EXTRA PROBLEM 6:
SOLVING DECISION TREES

Read the following decision problem and answer the questions below.
A manufacturer produces items that have a probability **p** of **being defective**. These items are formed into **batches of 150**. Past experience indicates that some **(batches)** are of **good quality (i.e. p=0.05)** and others are of **bad quality (i.e. p=0.25).** Furthermore, **80%** of the batches produced are of **good quality** and **20% of the batches are of bad quality.** These items are then used in an assembly, and ultimately their quality is determined before the final assembly leaves the plant. The manufacturer can either **screen each item** in a batch and replace defective items at a total average cost of $10 per item or **use the items directly without screening.** If **the latter action** is chosen, the cost of rework is ultimately $100 per defective item. For these data, the costs per batch can be calculated as follows:

	p = 0.05	p = 0.25
Screen	$1500	$1500
Do not Screen	$750	$3750

Because screening requires scheduling of inspectors and equipment, the decision to screen or not screen must be made 2 days before the potential screening takes place. However, the manufactures **may take one item taken from a batch** and sent it to a laboratory, and the test results (defective or non-defective) can be reported **before the screen/no-screen decision** must be made. After the laboratory test, **the tested item is returned to its batch.** The cost of this initial inspection is $125. Also note that the probability that a random sample item is defective is

$$0.8 * 0.05 + 0.2 * 0.25 = 0.09,$$

and **the probability that an item in a lot is of good quality given a randomly sampled item is defective is 0.444** and **the probability that an item in a lot is of good quality given a randomly sampled item is not defective is 0.835.** The manufactur wants to minimize his or her cost.

A. Derive the cost figures in the table above. Clearly show your calculations.

	p = 0.05	p = 0.25
Screen	150*$10 = $1500	150*$10 = $1500
Do not Screen	150*0.05*$100 = $750	150*0.25*$100 = $3750

B. What Law of Probability was used in deriving:

Pr(Randomly Sampled Item is Defective)

Pr(Sample Item is Defective|Batch of Good Quality) =0.05

Pr(Sample Item is Defective|Batch of Bad Quality) =0.25

Pr(Batch of Good Quality)=0.80 Pr(Batch of Bad Quality)=0.20

Pr(Randomly Sampled Item is Defective) =

Pr(Sample Item is Defective|Batch of Good Quality)Pr(Batch of Good Quality)+

Pr(Sample Item is Defective|Batch of Bad Quality)Pr(Batch of Bad Quality)=

0.05*0.8 + 0.25*0.2=0.09

Hence, THE LAW OF TOTAL PROBABILITY was used.

C. Show that:

Pr(Batch of Good Quality|Sampled Item is Defective) = 0.444
Pr(Batch of Good Quality|Sampled Item is Not Defective) = 0.835

$BGQ =$ **"Batch of Good Quality";** $SID =$ **"Sampled Item is Defective";**

$\overline{SID}$ **= "Sampled Item is Not Defective"**

$$Pr(BGQ|SID) = \frac{Pr(SID|BGQ)Pr(BGQ)}{Pr(SID)} =$$

$$\frac{0.05 \cdot 0.80}{0.09} = 0.444$$

$$Pr(BGQ|\overline{SID}) = \frac{Pr(\overline{SID}|BGQ)Pr(BGQ)}{Pr(\overline{SID})}$$

