Relay sensor placement in wireless sensor networks

**Xiuzhen Cheng · Ding-Zhu Du · Lusheng Wang ·
Baogang Xu**

Abstract This paper addresses the following relay sensor placement problem: given the set of duty sensors in the plane and the upper bound of the transmission range, compute the minimum number of relay sensors such that the induced topology by all sensors is globally connected. This problem is motivated by practically considering the tradeoff among performance, lifetime, and cost when designing sensor networks. In our study, this problem is modelled by a NP-hard network optimization problem named *Steiner Minimum Tree with Minimum number of Steiner Points and bounded edge length (SMT-MSP)*. In this paper, we propose two approximate algorithms, and conduct detailed performance analysis. The first algorithm has a performance ratio of 3 and the second has a performance ratio of 2.5.

X. Cheng (✉)
Department of Computer Science, The George Washington
University, Washington, DC 20052, USA
e-mail: cheng@gwu.edu

D.-Z. Du
Department of Computer Science, The University of Texas at
Dallas, Richardson, TX 75083, USA
e-mail: dzdu@utdallas.edu

L. Wang
Department of Computer Science, City University of Hong Kong,
Kowloon, Hong Kong
e-mail: lwang@cityu.edu.hk

B. Xu
School of Math. and Computer Science, Nanjing Normal
University, 122 Ninghai Road, Nanjing, 210097, P.R. China
e-mail: baogxu@njnu.edu.cn

1 Introduction

Wireless Sensor Networks (WSNs) are ad hoc multihop systems containing sensors connected by wireless links. The flourishing research on WSNs is driven by the advances in MEMS technology, CMOS logic, and wireless networking [5, 11]. WSNs have many possible applications, ranging from habitat monitoring to environment control [19]. WSN is used to produce macro-scale effects from micro-devices through coordinated activities of many sensors, thus *connectivity* is a very important issue in WSN architecture design.

Wireless links are mainly determined by transmit powers of sensors, and higher transmit power produces richer connectivity. However, "in the context of untethered nodes, the finite energy budget is a primary design constraint. Communications is a key energy consumer as the radio signal power in sensor networks drops off with r^4 [12] due to ground reflections from short antenna heights." (quoted from [4].) Here in this quote, r is the distance from the transmitter. This means to reach a slightly longer distance, the sensor needs to dispatch much higher transmit power. The second reason for the prohibitiveness of higher transmit power is the higher interference to on-going traffic. The higher the power a sensor transmits, the more the number of direct neighbors the sensor has, and the higher the negative influence the sensor has on the network throughput. The third reason is the lifetime of the network [14], which is determined by the lifetime of sensors as a whole. Wireless sensors are battery powered [10]. Either battery renewal is prohibited by economic considerations or it is impossible to recharge or replace a battery in a WSN. The forth reason, but not the last, is that the heat dissipated by higher-power transmission may meddle the sensing function (i.e. temperature sensors).

However, these observations do not mean that the lower the transmit power, the better. Very low transmit power may

cause disconnected topology, and thus network malfunction. It may cause rocketing hop count for message dissemination, thus ascending error rate and falling throughput. Economically deploying a sensor network with very low transmit power may be prohibitive, since the number of sensors needed may be doubled or tripled. Therefore a sensor network designer has to seek a tradeoff among performance, lifetime, and cost. A number of problems have been formulated to study this tradeoff. Many of them focus on topology control by minimizing the maximum transmit power [9, 15] or minimizing total transmit power [3, 18] to maintain global topology. Our research focus is different. We consider the placement of *relay sensors* to connect all sensors on duty, called *duty sensors*, with fixed transmit powers.

Our study is motivated by an important class of wireless sensor networks, in which the locations (i.e. monitored sites) of the sensors are fixed and their placements are predetermined. Further, we assume all sensors are placed in a 2-dimensional plane. For example, in desertification monitoring, a set of sites in the desert are preselected and different kinds of environmental sensors are placed in each site. The sensor transmit power is pre-computed by comprehensively considering factors on network lifetime, performance, cost, etc. Since no sensor can reach the main office directly, individual observations at each site need to be directed through multihop relaying for further processing. Based on this consideration, we study the following problem: *given a set of duty sensors (required sensors) in the plane, place minimum number of relay sensors to maintain global connectivity such that the transmission range of each sensor is at most R, where R is a constant.* This statement is modelled by the network optimization problem named *Steiner Minimum Tree with Minimum number of Steiner Points and bounded edge length (SMT-MSP)* [8]:

- Given a set of terminals (denoted by V) in the plane and a constant R, find a Steiner tree τ spanning V with minimum number of Steiner points such that every edge in τ has length at most R.

In this description, "terminals" refer to "duty sensors" while "Steiner points" refer to "relay sensors." SMT-MSP is a generalized *Steiner Minimum Tree (SMT) problem*. A Steiner tree for terminal set V is a spanning tree over $V \cup S$, where S contains all points not in V, which are called *Steiner points*. A SMT is a Steiner tree with minimum total edge length.

