Superimposed Code Based Channel Assignment in Multi-Radio Multi-Channel Wireless Mesh Networks

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ABSTRACT

Motivated by the observation that channel assignment for multi-radio multi-channel mesh networks should support both unicast and local broadcast[1], should be interference-aware, and should result in low overall switching delay, high throughput, and low overhead, we propose two flexible localized channel assignment algorithms based on s-disjunct superimposed codes. These algorithms support the local broadcast and unicast effectively, and achieve interference-free channel assignment under certain conditions. In addition, under the primary interference constraints[2], the channel assignment algorithm for unicast can achieve 100% throughput with a simple scheduling algorithm such as the maximal weight independent set scheduling, and can completely avoid hidden/exposed terminal problems under certain conditions. Our algorithms make no assumptions on the underlying network and therefore are applicable to a wide range of MR-MC mesh network settings. We conduct extensive theoretical performance analysis to verify our design.

Categories and Subject Descriptors

C.2.1 [**Network Architecture and Design**]: Wireless Communication

General Terms

Algorithms, Design

Keywords

Multi-radio multi-channel wireless mesh networks, interference, channel assignment, superimposed codes

[1]A broadcast to be heard by all immediate neighbors.

[2]Under the primary interference constraints, each radio can talk with at most one single neighbor at any instant of time. Namely the set of active links supported the same channel at any point of time is a matching.

MobiCom'07, September 9–14, 2007, Montréal, Québec, Canada.

1. INTRODUCTION

With recent advances in wireless technology, the utilization of multiple radios as well as non-overlapping channels provides an opportunity to reduce interference and increase network capacity. Equipped with multiple radios, nodes can communicate with multiple neighbors simultaneously over different channels, and thus can significantly improve the network performance by exploring concurrent transmissions [1].

In a multi-radio multi-channel (MR-MC) mesh network, a key challenging problem for capacity optimization is *channel assignment*. Since practically the number of radios at each node is always much smaller compared to that of orthogonal channels due to reasons such as cost and small form factors, it may be prohibitive to assign one fixed channel to each radio. In other words, a radio may need to switch to different channels as time goes for better performance. This radio constraint makes the channel assignment in MR-MC mesh networks much harder. In this paper, we propose two channel assignment algorithms for interference mitigation and throughput maximization. Our research is motivated by the following observations.

- Current channel assignment approaches lack a support to local broadcast in MR-MC mesh networks. As neighboring nodes tend to use different channels for transmissions, the broadcast packet has to be separately transmitted by the sender on multiple channels. Thus, broadcast can be more expensive than that in single-radio single-channel (SR-SC) networks.
- A number of current channel assignment approaches rely heavily on solving complex optimization problems, which might be impractical for many MR-MC mesh network scenarios. In addition, techniques based on default radio/channel degrade network throughput when the number of radios is much smaller than that of channels.
- Channel switching delay is an important parameter that should be counted in channel assignment. Since the number of radios per node is usually much smaller than that of orthogonal channels, allowing a radio switch among the full range of channels results in higher overall delay since the radio may switch back and forth frequently when multiple different flows traverse the same node simultaneously.
- CSMA/CA is believed to be inadequate to meet the high traffic demand in mesh networks [2]. Any channel assignment that requires RTS/CTS for channel reservation is unfavored due to the high overhead. Since co-channel interference is one of the major reasons for capacity degradation in MR-MC mesh networks, interference-aware channel assignment for throughput optimization should be sought.

In this paper, we propose two channel assignment algorithms based on s-disjunct superimposed codes. The basic idea is sketched as follows. For each node, all available orthogonal channels are labelled as either primary or secondary via a binary channel codeword. This labelling is controlled by an s-disjunct superimposed $(s, 1, N)$-code. The codeword of the transmitting node, together with those of the interferers, determine the channel. Note that primary channels are always preferred during channel assignment. Our analysis indicates that by exploring the s-disjunct property of the $(s, 1, N)$-code, it is possible to achieve interference-free channel assignment for both unicast and broadcast. Comparing with the related literature in Section 2, we have identified the following unique contributions of our paper.

- We have designed two localized simple algorithms that can effectively support both local broadcast and unicast. Under certain conditions, interference-free broadcast and unicast can be achieved.

- Since our algorithms assign channels to transmitters for both unicast and broadcast, and because the channels are selected from a small subset of primary channels whenever possible, our algorithms can effectively decrease the overall switching delay caused by the oscillation of switching back and forth due to the large difference between the numbers of radios and channels.

