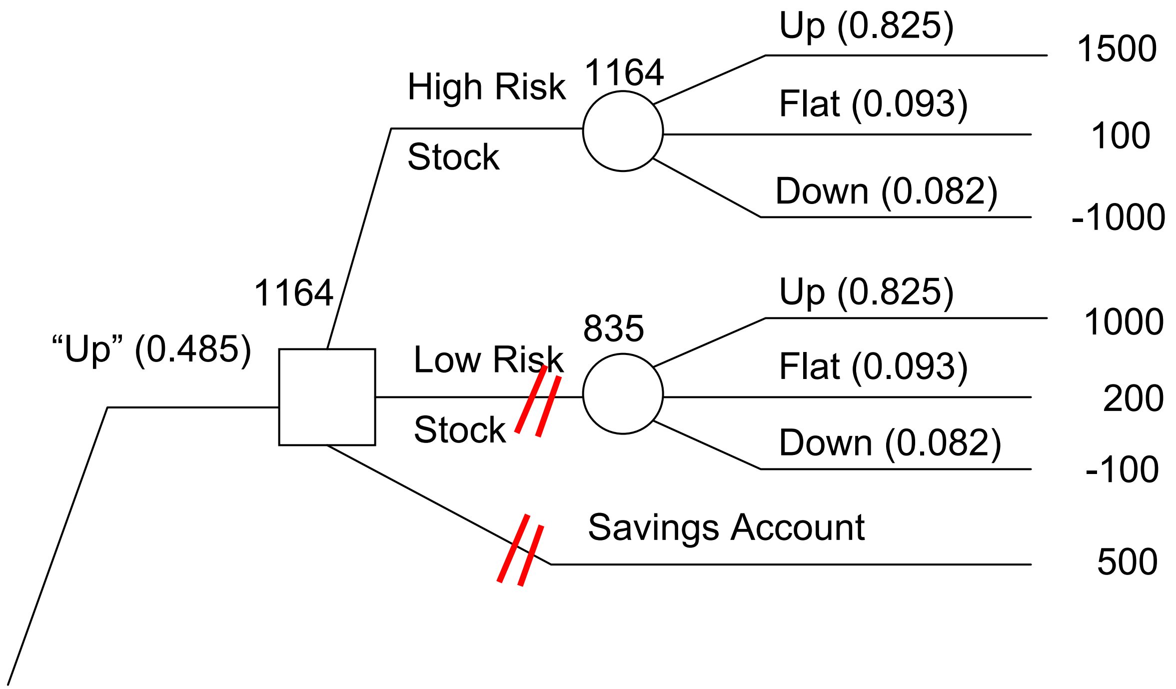
Expected Value of Imperfect Information

STEP 2: Insert the probabilities in the probability tree

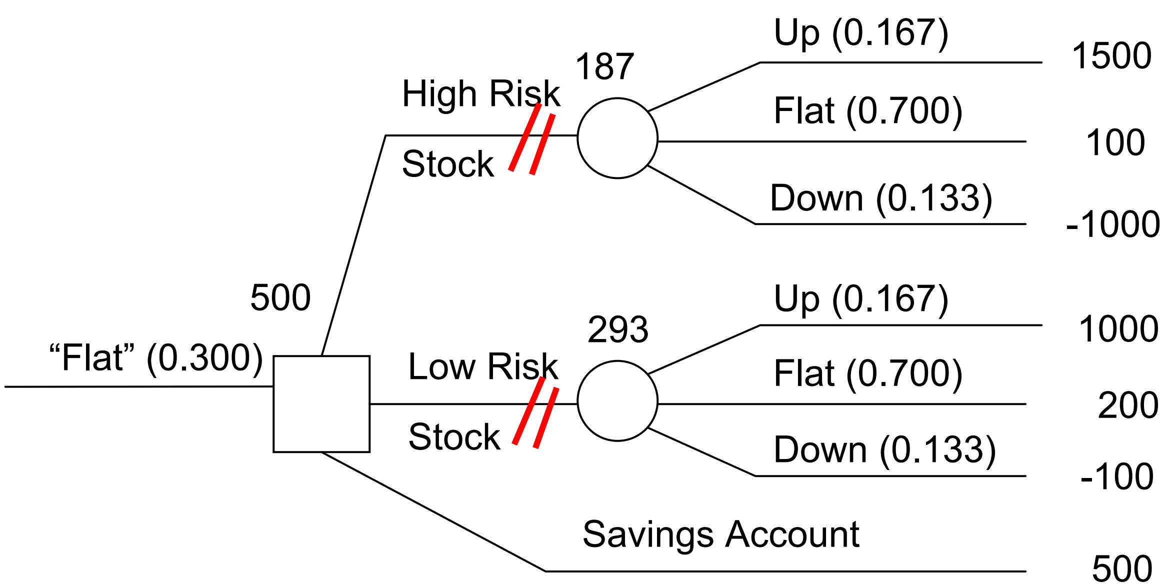


STEP 3: Calculate EMV after consulting the expert