$$= \frac{(1-0.05) \cdot 0.80}{(1-0.09)} = 0.835$$

D. Calculate and show your calculations:

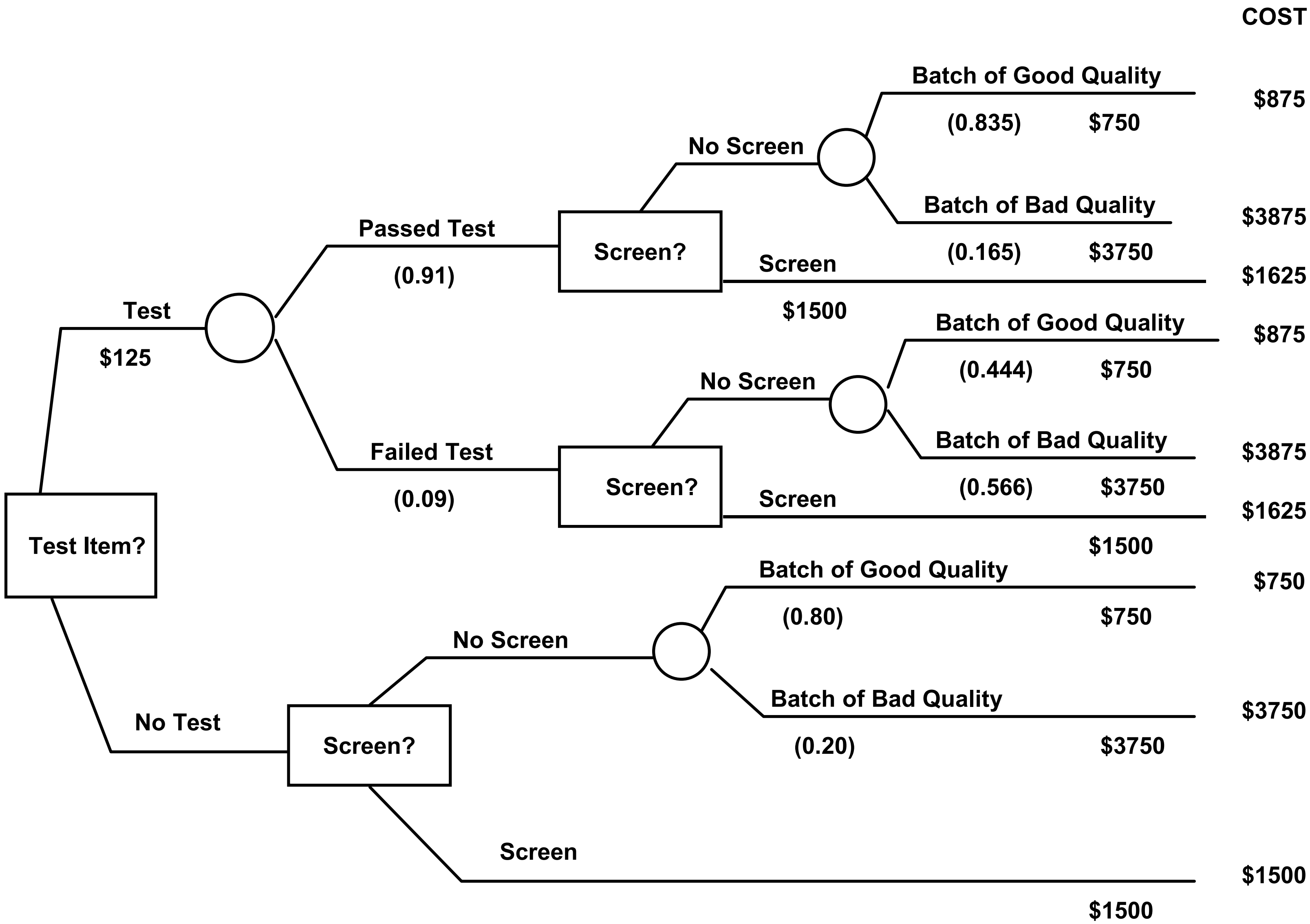
$\overline{BGQ} =$ **"Batch of Bad Quality"**; $\overline{SID} =$ **"Sampled Item is Not Defective"**

Pr(Bad Quality|Sampled Item is Defective) = ?
Pr(Bad Quality|Sampled Item is Not Defective) = ?