- With a very simple scheduling algorithm, our channel assignment for unicast is proved to be able to achieve 100% throughput under the primary interference constraints. We also identifies the conditions when hidden and exposed terminal problems are completely avoided with our channel assignment.

- We have conducted extensive theoretical performance analysis to verify our algorithm design. In addition, our algorithms are localized, and have low computation and communication overheads.

- Our algorithms support dynamic, static, and adaptive channel assignment without requesting any complex scheduling and/or channel coordination. These algorithms make no assumptions on the underlying network settings such as traffic patterns and MAC/routing protocols. Therefore they are applicable to a wide range of mesh networks.

The rest of the paper is organized as follows: Section 2 discusses the related work in channel assignment for MR-MC mesh networks. In Section 3, we present our network model and assumptions. Section 4 introduces the s-disjunct superimposed code and links it to the problem of channel assignment in MR-MC mesh networks. In Section 5, we present our channel assignment algorithms for both unicast and broadcast, and analyze their performance theoretically. In Section 6, we discuss a number of related issues. Section 7 summarizes the work and concludes the paper.

2. RELATED WORK

In this section, we survey the most related research in channel assignment for MR-MC mesh networks.

The benefits of using multiple radios and channels have been theoretically studied in [1,3–5] by jointly considering routing, scheduling, and channel assignment. Load-aware channel assignment is studied in [6, 7]. Marina and Das jointly consider channel assignment and topology control in [8].

In Kyasanur and Vaidya [9], the multiple radios at each node are divided into two groups, with one assigned fixed channels for packet reception and ensuring connectivity, and the other assigned switchable channels for capacity increase. This multiple channel management actually handles the channel allocation at the receiver side. Each switchable radio switches to the fixed channel of the destination radio when data transmission needs to be launched. For fixed channel assignment, a node selects random channels for its fixed interfaces initially. To balance the utilization of all channels, nodes collect two-hop neighborhood information and change their fixed channels accordingly. Obviously this fixed channel assignment takes time to converge. In addition, the number of switchable channels is relatively large when the number of radios per node is small, which may cause a large overall switching delay when the node has to switch back and forth in order to simultaneously relay multiple flows to different neighbors. Furthermore, the receiver-based channel assignment does not support broadcast efficiently and each broadcast packet has to be transmitted separately on one of the fixed channels for each neighbor. Our work differs in that we consider transmitter channel assignment, which is expected to incur low overall switching delay and can trivially support efficient broadcast.

A common default channel is introduced in [10–14] to handle the network partition caused by dynamic channel assignment, and to facilitate channel negotiation for data communications. To assign channels to the interfaces other than the default radio, [10] presents a localized greedy heuristic based on an interference cost function defined for pairs of channels. Refs. [11, 12] consider the mesh networks with main traffic flowing to and from a gateway, which is also in charge of the channel computation. In their channel assignment to a non-default radio, nodes closer to the gateway and/or bearing higher traffic load get a better quality channel. In DCA [14], the default channel is used as a control channel. For each node, one of the radios stays on the control channel for exchanging control messages, and other radios dynamically switch to the data channels for transmission. In this case, the utilization of the control channel could be small even though the data channels can be fully utilized. A multi-channel MAC is proposed in [13] for single-radio networks. This MAC protocol requires all nodes to meet at the common channel periodically to negotiate the channels for data communication.

The default channel does not have to be the same for all nodes in the network. In [15], each node fixes one radio on some channel but different nodes possibly use different fixed channels. This channel assignment actually fixes the reception channel for each node, and therefore the remaining radios of the node dynamically switch to its neighbors' fixed channels for data transmission. The same idea is adopted in [9]. In SSCH [16], radios switch among channels following some pseudo-random sequences such that neighboring nodes meet periodically at a common channel. This approach is simple but it requires clock synchronization.

Compared to the works mentioned above, our work does not require any special radio. We consider the channel assignment to all radios in a static fashion. In addition, our channel assignment algorithms are localized and are designed for a mesh network with a more general peer-to-peer traffic pattern.