The main reason for minimizing the number of Steiner points is to decrease system cost, as these relay sensors may have higher capability, thus higher cost. For example, in the desertification monitoring WSN, the relay sensors can be simple base stations with higher processing and relaying ability, compared with those environmental sensors. SMT-MSP is NP-hard [8]. Lin and Xue [8] have proved that *steinerized minimum spanning tree* (adding minimum number of

Steiner points on the edges of a minimum spanning tree to upper-bound the edge length to R) has performance ratio 5. The *performance ratio of an approximate algorithm $\mathcal{A}$ to a minimization problem $\mathcal{P}$* is defined to be $sup_I \frac{A_I}{OPT_I}$, where I is an instance of problem $\mathcal{P}$, A_I is the output from $\mathcal{A}$ for instance I, and OPT_I is the optimal solution for instance I. In [2], Chen et al. show that steinerized Spanning tree actually has performance ratio exactly 4. They also present a new $O(n^4)$-time approximate algorithm with performance ratio at most 3, where n is the number of given terminals. The first algorithm proposed in this paper also has a performance ratio of 3 but its time complexity is $O(n^3)$. The second algorithm proposed in this paper is a randomized one with performance ratio at most $\frac{5}{2}$.

In the following two sections we are going to propose two approximate algorithms[1] for SMT-MSP, together with their theoretical performance analysis. We will briefly summarize the related work in Section 4 and conclude our paper in Section 5.

2 A ratio 3 algorithm for STP-MSP

For a given set P of terminals, a *minimum spanning tree* is a tree interconnecting the terminals in P with edge between terminals. For a given constant R, a *steinerized minimum spanning tree* is a tree obtained from a minimum spanning tree by inserting $\lceil \frac{|ab|}{R} \rceil - 1$ Steiner points to break each edge ab into multiple pieces of length at most R. Edge ab is a *steinerized edge*. A *full component* of a Steiner tree is a subtree in which each terminal is a leaf and each internal node is a Steiner point.

Given a set P of n terminals in the Euclidean plane, and a positive constant R, we want to find a Steiner tree with minimum number of Steiner points such that each edge in the tree has length at most R. In [2], Chen et al. present an $O(n^4)$-time approximation with performance ratio at most 3. With a slight modification, we may reduce the running time to $O(n^3)$.

Our algorithm is given in Fig. 1. Since we construct 3-stars in Step 2, the algorithm runs in $O(n^3)$ time. Now we analyze this algorithm theoretically.

Let T be a Steiner tree and e be a line segment. $C(T)$ and $C(e)$ denote the numbers of Steiner points in T and e, respectively. $|e|$ denotes the length of e.

Lemma 2.1. [2] *Every steinerized minimum spanning tree has the minimum number of Steiner points among all steinerized spanning trees.*

---

[1] We use *terminals* and *Steiner points* to refer to the duty sensors and relay sensors respectively.

Fig. 1 The ratio-3 algorithm

Algorithm A: A 3-approximate Algorithm for STP-MSP

Input: A set P of n terminals, a positive constant R.
Output: A Steiner tree T_A in which each edge has length at most R.

0. Sort all $\frac{n(n-1)}{2}$ possible edges between the n terminals of P in length increasing order $e_1, e_2, \ldots, e_{\frac{n(n_1)}{2}}$, and set $T_A = (P, \emptyset)$;
1. **for** each e_i such that $|e_i| \leq R$ **do**
       if e_i connects two different connected components of T_A
       then put e_i into T_A;
2. **for** each subset of three terminals a, b, c, where a, b, c reside in three different connected components of T_A, **do**
       if there exists a point s within distance R from a, b and c
       then put the 3-star, consisting of the three edges sa, sb, sc, into T_A;
3. **for** each e_i with $|e_i| > R$ in the increasing order **do**
       if e_i connects two different connected components of T_A
       then put the steinerized e_i into T_A.

Lemma 2.2. [8] *There exists a shortest optimal Steiner tree T^* for STP-MSP such that every vertex in T^* has degree at most five.*

Lemma 2.3. [2] *Let T^* be a shortest optimal tree for STP-MSP such that every Steiner point has degree at most five. Let T_j be a full component of T^*. Then the following hold:*

(1) The steinerized minimum spanning tree on terminals in T_j has at most $3 \cdot C(T_j) + 1$ Steiner points.

(2) If T_j contains a Steiner point of degree at most four, then the steinerized minimum spanning tree on terminals in T_j has at most $3 \cdot C(T_j)$ Steiner points.

(3) If the steinerized minimum spanning tree on terminals in T_j has an edge (of length at most R) between two terminals, then it contains at most $3 \cdot C(T_j)$ Steiner points.

From (3) of Lemma 2.3, we know that if the number of Steiner points contained in a steinerized spanning tree on terminals in a full component T_j reaches the upper bound $3 \cdot C(T_j) + 1$, then any two terminals can not be connected directly by a single edge of length at most R, i.e., there must be a Steiner point between them.