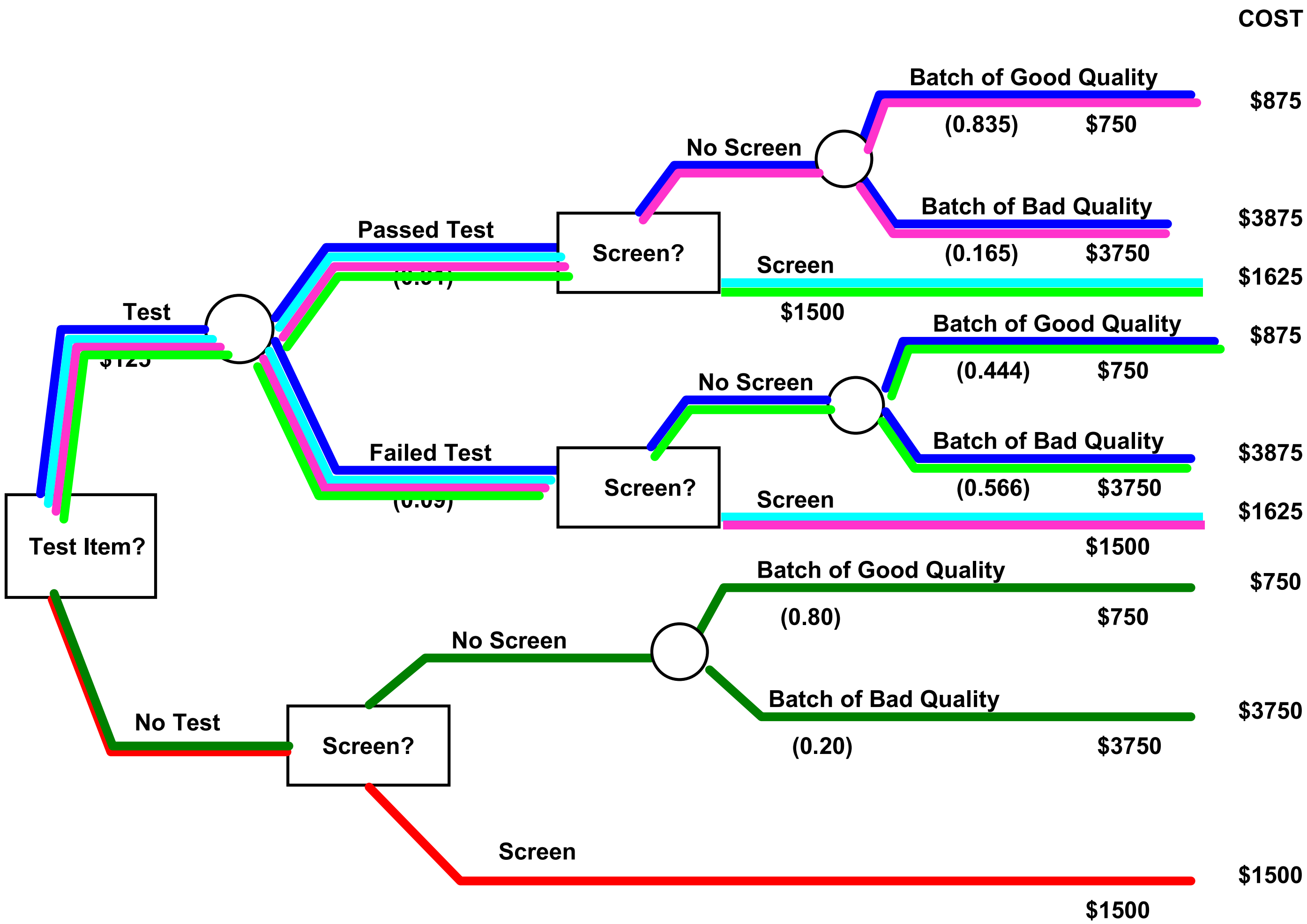
$$Pr(\overline{BGQ}|SID) = 1 - Pr(BGQ|SID) = 1 - 0.444 = 0.556$$

$$Pr(\overline{BGQ}|\overline{SID}) = 1 - Pr(BGQ|\overline{SID}) = 1 - 0.835 = 0.165$$