Another important category of related work is code assignment for hidden terminal interference avoidance in CDMA packet radio networks. Bertossi and Bonuccelli [17] presents a centralized greedy algorithm to assign CDMA codes to vertices such that every pair of nodes at two-hop distance is assigned with a couple of different codes and the number of orthogonal codes utilized is minimized. This is a NP-Complete problem, and therefore the proposed algorithm is an approximate heuristic. The distributed implementation of the algorithm, which results in a high overhead, is also proposed in [17]. The same code assignment problem is considered in [18] too, where a distributed heuristic is proposed. Note that to ensure hidden terminal interference-free communications, different codes should be assigned to every pair of nodes that are two-hop away. Our work differs from [17, 18] in that we intend to assign channels to nodes with an objective of interference-free unicast and broadcast to their immediate neighbors. In addition, the number of available orthogonal channels in our study is much smaller than that of the CDMA codes in a packet radio network. Furthermore, our localized algorithms are much simpler and results in much lower overhead.

Our work focuses on channel assignment for general MR-MC mesh networks. Each node is associated with a binary channel codeword, and computes its channels based on the codewords of the interferers. The algorithms involved are simple, has very low computation and communication overheads, and can support both unicast and local broadcast effectively.

3. NETWORK MODEL

In this section, we introduce the underlying network model, assumptions, and terminologies employed in the paper.

3.1 Basics

We consider a stationary multi-radio multi-channel (MR-MC) wireless mesh network with $|V|$ nodes. There exist N orthogonal (non-overlapping) frequency channels labelled by $k_1, k_2, \cdots, k_N$. Each node is equipped with Q radio interfaces. In our consideration, $Q \ll N$. This is a practical assumption since the number of radios per node is constrained by cost and form factors. For example, in an IEEE 802.11a based mesh network, each node may have 2 or 3 radios but the number of orthogonal channels is 12. We assume that the footprint of a radio is a disk resulting from an omni-directional antenna. In addition, we assume that each radio supports the same set of non-overlapping channels. Note that the number of radios equipped on each mesh node could be different.

For each node, the N available orthogonal channels are divided into two categories: *primary channels* and *secondary channels*. A binary column vector $\vec{c_u}$ of length N, called a *channel codeword*, is associated with each node u to label its channels, with a value 1 representing a primary channel and a value 0 secondary. For example, $\vec{c_u} = (1, 0, 0, 1, 0, 0, 0, 1, 0, 1, 0, 0)'$ means that channels k_1, k_4, k_8, and k_{10} are primary to u, and k_2, k_3, k_5, k_6, k_7, k_9, k_{11} and k_{12} are secondary to u for a network that can support 12 orthogonal channels. Note that partitioning the channels into two sets can facilitate our algorithm design. Intuitively, a node should favor a channel that is secondary to all its interferers. Therefore for each node, the number of primary channels should be smaller than that of the secondary.

We require that for any two channel codewords $\vec{c_u}$ and $\vec{c_v}$, there exist at least two channels k_1 and k_2 such that k_1 is primary to u but secondary to v, and k_2 is secondary to u but primary to v. In other words, we can always find out a channel that is primary to one node and secondary to another node when the two corresponding channel codewords are different. For simplicity, we assume all nodes have the same number of primary channels. Let this number be w. Then the number of channel codewords satisfying the above condition is $\binom{N}{w}$ for N available orthogonal channels, which reaches its maximum when $w = \frac{N}{2}$. For example, when $N = 12$, there are 66, 495, and 924 available channel codewords for $w = 2, 4, 6$ respectively. We assume that the channel codewords assigned to each node is unique. As explained in Section 6, this assumption can be relaxed when the cellular grid architecture is introduced for salability considerations.

In our study, the network is modelled by a directed graph $G(V, E)$, where V is the set of nodes, and E is the set of directed links. A *channel code*, denoted by a $N \times |V|$ binary matrix $\mathcal{C}$, is associated with G. Therefore sometime G is denoted by $G(V, E, \mathcal{C})$. Each column of $\mathcal{C}$ represents a channel codeword pertaining to a node in the network. For example, the uth column is the channel codeword $\vec{c_u}$ for node u. The purpose of this paper is to assign channels to a node u based on $\vec{c_u}$ and the channel codewords of its interferers in order to mitigate co-channel interference for network capacity maximization, an optimization problem requiring the joint consideration of routing, channel assignment, and packet scheduling. Nevertheless, we focus on channel assignment in this paper, and propose to study joint routing and scheduling based on our channel assignment as a future research.