Theorem 2.4. *Let T^* be an optimal tree for STP-MSP and T_A an approximation produced by Algorithm A. Then $C(T_A) \leq 3C(T^*)$.*

Proof: Let T^S be a steinerized minimum spanning tree on all terminals, and let k be the number of 3-stars produced by Step 2 of **Algorithm** A. Then

$$C(T_A) \leq C(T^S) - k.$$

By Lemma 2.2, we assume that each Steiner point of T^* has degree at most five. Assume that T^* has h full components $T_1, T_2, \ldots, T_h$. For $i = 1, 2$, let $T^{(i)}$ be the components produced by Step i of **Algorithm** A. We construct a steinerized spanning tree T as follows: Initially, set $T := T^{(1)}$, then for each full component T_j ($1 \leq j \leq h$), add to T the steinerized minimum spanning tree H_j on terminals of T_j. If the resulted tree has a cycle, then destroy the cycle by deleting some edges of H_j. Without loss of generality, suppose that $T_1, T_2, \ldots, T_g$ ($g \leq h$) are the full components in T^* such that every Steiner point has degree five and $T^{(1)} \cup T_j$ has no cycle. Combining Lemma 2.1 and Lemma 2.3 with the fact that for destroying a cycle from $T \cup H_j$, a Steiner point must be removed unless H_j contains an edge between two terminals, we have

$$C(T^S) \leq C(T) \leq 3C(T^*) + g,$$

i.e.,

$$C(T_A) \leq 3C(T^*) + g - k.$$

Suppose that $T^{(1)}$ has p components. Then, $T^{(2)}$ has $p - 2k$ components $C_1, C_2, \ldots, C_{p-2k}$. Now we construct another graph H on all terminals as follows: Initially put all edges of $T^{(1)}$ into H, then consider every T_j ($1 \leq j \leq g$). If T_j has a unique Steiner point (this Steiner point connects five terminals which must lie in at most two $C_i's$), then among the five terminals there are three pairs (edges) of terminals, each pair (edge) lies in the same C_i. We add the three edges into H. If T_j has at least two Steiner points, then there are two Steiner points each connecting four terminals, and we can also find three pairs (edges) of terminals such that each pair

(edge) lies in the same C_i. Thus, we can add the three edges into H. It is clear that H has at most $p - 3g$ components. Since each component of H is contained by a C_i, we have $p - 2k \le p - 3g$, then $g - k \le \frac{3g}{2} - k \le 0$. This ends the proof.   □

3 A 2.5-approximate algorithm of STP-MSP

In this section, we give a randomized algorithm of ratio-$\frac{5}{2}$ for the STP-MSP problem. The following are some useful terminologies and Lemmas.

A Steiner tree for n terminals is a *k-restricted Steiner tree* if each full component spans at most k terminals. A path $q_1 q_2 \ldots , q_m$ in a tree T is called a *convex* path if for every $i = 1, 2, \ldots , m - 3$, $q_i q_{i+2}$ intersects $q_{i+1} q_{i+3}$. An angle of degree more than $120°$ is called a *big* angle. An angle of degree less than or equal to $120°$ is called a *small* angle.

Lemma 3.1. [2] *Let $q_1 q_2 \ldots , q_m$ be a convex path. Suppose there are b big angles among $m - 2$ angles $\angle q_1 q_2 q_3, \angle q_2 q_3 q_4, \ldots , \angle q_{m-2} q_{m-1} q_m$. Then, $|q_1 q_m| \le (b + 2)R$.*

Note that if there is no small angle, $|q_1 q_m| \le (b + 1)R$. Thus this lemma is useful only when there are many small angles in the convex path.

Lemma 3.2. [2] *In a shortest optimal tree T for STP-MSP, there are at most two big angles at a point of degree three, there is at most one big angle at a point of degree four, and there is no big angle at a point of degree five.*

Let T^* be a full Steiner tree that is a shortest optimal tree for STP-MSP on n terminals. Let s_i denote the number of Steiner points of degree i in T^*.

Lemma 3.3. [2] $3s_5 + 2s_4 + s_3 = n - 2.$

Theorem 3.4. [13] *There exists a randomized algorithm for the minimum spanning tree problem in 3-hypergraphs running in $poly(n, w_{max})$ time with probability at least 0.5, where n is the number of nodes in the hypergraph and w_{max} is the largest weight of edges in the hypergraph.*

Lemma 3.5. *Consider a clockwise tour F of T^* that visits the n terminals in the order of $t_1, t_2, \ldots , t_n, t_1$ (see Fig. 2). Then,*

(i) *the tour F has exactly n convex paths $P_1, P_2, \ldots , P_n$ such that P_i connects two terminals t_i and t_{i+1} ($t_{n+1} = t_1$);*
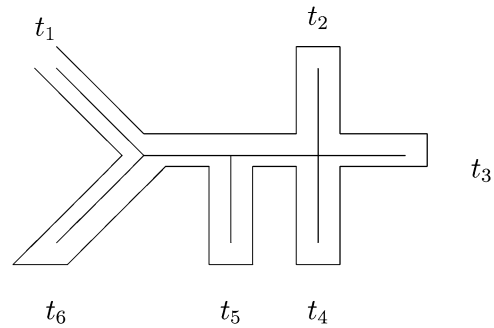

Fig. 2 The tour F that visits all the terminals

(ii) *each angle at a Steiner point appears in these n convex paths exactly once.*

(iii) *connect the two ends of P_i by an edge e_i and then steiner-ize e_i, $i = 1, 2, \ldots , n$. The total number of Steiner points in any $n - 1$ e_i's $\bar{C} = \sum_{i=1}^{n-1} C(e_i)$, is upper bounded as follows:*

$$\bar{C} \le s_4 + 2s_3 + 2s_2 + n - C(e_n)$$
$$= 3(s_5 + s_4 + s_3) + 2s_2 + 2 - C(e_n) \quad (1)$$

We denote by T_F the tree consisting of n terminals and $(n - 1)$ edges $e_1, e_2, \ldots , e_{n-1}$.