E. Model the decision problem in a decision tree and fill in ALL the details

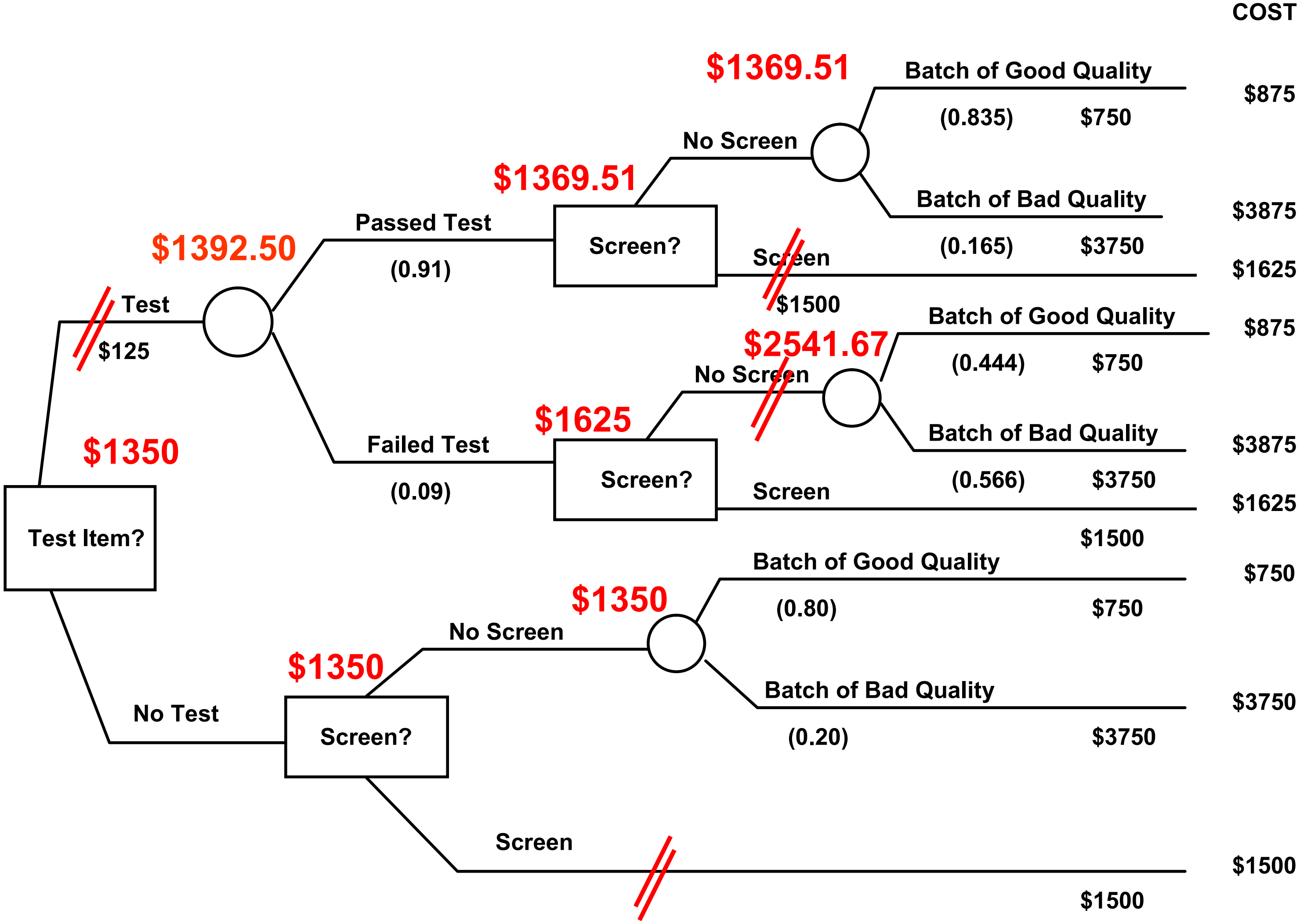


F. How many cumulative risk profiles can be drawn for the decision tree under E? Provide an Explanation.



Upper Part: 4 (T,{NS,NS}), (T,{S,S}), (T,{NS,S}), (T,{S,NS})
Lower Part: 2 (NT,NS), (NT, S) Total: 6 Strategies