We assume that a DATA packet sending from u to v is acknowledged with an ACK message from v to u. Therefore even though we use a directed graph to model the network, only bidirectional links are considered. A directed link from node u to v is denoted by $(u \rightarrow v)$. In addition, we use $N_1(u)$ and $N_2(u)$ to represent the sets of neighbors of u within one-hop and two-hop away. We have $u \notin N_1(u)$ and $u \notin N_2(u)$.

3.2 Interference Model

For any node $u \in V$, denoted by $\mathcal{N}(u)$ the set of interferers of u. A node $v \in V$ is an *interferer* of u if v's transmission interferes with u's transmission. Therefore when two-way handshake (DATA-ACK) is adopted for successful packet delivery, the interferers for the unicast from u to v include $N_1(u)$ and $N_1(v)$. For a local broadcast by u, the interferers include all nodes in $N_2(u)$.

4. LINKING SUPERIMPOSED CODES WITH MR-MC NETWORKS

In this section, we first give a brief introduction on *superimposed codes*. Then we link the superimposed $(s, 1, N)$-code, also called the *s-disjunct code*, to channel assignment in MR-MC mesh networks.

4.1 Superimposed codes

Superimposed codes were introduced by Kautz and Singleton [19] in 1964. Since then, they have been extensively studied and applied to various fields, such as multi-access communications [20], [21], cryptography [22], pattern matching [23], circuit complexity [24], and many other areas of computer science. For convenience, we first introduce the basic definitions and properties of superimposed codes.

Let N, t, s, and L be integers such that $1 < s < t$, $1 \leq L \leq t - s$, and $N > 1$. Given a $N \times t$ binary matrix $\mathcal{X}$, denote the ith column of $\mathcal{X}$ by $X(i)$, where $X(i) = (x_1(i), x_2(i), \cdots, x_N(i))'$. We call $X(i)$ a codeword i of $\mathcal{X}$ with a length N. In other words, $\mathcal{X}$ is a *binary code* with each column corresponding to a codeword. Let w and λ be defined as:

$$w_i = \sum_{k=1}^{N} x_k(i), \quad (1)$$

$$\lambda_j = \sum_{k=1}^{t} x_j(k). \quad (2)$$

Therefore w and λ are called the *column weight* and *row weight* of $\mathcal{X}$, respectively. We have $w_{min} = \min_{i=1}^{t} w_i$, $w_{max} = \max_{i=1}^{t} w_i$, $\lambda_{min} = \min_{j=1}^{N} \lambda_j$, and $\lambda_{max} = \max_{j=1}^{N} \lambda_j$. Note that w_i and

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}$$

Figure 1: An example of a superimposed $(3, 1, 13)$-code of size 13

λ_j record the number of 1's in column i and in row j of $\mathcal{X}$, respectively. Hence w_{min} and w_{max} are the minimum and the maximum column weights of $\mathcal{X}$, respectively; and λ_{min} and λ_{max} are the minimum and the maximum row weights of $\mathcal{X}$, respectively.

The Boolean sum

$$Y = \bigvee_{i=1}^{s} X(i) = X(1) \bigvee X(2) \bigvee \cdots \bigvee X(s)$$

of codewords $X(1), X(2), \cdots, X(s)$ is the binary codeword $Y = (y_1, y_2, \cdots, y_N)'$ such that

$$y_j = \begin{cases} 0, & \text{if } x_j(1) = x_j(2) = \cdots = x_j(s) = 0, \\ 1, & \text{otherwise,} \end{cases}$$

for $j = 1, 2, \cdots, N$. We say that *a binary codeword Y covers a binary codeword Z if the Boolean sum $Y \bigvee Z = Y$.*

Superimposed code (SC): A $N \times t$ binary matrix $\mathcal{X}$ is called a superimposed code of length N, size t, strength s, and $listsize \leq L-1$ if the Boolean sum of any s-subset[3] of the codewords of $\mathcal{X}$ covers no more than $L-1$ codewords that are not components of the s-subset. This code is also called a (s, L, N)-code of size t. Fig. 1 shows an example of a superimposed $(3, 1, 13)$-code of size 13.

s-disjunct Code: A binary matrix $\mathcal{X}$ is called an s-disjunct code if and only if it has the property that the Boolean sum of any s codewords in $\mathcal{X}$ does not cover any codeword not in that set of s codewords.