Proof: (i) and (ii) are very easy to see from the structure of T^*. Now, we prove (iii). Consider the tour F. By Lemma 3.1, if there are a_i big angles in P_i, then there are at most $a_i + 1$ Steiner points on e_i, and so the total number of Steiner points in F is at most n plus the number of big angles in T^*. By Lemma 3.2, there are at most $2s_2 + 2s_3 + s_4$ big angles in T^*. From Lemma 3.3, we know that (iii) is valid.   □

As defined previously, T^* is a full Steiner tree that is a shortest optimal tree for STP-MSP on n terminals. Without loss of generality, we assume that T^* has Steiner points of degree at least three. Selecting an arbitrary Steiner point of degree at least three as the root of T^*, we get a rooted tree. A *good point t* in T^* is a Steiner point that is adjacent to some terminals and satisfies one of the following:

(i) (type (1)) t has three or more terminals as children;
(ii) (type (2)) t has two terminals as children and the degree of t is 4;
(iii) (type (3)) t is a point of degree 3.

Note that a good point is of degree at least 3. A *bad* point is a Steiner point of degree at least 3 in T^* that is not a good point.

Theorem 3.6. *There is a 3-restricted Steiner tree such that each edge has length at most R and the number of Steiner points is at most $\frac{5}{2}$ times the optimum.*

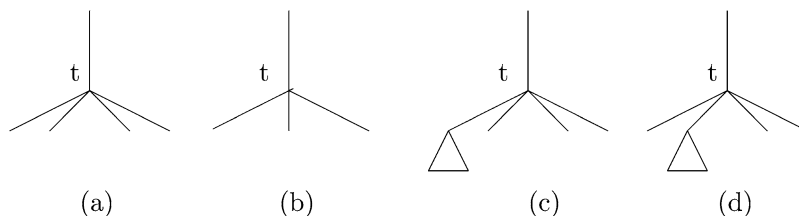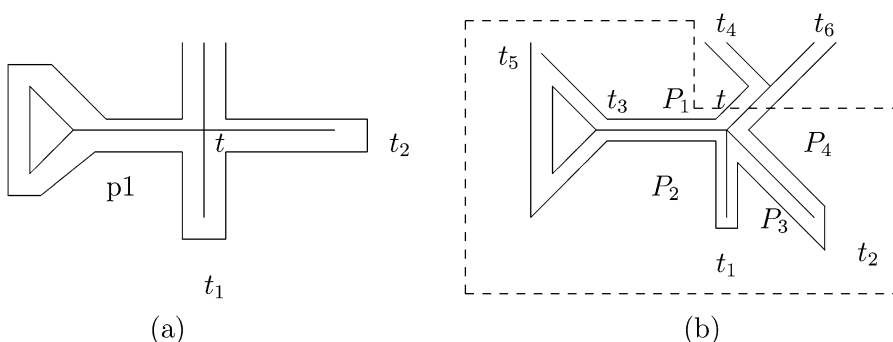**Fig. 3** Four possible cases of
type (1) good point

(a)   (b)   (c)   (d)

Fig. 4 Two cases of type (2)
good point. (a) Case 1. (b) Case
2

(a)   (b)

Proof: Let F and T_F be defined as in Lemma 3.5. From Lemma 3.5, we know that in T_F, (1) each degree 2 Steiner point is used at most twice, and (2) each Steiner point of degree at least 3 is used at most three times.

We modify T_F to get a 3-restricted tree T such that each of the good points of type (1) or type (3) in T^* is used at most twice, at least half of the type (2) good points are used at most twice, and each of the rest of the Steiner points of degree at least 3 in T^* is used at most three times.

Our modification is sketched below:

(i) Let t be a type (1) good point of degree d. We have to consider the four cases as shown in Fig. 3.

There are d convex paths in F that go through t. We can use a 3-star connecting the three terminals to replace two of the d convex paths in the tour F. It is easy to verify that the number of times that t is used in the new 3-restricted T is reduced by 1, i.e., t used at most twice (instead of three times).

(ii) Let t be a type (2) good point. Thus, t is of degree four. If there is no big angle at t, by Lemmas 3.1 and 3.2, the total number of big angles in T^* is reduced by 1, and t is used at most twice. So, we can assume that there is a big angle at t.

Let P_1 be the convex path in the tour F having the big angle at t. Let t_1 and t_2 be the two terminals adjacent to t. Two cases arise.