Expected Value of Imperfect Information

STEP 3B: Calculate EMV after consulting the expert



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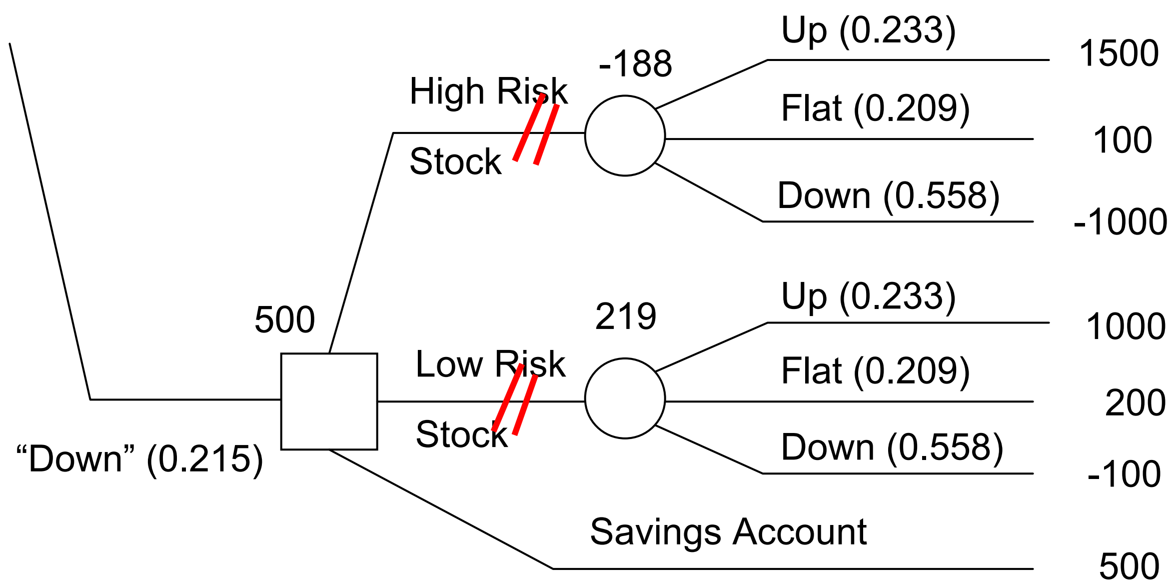
Expected Value of Imperfect Information