Based on the definitions, a superimposed $(s, 1, N)$-code is a s-disjunct code. Taking the $(3, 1, 13)$-code shown in Fig. 1 as an example, the Boolean sum of the first 3 codewords of $\mathcal{X}$ is $X(1) \bigvee X(2) \bigvee X(3) = (1, 1, 1, 1, 1, 1, 0, 0, 0, 1, 1, 1, 0)'$, which doesn't cover any other codeword of $\mathcal{X}$ but themselves.

According to the s-disjunct characteristic of the superimposed $(s, 1, N)$-code, we can derive the following important property:

LEMMA 4.1. *Given an $(s, 1, N)$ superimposed code $\mathcal{X}$, for any s-subset of the codewords of $\mathcal{X}$, there exists at least one row at which all codewords in the s-subset contains the value 0.*

PROOF. For contradiction we assume that there is no row at which all codewords in the s-subset contain a common value 0. Then the Boolean sum of the s codewords equals $(1, 1, \cdots, 1)'$, which can cover all other codewords in $\mathcal{X}$, contradicting to the fact that $\mathcal{X}$ is a superimposed s-disjunct code. □

[3] An s-subset is a subset of s codewords.

4.2 Superimposed $(s, 1, N)$-codes and Channel Assignment in MR-MC Networks

As elaborated in Subsection 3.1, an MR-MC network is modelled by a directed graph $G(V, E, \mathcal{C})$, where $\mathcal{C}$ is the corresponding channel code. For any given node $u \in V$, $\vec{c}_u \in \mathcal{C}$ is a binary vector with each element corresponding to a channel and its $1/0$ value representing this channel being a primary channel or a secondary channel of node u. This observation helps us to build a direct mapping between a superimposed s-disjunct code $\mathcal{X}$ (represented by a $N \times t$ matrix), and the channel code $\mathcal{C}$ of a network G: N represents the number of available orthogonal channels, and each codeword of $\mathcal{X}$ indicates a possible channel codeword to a node in G. Then the column weight w_i of $\mathcal{X}$ represents the number of primary channels a node i has, and the row weight λ_j represents the number of nodes that take channel k_j as a primary channel.

In this paper, we will design algorithms for channel assignment based on superimposed codes. This research is motivated by the following observation: if the channel code $\mathcal{C}$ of a network G is a superimposed s-disjunct code $\mathcal{X}$, the nice s-disjunct property of $\mathcal{X}$ can be applied to derive the conditions for interference-free channel assignment.

Therefore we assume that the channel code $\mathcal{C}$ of network G is an s-disjunct superimposed code. From now on, we will use $\mathcal{X}$ to represent the channel code. We require that each node gets a unique codeword from $\mathcal{X}$ before participating in the network. In our algorithms, codewords from one-hop or two-hop neighbors are required for channel computation. A natural question is: how to obtain the codewords from neighboring nodes before channel assignment is complete? In this study, we assume that each node broadcasts its channel codeword once on each of its primary channels, or on all channels, to inform the neighbors of its codewords.

5. CHANNEL ASSIGNMENT BASED ON SUPERIMPOSED CODES

In this section, we first propose a generic channel assignment algorithm for MR-MC mesh networks. The generic algorithm assigns channels to nodes instead of links. This can facilitate channel selection for broadcast traffic. Then we propose an algorithm for link channel assignment targeting the unicast traffic. We also analyze the performances of both algorithms in detail.

5.1 The Generic Channel Assignment Algorithm

Let G be an MR-MC wireless mesh network with N available orthogonal channels, and $\mathcal{X}$ be the superimposed $(s, 1, N)$-code for its channel assignment. For any node u in G, a unique codeword $X(u) \in \mathcal{X}$ is associated with u indicating u's primary and secondary channel sets. Denote by $\mathcal{N}(u)$ the set of interferers of u. Algorithm 1 is a generic one that computes a set of channels for node u's transmissions.

Intuitively, u should choose only those channels not being used by any of its interferers from its primary channel set. If none of these primary channels is available, u should choose the secondary channels that are not primary to any of the nodes in $\mathcal{N}(u)$, the set of interferers of u. Since all nodes intend to utilize their primary channels whenever possible, choosing a channel that is secondary to all interferers is a reasonable choice. If u can not find out a channel that is secondary to all interferers, it picks up the primary channels that are primary to the least number of nodes in $\mathcal{N}(u)$.