G. Solve the tree under E using EMV and clearly show your calculations in the tree under E

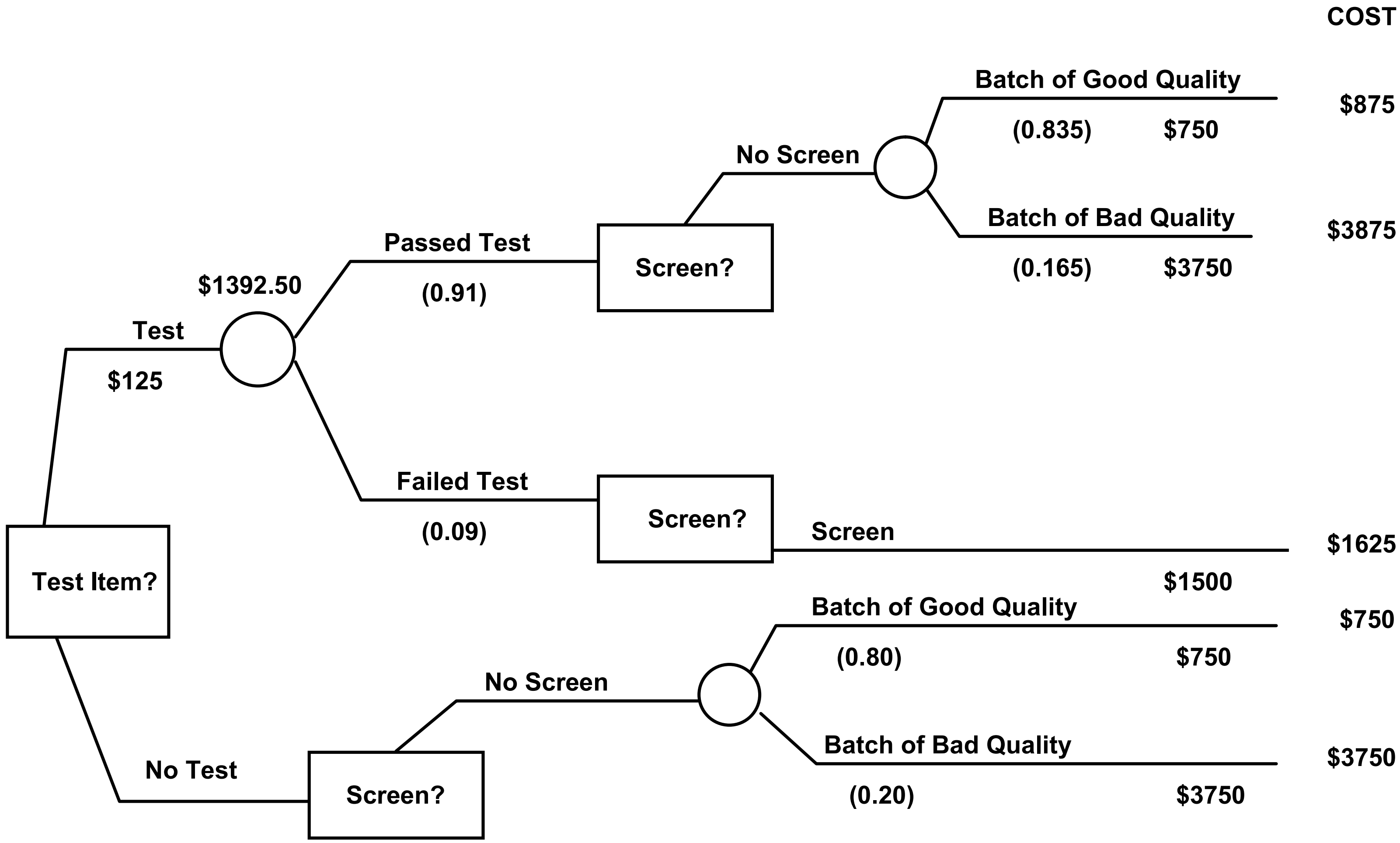


H. Describe in words the optimal decision strategy.

Do not Sample an Item Randomly from a Batch for Testing, and Do Not Screen the entire Batch

(No Test, No Screen in the Decision Tree)

I. Draw the Cumulative Risk Profile for each alternative of the immediate decision, taking optimal decisions from thereon. What can you conclude with respect to dominance considerations? (Hint: You should be drawing 2 Cumulative Risk Profiles).



Strategy: Test Random Item and If Item Passed Test then do not Screen the entire Batch and if Item Failed Test then Screen the entire Batch