STEP 3C: Calculate EMV after consulting the expert



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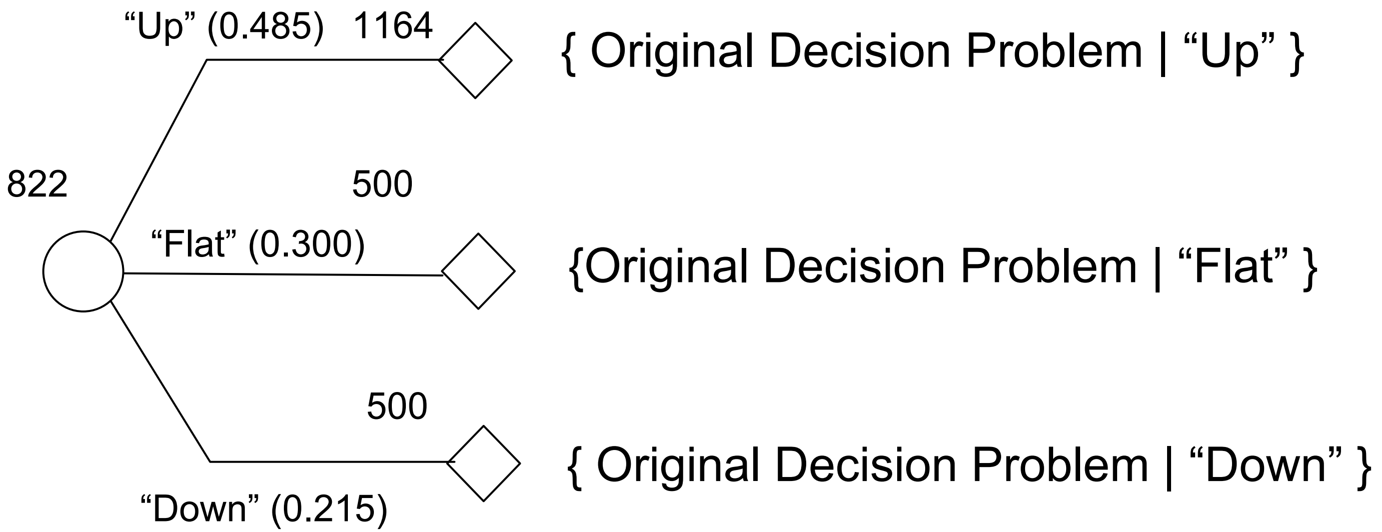
Expected Value of Imperfect Information

STEP 3D: Calculate EMV after consulting the expert



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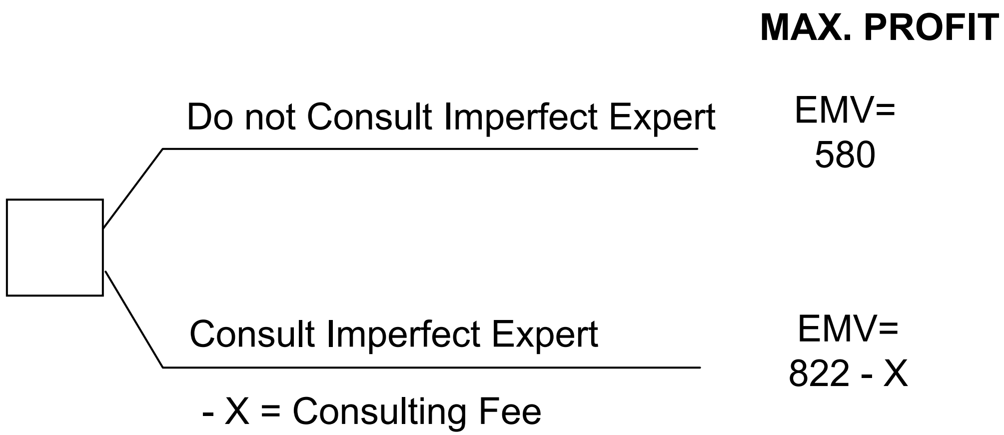
STEP 4: Calculate EVII for consulting the expert



Conclusion:

You would be willing to consult the **clairvoyant expert** if:

$$822 - X \geq 580 \Leftrightarrow X \leq 822 - 580 = 242 \text{ (=EVII)}$$

EVII = Expected Value of Imperfect Information

Expected Value of Imperfect Information

Interpretation:

EVII is the maximum amount of money you would be willing to pay for the services of the **imperfect expert.** If he charges more than $242 you would not consult the expert.

Note:

- EVPI ≥ EVII. Interpretation: **Perfect Information is always better than imperfect information.**
- When performing sensitivity analysis **EVPI calculation** of every uncertain event should be **considered.** When EVPI is high for a particular uncertain event, investment to reduce uncertainty may be warranted.