These primary channels have the smallest row weight in $\mathcal{X}(\mathcal{N}(u))$, the set of codewords of $\mathcal{N}(u)$. Let $CH(u)$ be the set of channels assigned to u.

Algorithm 1 Channel Assignment for Node u

Input: Codewords $X(u)$ and $\mathcal{X}(\mathcal{N}(u))$.
Output: $CH(u)$, the set of channels assigned to u.

function $CH(u)$=**ChannelSelect**($X(u)$, $\mathcal{X}(\mathcal{N}(u))$)
  $CH_1(u) \leftarrow Channels(BoolSum(\mathcal{X}(\mathcal{N}(u) \cup \{u\})) \oplus BoolSum(\mathcal{X}(\mathcal{N}(u))))$ ▷ Find the set of primary channels that are secondary to all nodes in $\mathcal{N}(u)$.
  if $CH_1(u) \neq \emptyset$ **then**
    $CH(u) \leftarrow CH_1(u)$
  else
    $CH_2(u) \leftarrow Channels(\overline{BoolSum(\mathcal{X}(\mathcal{N}(u) \cup \{u\}))})$ ▷ Find the set of secondary channels that are secondary to all nodes in $\mathcal{N}(u)$.
    if $CH_2(u) \neq \emptyset$ **then**
      $CH(u) \leftarrow CH_2(u)$
    else
      $CH_3(u) \leftarrow$ Select $Channels(X(u))$ with the smallest row weight in $\mathcal{X}(\mathcal{N}(u))$ ▷ Select the primary channels with the least row weight in $\mathcal{N}(u)$.
      $CH(u) \leftarrow CH_3(u)$
    end if
  end if
end function

The basic idea for Algorithm 1 can be sketched below. Given $X(u)$ and $\mathcal{X}(\mathcal{N}(u))$, the Boolean sum of $\mathcal{X}(\mathcal{N}(u))$ and $\mathcal{X}(\mathcal{N}(u) \cup \{X(u)\})$ are first computed. Then the algorithm computes $CH_1(u)$, the set of u's primary channels that are secondary to all nodes in $\mathcal{N}(u)$. If $CH_1(u) \neq \emptyset$, assign $CH_1(u)$ to u; Otherwise, check $CH_2(u)$, the set of channels that are secondary to all nodes in $\mathcal{N}(u) \cup \{u\}$. If $CH_2(u) \neq \emptyset$, assign $CH_2(u)$ to u; otherwise, assign $CH_3(u)$, the set of primary channels whose corresponding row weights in the set $\mathcal{X}(\mathcal{N}(u))$ are minimum, to u.

Note that the set of primary channels of u are those favored by u. Therefore, $CH_1(u)$ contains the channels favored by u only, and $CH_3(u)$ is the set of channels favored by u and the least number of interferers of u. For $CH_2(u)$, since it contains the set of channels nobody likes to utilize in u's interference range, u should take this advantage. These channel assignment criterions reflect our design principle: *a node always selects a channel that causes the least interference to its neighborhood.*

Also note that Algorithm 1 is a localized one with each node u running a copy and making its channel assignment independently. We will prove in Lemma 5.1 that if there is an unused channel in $CH_1(u)$ for a radio r of u, r's transmission is guaranteed to be interference free.

Since each node may be equipped with multiple radios, the channels in $CH_1(u)$ may not be enough. In this case, assign all channels from $CH_1(u)$ first, then use the channels from $CH_2(u)$, and then from $CH_3(u)$.

Remarks: Algorithm 1 is a generic one that takes the codewords of u and its interferers as inputs. Therefore, Algorithm 1 does not rely on any interference model, as long as the set of u's interferers can be defined. Additionally, since Algorithm 1 assigns channels to the node, or the transmitters of the node, Algorithm 1 is a static channel allocation method. If roles of radios (the role of transmission or reception) are fixed, Algorithm 1 can help to decrease the number of channel switchings significantly compared to dynamic channel assignment. However, Algorithm 1 is dynamic when the set of interferers are collected on-line. Therefore, Algorithm 1 is flexible in that it can support both static and dynamic channel assignments.

Note that the channels determined by Algorithm 1 can be used for both unicast and local broadcast simultaneously. Since Algorithm 1 intends to pick up channels that may not be used by the interferers based on the local knowledge, it is superior in supporting local broadcast compared to existing research (Section 2). We plan to conduct extensive simulations to study the performance of Algorithm 1 when utilized to support broadcast in MR-MC mesh networks.