Case 1. t_1 or t_2, say, t_1, is in the convex path P_1 connecting t_1 and another terminal t_3 (see Fig. 4(a)). If $t_3 = t_2$, then the convex path in T_F connecting t_1 and t_2 has no small angle. Thus, the length $|t_1 t_2| \leq (b+1)R$, i.e., the upper bound $(b+2)R$ in Lemma 3.1 is not tight. Therefore, the number of times that t is used in T_F is at most 2 (not 3). In this case, we do not have to modify the tree. If $t_3 \neq t_2$, we connect t_2

to the convex path P_1 at point t. This forms a 3-star with t as the center connecting three terminals t_1, t_2 and t_3. We then use the three line segments tt_1, tt_2 and tt_3 (not the 3 convex paths) to form the three edges of the 3-star. Let t_i and t_j be two points. We use $t_i t_j$ to represent the line segment connecting t_i and t_j. From Lemma 3.1, the big angle at t ensures that

$$C(tt_1) + C(tt_3) = C(tt_3).$$

Moreover, $C(tt_2) = b$ (t is not counted and there is no small angle in tt_2), where b is the number of big angles in edge tt_2 in T_F. Note that $C(t_1 t_2)$ is estimated as at least $b+1$ in Lemma 3.5. Thus, the number of times that t is used in T is reduced from at most 3 to at most 2.

Case 2. Neither t_1 nor t_2 is in the convex path that has the big angle. Let t_3 be the child of t other than t_1 and t_2. See Fig. 4(b). In this case, t_3 is either the leftmost child or the rightmost child of t. Without loss of generality, we assume that t_3 is the leftmost child, and let P_1 be the convex path in F which connects two terminals t_4 and t_5 and contains the big angle at t (see Fig. 4(b)). Note that all the descendent terminals of t (in the dashed circle in Fig. 4(b)) are connected with paths P_1, P_2, P_3 and P_4. Thus, we can shorten P_1 to obtain P_1' by cutting off the part from t to t_5. By doing this, we get a 3-star with t as the center connecting t_1, t_2 and t_4. Since P_1 has a big angle at t, from Lemma 3.1, we know that the number of times that t is used in T is at most 2 (not 3).

In above discussion, we just consider the case where there is only one type (2) good point in the convex path P_1. Now, consider the case where there is more than one type (2) good
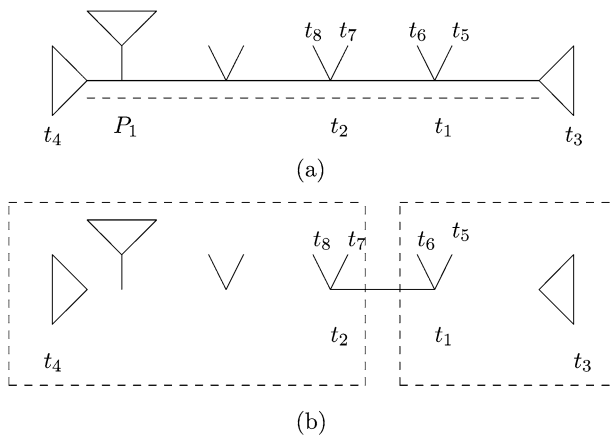
Fig. 5 (a) The original T^*. The dashed line stands for P_1. (b) After deleting P_1, we have two components in the dashed boxes. We use an edge connecting t_1 and t_2 to form a tree again

point in P_1. See Fig. 5(a). Note that T_F is a tree having $n - 1$ edges (corresponding to convex paths) connecting the n terminals. Let e be the edge in T_F which corresponds to P_1. Deleting the edge e forms two components (inside a dashed box), with each containing an end of e (t_3 or t_4). See Fig. 5(b). The terminals in each of the two components are connected by the rest of the $(n-2)$ edges (possibly replaced by some 3-stars in the modification process).

Let t_1 and t_2 be the two type (2) good points in P_1 that are the leftmost and rightmost in the two components, respectively. See Fig. 5(b).

We replace e by a segment (see the line in Fig. 5(b) connecting the two boxes) connecting t_1 and t_2 directly. Thus, we get a tree again. This makes all type (2) good points in P_1 other than t_1 and t_2 appear in T at most twice (not three times). Moreover, we can form a 3-star with t_1 as the center connecting t_5, t_6 and t_7, or with t_2 as the center connecting t_5, t_7 and t_8. By doing this, one of t_1 and t_2 appears in T at most twice instead of three times.

Thus we can conclude that at least half of the type (2) good points are used in T at most twice instead of three times.

(iii) t has at least one child, say, t_1, and the degree of t is 3 (see Fig. 6). If there is at most one big angle at t, then in Lemma 3.5, t is overestimated, i.e., T is used in the tour F at most twice (not three times). So, we assume that there are two big angles at t. Thus, at least one of the two convex paths P_2 and P_3, say, P_3, (See Fig. 6) has a big angle at t. We then can shorten the edge e corresponding to P_3 in T_F by cutting off the part from t to t_1 and form a 3-star with t as center connecting t_1, t_2 and t_3 (the other end of P_3). By doing this, we save at least one Steiner point, and thus the number of times that t is used in T is at most 2 (not 3).