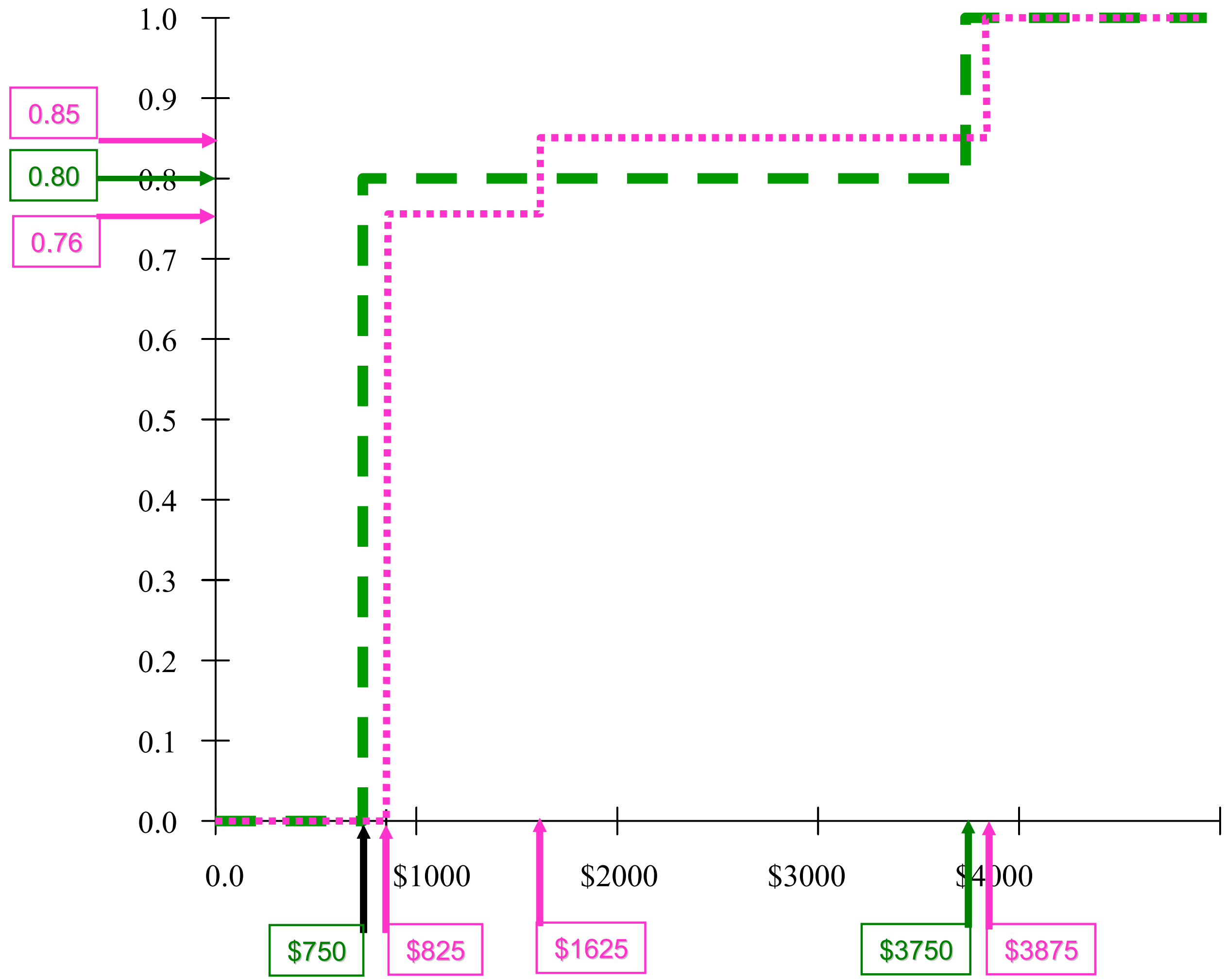
$$Pr(Cost = 875) = 0.91*0.835 = 0.760$$
$$Pr(Cost = 1625) = 0.09$$
$$Pr(Cost = 3875) = 0.91*0.165 = 0.150$$

$$Pr(Cost \leq 875) = 0.760$$
$$Pr(Cost \leq 1625) = 0.760 + 0.09 = 0.85$$
$$Pr(Cost \leq 3875) = 0.85 + 0.15 = 1.000$$

Strategy: Do not Test Random Item and Do not Screen the Entire Batch

$$Pr(Cost = 750) = 0.80$$
$$Pr(Cost = 3750) = 0.20$$

$$Pr(Cost \leq 750) = 0.80$$
$$Pr(Cost \leq 3750) = 0.80 + 0.20 = 1.00$$



CUMULATIVE RISK PROFILES CROSS. HENCE, NO DETERMINISTIC DOMINANCE AND NO STOCHASTIC DOMINANCE