Example: Take the superimposed 3-disjunct code $\mathcal{X}$ in Fig. 1 as an example. Given a node u and $\mathcal{N}(u) = \{v, w, y\}$. Let $X(u) = X(1)$. If $X(v) = X(2)$, $X(w) = X(3)$, and $X(y) = X(4)$, Algorithm 1 yields $CH_1(u) = \{1, 10\}$, which means that channels 1 and 10 can be assigned to u. In this case, u picks up its primary channels. Since both channels are primary to u, based on Lemma 5.1, the transmission from u will not interfere with any other on-going traffic. If $\mathcal{N}(u) = \{v, w, y, z\}$, and $X(v) = X(3)$, $X(w) = X(10)$, $X(y) = X(12)$, and $X(z) = (13)$, no primary channels of u can be assigned to u but u can get channels $\{5, 7\}$ that are secondary to all nodes in $\mathcal{N}(u) \cup \{u\}$. When $\mathcal{N}(u) = \{v, w, y, z\}$, and $X(v) = X(4)$, $X(w) = X(10)$, $X(y) = X(12)$, and $X(z) = X(13)$, no channel that is secondary to all nodes in $\mathcal{N}(u)$ can be assigned to u. Therefore u picks up channels from its primary channel set $\{1, 2, 4, 10\}$ since all of them have the same row weight of 1 in $\mathcal{N}(u)$.

5.1.1 Conditions for Interference-Free Channel Assignment

In this subsection, we study the conditions for interference-free channel assignment based on Algorithm 1. Note that Algorithm 1 does not require a node u to collect the codewords of all interferers. If u knows nothing about its neighborhood, one of its primary channels will be picked for transmission. However, if $\mathcal{N}(u)$ is the complete set of interferers of node u, interference-free channel assignment is possible. In the following, we will first study the two scenarios when the channels assigned to u based on Algorithm 1 do not conflict with those of any other node in $\mathcal{N}(u)$. Then we study the conditions when interference-free communication in the whole network can be achieved. For simplicity, we assume that each node u in the network is equipped with two radios: one for transmission and one for reception. The results can be generalized to the case of more than two radios.

LEMMA 5.1. *If $CH_1(u) \neq \emptyset$, node u does not interfere with any other node in $\mathcal{N}(u)$.*

PROOF. When $CH_1(u) \neq \emptyset$, node u picks up channels from $CH_1(u)$, a subset of u's primary channel set, for transmission. $CH_1(u)$ contains channels that are primary to u but secondary to all nodes in $\mathcal{N}(u)$. For $\forall v \in \mathcal{N}(u)$, v can't use any channel from $CH_1(u)$ based on Algorithm 1 since v is assigned with either its own primary channels (from $CH_1(v)$ or $CH_3(v)$), which can't be in $CH_1(u)$, or channels that are secondary to all interferers in $\mathcal{N}(v)$ ($CH_2(v)$), which are secondary to u too since $u \in \mathcal{N}(v)$. □

Note that based on Lemma 5.1, if $\mathcal{N}(u)$ is the complete set of interferers of node u, u's transmissions on the channels from $CH_1(u)$ do not cause any interference to other on-going traffic.

THEOREM 5.1. *If $CH_1(u) \neq \emptyset$ holds for $\forall u \in V$ and $\mathcal{N}(u)$ is the complete set of interferers of u in the network $G(V, E)$, the channel assignment based on Algorithm 1 guarantees interference free communications in the network.*

PROOF. The theorem holds from Lemma 5.1. □

Theorems 5.1 indicates that if each node can compute a primary channel that is secondary to all its interferers based on Algorithm 1, interference-free communications in the whole network can be achieved. In the following, we identify another scenario to accomplish interference-free transmission.