Note that in the above modification, we merge P_2 and P_3 into a 3-star and save one Steiner point by taking the advantage of a big angle at t. Each convex path can only be

Fig. 6 The type (3) Steiner point
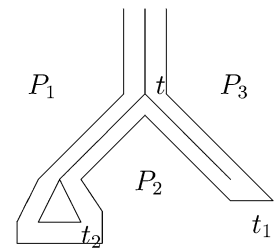
used to form a 3-star once. Otherwise, we get an i-star for $i > 3$. Thus, we have to make sure that each type (3) good point t can match a unique convex path that has a big angle at t. This can be done since each type (3) good point has degree 3 and there would be two big angles at t. (If there is only one or zero big angle at t, then t is used only once or twice in T_F. Thus we do not have to do any modification.)

Consider the case that in the convex path P_1 there are many type (2) good points and type (3) good points. Using the same argument as in (ii) demonstrated in Fig. 5, we can replace the edge corresponding to P_1 in T_F by the segment connecting t_1 and t_2 as in Fig. 5. Thus, every type (2) and type (3) good point other than t_1 and t_2 is used at most twice (not three times). In this case, t_1 and t_2 compete the edge t_1t_2 to form a 3-star. We assign t_1t_2 to either t_1 or t_2 using the following strategy: If a type (2) good point competes t_1t_2 with a type (3) good point, say, t, we always let the type (2) good point have t_1t_2 since the type (3) good point has another big angle at it, but the type (2) good point does not. If next time t competes with another type (2) good point or type (3) good point, we let t win. Thus, every type (3) good point appears at most twice in T and at least half of the type (2) good points appear in T at most twice.

Now, we can make sure that in T each good point of type (1) and (3) appears at most twice, and at least half of the type (2) good points appear twice. From Lemma 3.5, we have

$$C(T) \leq 2g_1 + 2.5g_2 + 3b + 2s_2 + 2 - C(e_n),$$

where g_1 is the number of good points of type (1) and (3), g_2 is the number of type (2) good points, and b is the number of bad points.

We can delete a convex path P_n from tour F to form T_F. We always delete the convex path such that the corresponding edge e_n is the longest.

In the following, we show that $b < g_1$. Let T' be the tree obtained from T^* by deleting all terminals. Obviously, each leave in T' is a good point. Therefore, the number of bad points is the number of points of degree at least 3 in T'. Thus, $b < g_1$.

If $C(e_n) \geq 2$, we have

$$C(T) \leq 2g_1 + 2.5g_2 + 3b + 2s_2. \qquad (2)$$

Therefore,

$$C(T) \le 2g_1 + 2.5g_2 + 2.5b + 0.5b + 2s_2 \le 2.5g_1 + 2.5g_2$$
$$+ 2.5b + 2s_2 \le 2.5C(T^*).$$

If $C(e_n) = 1$, there is no big angle in T^*. Thus, there is no degree-3 point in T^*. Suppose that there are degree-4 points in T^*. Since there is no big angle in T^*, the number of times of degree-4 Steiner points is overestimated. Thus (2) still holds.

Now, we only have to consider the case where each Steiner point in T^* has degree 5. In this case, each point in T' is either of degree 1 or degree at least 3. Thus, the root of T' is either of degree 1 or degree at least 3. In this case, it is easy to see that the number of leaves of T' is at least two more than the number of points of degree at least 3, i.e., $g_1 \ge b + 2$. Therefore,

$$C(T) \le 2g_1 + 2.5g_2 + 2.5b + 0.5b + 2s_2 + 1 \le 2.5g_1$$
$$+ 2.5g_2 + 2.5b + 2s_2 \le 2.5C(T^*).$$

□

Now, we focus on the computation of an optimal 3-restricted tree.

Let $H_3(V, F, W)$ be a weighted 3-hypergraph, where $V = P$, $F = \{(a, b)|a \in V$ and $b \in V\} \cup \{(a, b, c)|a \in V, b \in V$ and $c \in V\}$, and for each edge $e \in F$, $w(e)$ is the smallest number of Steiner points to form an optimal solution of the STP-MSP problem on the terminals in e.

Given three points a, b and c on the Euclidean plane, let s be the Steiner point that minimizes $(|sa| + |sb| + |sc|)$, and let k be the number of Steiner points in an optimum solution T of STP-MSP on $\{a, b, c\}$ with constant R.

Lemma 3.7.

$$\left\lceil \frac{|sa|}{R} \right\rceil + \left\lceil \frac{|sb|}{R} \right\rceil + \left\lceil \frac{|sc|}{R} \right\rceil - 2 \ge k \ge \left\lfloor \frac{|sa|}{R} \right\rfloor$$
$$+ \left\lfloor \frac{|sb|}{R} \right\rfloor + \left\lfloor \frac{|sc|}{R} \right\rfloor - 2. \tag{3}$$

Proof: By Steinerizing the optimum Steiner tree, we get a solution of STP-MSP on $\{a, b, c\}$ with exactly $\lceil \frac{|sa|}{R} \rceil + \lceil \frac{|sb|}{R} \rceil + \lceil \frac{|sc|}{R} \rceil - 2$ Steiner points.

Let $|T|$ be the total length of T, which is the sum of the length of edges of T. Then

$$(k - 1) \cdot R + 3R \ge |T| \ge |sa| + |sb| + |sc|,$$

i.e.,

$$k + 2 \ge \frac{|sa| + |sb| + |sc|}{R} \ge \left\lfloor \frac{|sa| + |sb| + |sc|}{R} \right\rfloor$$
$$\ge \left\lfloor \frac{|sa|}{R} \right\rfloor + \left\lfloor \frac{|sb|}{R} \right\rfloor + \left\lfloor \frac{|sc|}{R} \right\rfloor.$$

Therefore, (3) holds.   □

Lemma 3.7 gives an upper bound on the cost of (a, b, c).

Lemma 3.8. [7] *Testing whether three circles has a point in common can be done in constant time.*

For given points a, b and c on the Euclidean plane, one can find the minimum Steiner tree on $\{a, b, c\}$ in constant time. Let $q_{a,b,c}$ be the number of Steiner points used to steinerize the optimum Steiner tree on $\{a, b, c\}$, and $q_P = \max\{q_{a,b,c}|\{a, b, c\} \subset P\}$. Then, by Lemma 3.7 and Lemma 3.8, the weight W of $H_3(V, F, W)$ can be calculated in $O(n^3 q_P^2)$ time. By Theorem 3.4 and Theorem 3.6, we have

Theorem 3.9. *Given a set P of n terminals and a positive constant R, there exists a randomized algorithm that computes a solution of STP-MSP on P such that the number of Steiner points is at most $\frac{5}{2}$ times of the optimum running in $poly(n, q_P)$ time with probability at least 0.5.*

The complete algorithm is given in Fig. 7.

4 Related work

Note that our starting point on *topology control* is very different than those in the literature. We maintain global connectivity by introducing relay sensors to keep transmission range moderate while most related research works focus on algorithm design to control the transmit power dissipated by each sensor [6, 9, 15, 16, 18]. For example, Ramanathan and Rosales-Hain [15] show that one can efficiently minimize the maximum per node transmit power and maintain global connectivity and biconnectivity. Rodoplu and Meng [16], and Wattenhofer et al. [18], propose heuristics independently with different assumptions to minimize the total power from every sensor to a master site. Hu [6] first determines a topology according to *Delaunay triangulation*. Then the degree of each node is adjusted through neighbor negotiation such that each node has similar number of neighbors. Computing a minimum energy topology for a WSN is NP-hard [3]. Even though our starting point is different than the works mentioned above, we claim that these techniques and our work can be combined coherently. For example, when

Fig. 7 The complete
2.5-approximate algorithm

$\frac{5}{2}$-approximate Algorithm for STP-MSP

Input: A set P of n terminals in the Euclidean plane, a positive constant R
Output: A 3-restricted Steiner tree T in which each edge has length at most R.

1. Construct a weighted hypergraph $H_3(V, F, w)$.
2. Call the randomized algorithm in [13] to compute a minimum spanning tree T for $H_3(V, F, w)$;
3. Replace every edge f of the minimum spanning tree T on $H_3(V, F, w)$ with a Steiner tree of $w(f)$ Steiner points such that the maximum length of each edge in the tree is at most R and output the obtained tree.

designing a WSN, one can first apply any of the two algorithms proposed in this paper to compute all relay sensors based on restrictions on power dissipation or transmission range, then apply any of these techniques in literature based on given resources and optimization objectives to compute the global topology for further transmit power reduction.

An interesting work that also considers relay sensor placement is reported in [17]. This work computes the minimum set of relay nodes such that each sensor can connect to 1 or 2 relay nodes and all relay nodes are either connected or 2-connected. In other words, the algorithms in [17] consider a hierarchical topology while our work sticks to the flat topology. Sensor placement that induces regular topologies such as circular, star, and hexagonal are presented and analyzed in [1].

## 5 Conclusion and future work

This paper tackles the problem of computing relay sensors to maintain global connectivity in WSNs when transmission range of all sensors are restricted. Our objective is to minimize the number of relay sensors needed, as they contribute to the overall cost of a WSN. We model this problem by a network optimization problem named SMT-MSP – Steiner Minimum Tree with Minimum number of Steiner Points and bounded edge length. SMT-MSP is NP-Complete. We propose two approximate algorithms for SMT-MSP and give detailed theoretical performance analysis.

Note that our algorithms in this paper compute relay sensors to globally network all duty sensors. As a future work we will consider the optimal relay sensor placement for k-connectivity, where $k > 1$, to improve fault tolerance in sensor networks. We also will consider the design tradeoff between transmit power per sensor and the number of sensors in the network for topology control.

References

1. E.S. Biagioni and G. Sasaki, Wireless sensor placement for reliable and efficient data collection, *IEEE HICSS'03*, Big Island, Hawaii (2003).
2. D. Chen, D.-Z. Du, X. Hu, G. Lin, L. Wang and G. Xue, Approximations for Steiner trees with minimum number of Steiner points, Journal of Global Optimization 18 (2000) pp. 17–33.
3. X. Cheng, B. Narahari, R. Simha, M.X. Cheng and D. Liu, Strong minimum energy topology in wireless sensor networks: NP-completeness and heuristics, *IEEE Transactions on Mobile Computing* 2(3) (July-September 2003) 248–256.
4. D. Estrin, L. Girod, G. Pottie and M. Srivastava, Instrumenting the world with wireless sensor networks, in: *Proceedings of International Conference on Acoustics, Speech, and Signal Processing* 4 (2001) pp. 2033–2036.
5. J. Hill, R. Szewczyk, A. Woo, S. Hollar, D. Culler and K. Pister, System architecture directions for networked sensors, *ACM SIGPLAN Notices* 35(11) (2000) 93–104.
6. L. Hu, Topology control for multihop packet radio networks, *IEEE Transactions on Communications* 41(10) (1993) 1474–1481.
7. Z.-M. Li, D.-M. Zhu and S.-H. Ma, An approximation algorithm for the bottleneck Steiner tree problem in the euclidean plane, Journal of Computer Science and Technology 19(6) (2004) 791–794.
8. G. Lin and G. Xue, Steiner tree problem with minimum number of Steiner points and bounded edge-length, *Information Processing Letters* 69 (1999) pp. 53–57.
9. E.L. Lloyd, R. Liu, M.V. Marathe, R. Ramanathan and S.S. Ravi, Algorithmic aspects of topology control problems for ad hoc networks, *ACM Mobile Networks and Applications* 10(1–2) (February 2005) 19–34.
10. R. Min, M. Bhardwaj, S.-H. Choi, N. Ickes, E. Shih, A. Sinha, A. Wang and A. Chandrakasan, Energy-centric enabling technologies for wireless sensor networks, *IEEE Wireless Communications* (August 2002) pp. 28–39.
11. G.J. Pottie, Wireless sensor networks, *Information Theory Workshop* (1998) pp. 139–140.
12. G. Pottie and W. Kaiser, Wireless sensor networks, *Communications of the ACM* 43(5) (2000) 51–58.
13. H.J. Prömel and A. Steger, A new approximation algorithm for the Steiner tree problem with performance ratio 5/3, Journal of Algorithms 36 (2000) pp. 89–101.
14. V. Raghunathan, C. Schurgers, S. Park and M. Srivastava, Energy-aware wireless sensor networks, *IEEE Signal Processing* 19(2) (2002) 40–50.
15. R. Ramanathan and R. Rosales-Hain, Topology control of multihop wireless networks using transmit power adjustment, *INFOCOM 2000*, 2 (2000) pp. 404–413.

16. V. Rodoplu and T.H. Meng, Minimum energy mobile wireless networks, IEEE Journal of Selected Areas in Communications 17(8) (1999) 1333–1344.
17. J. Tang, B. Hao and A. Sen, Relay node placement in large scale wireless sensor networks, *Computer Communications* 29 (2006) pp. 490–501.
18. R. Wattenhofer, L. Li, P. Bahl and Y.-M. Wang, Distributed topology control for power efficient operation in multihop wireless ad hoc networks, *INFOCOM 2001*, 3 (2001) pp. 1388–1397.
19. N. Xu, A Survey of Sensor Network Applications, http://enl. usc.edu/~ningxu/papers/survey.pdf.
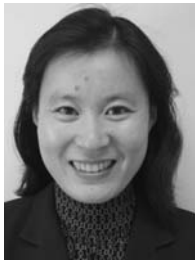
Xiuzhen Cheng is an Assistant Professor in the Department of Computer Science at the George Washington University. She received her MS and PhD degrees in Computer Science from the University of Minnesota - Twin Cities in 2000 and 2002, respectively. Her current research interests include Wireless and Mobile Computing, Sensor Networks, Wireless Security, Statistical Pattern Recognition, Approximation Algorithm Design and Analysis, and Computational Medicine. She is an editor for the International Journal on Ad Hoc and Ubiquitous Computing and the International Journal of Sensor Networks. Dr. Cheng is a member of IEEE and ACM. She received the National Science Foundation CAREER Award in 2004.

Ding-Zhu Du received his M.S. degree in 1982 from Institute of Applied Mathematics, Chinese Academy of Sciences, and his Ph.D. degree in 1985 from the University of California at Santa Barbara. He worked at Mathematical Sciences Research Institutea, Berkeley in 1985-86, at MIT in 1986-87, and at Princeton University in 1990-91. He was an associate-professor/professor at Department of Computer Science and Engineering, University of Minnesota in 1991-2005, a professor at City University of Hong Kong in 1998-1999, a research professor at Institute of Applied Mathematics, Chinese Academy of Sciences in 1987-2002, and a Program Director at National Science Foundation of USA in 2002-2005. Currently, he is a professor at Department of Computer Science, University of Texas at Dallas and the Dean of Science at Xi'an Jiaotong University. His research interests include design and analysis of algorithms for combinatorial optimization problems in communication networks and bioinformatics. He has published more than 140 journal papers and 10 written books. He is the editor-in-chief of Journal of Combinatorial Optimization and book series on Network Theory and Applications. He is also in editorial boards of more than 15 journals.

Lusheng Wang received his PhD degree from McMaster University in 1995. He is an associate professor at City University of Hong Kong. His research interests include networks, algorithms and Bioinformatics. He is a member of IEEE and IEEE Computer Society.

Baogang Xu received his PhD degree from Shandong University in 1997. He is a professor at Nanjing Normal University. His research interests include graph theory and algorithms on graphs.