LEMMA 5.2. *Given a node u with $CH_1(u) = \emptyset$ and $CH_2(u) \neq \emptyset$, if $CH_1(v_i) \neq \emptyset$ holds for all its interferers $v_1, v_2, \cdots, v_{|\mathcal{N}(u)|}$, node u's transmissions do not interfere with any other node in $\mathcal{N}(u)$.*

PROOF. Since $CH_1(u) = \emptyset$ and $CH_2(u) \neq \emptyset$, the set of channels assigned to u contains u's secondary channels that are secondary to all other nodes in $\mathcal{N}(u)$. If $CH_1(v_i) \neq \emptyset$ holds for all its interferers $v_1, v_2, \cdots, v_{|\mathcal{N}(u)|}$ in $\mathcal{N}(u)$, the set of channels assigned to v_i for $i = 1, 2, \cdots, |\mathcal{N}(u)|$ include v_i's primary channels only. Therefore, u's and its interferers' transmission channels do not overlap, and thus u's transmissions do not interfere with its interferers, and are not interfered by its interferers. □

Note that Theorem 5.1 does not place any restrictions on the size of the interferer set for any node. In the following, we prove that when $s \geq |\mathcal{N}(u)|$ holds for $\forall u \in V$ in the network $G(V, E)$, interference-free communication is guaranteed.

THEOREM 5.2. *If $s \geq |\mathcal{N}(u)|$ and $\mathcal{N}(u)$ is the complete set of interferers of u for $\forall u$ in G, the channel assignment based on Algorithm 1 guarantees interference free communications in the network.*

PROOF. Since $\mathcal{X}$ is an s-disjunct code, $BoolSum(\mathcal{X}(\mathcal{N}(u)))$ does not cover $X(u)$, which means that there exists at least one row in $\mathcal{X}$ at which $X(u)$ has the value 1 and all $\mathcal{X}(\mathcal{N}(u))$ have the value 0 (see Lemma 4.1). Therefore condition $CH_1(u) \neq \emptyset$ holds. Based on Theorem 5.1, the claim holds. □

Theorem 5.2 reports another condition for interference-free communications in the whole network based on Algorithm 1. In other words, if s upper-bounds the cardinality of the complete interferer set of each node in the network, interference-free communications can be achieved. This condition sounds very rigorous. However, for a stationary multi-radio multi-channel mesh network where the mesh routers can be carefully placed, the set of interferers could be small to provide sufficient coverage. In this scenario, channel assignment based on Algorithm 1 yields an interference-free network.

5.1.2 Probabilities for interference-Free Channel Assignment

Note that Lemma 5.1 and Lemma 5.2 report two conditions to achieve interference-free communications with no restrictions on the size of $\mathcal{N}(u)$. In this subsection, we conduct further analysis to derive the probabilities for interference-free channel assignment when $|\mathcal{N}(u)| > s$ based on Algorithm 1. In other words, we will study the probability that a node u can find out a channel to achieve interference-free communication in its local neighborhood when $s' > s$, where $s' = |\mathcal{N}(u)|$.

Let P_1 be the probability that Lemma 5.1 holds for some node u, and P_2 be the probability that Lemma 5.2 holds. Let $\mathcal{N}(u)$ be the complete set of interferers of node u. Under the protocol interference model, $\mathcal{N}(u) = N_2(u)$. We have

$$P_1 = p(CH_1(u) \neq \emptyset), \tag{3}$$

$$\begin{aligned} P_2 &= p(CH_2(u) \neq \emptyset, CH_1(u) = \emptyset, \\ &\quad CH_1(v_i) \neq \emptyset, \forall v_i \in \mathcal{N}(u)) \\ &= p(CH_2(u) \neq \emptyset, CH_1(u) = \emptyset) \cdot \\ &\quad p(CH_1(v_i) \neq \emptyset, \forall v_i \in \mathcal{N}(u)) \\ &= p(CH_2(u) \neq \emptyset, CH_1(u) = \emptyset) \cdot \\ &\quad \prod_{i=1}^{|\mathcal{N}(u)|} p(CH_1(v_i) \neq \emptyset) \end{aligned} \tag{4}$$

The last two equalities hold because the channel codeword for each node is randomly and independently assigned. Based on Eq. (3) and (4), to compute P_1 and P_2, we need to first compute the probability that $CH_1(u) \neq \emptyset$ for $\forall u \in V$, and the probability that $CH_1(u) = \emptyset$ and $CH_2(u) \neq \emptyset$ hold simultaneously.

Let m be the number of rows in $BoolSum(\mathcal{X}(\mathcal{N}(u)))$ with a value 0. Given the condition $CH_1(u) \neq \emptyset$ or $CH_2(u) \neq \emptyset$, it implies that $m > 0$. Denote these m rows by row_1, row_2, $\cdots$, row_m. Let λ_{max} be the maximum row weight among row_1, row_2, $\cdots$, row_m. We have $t - s' - \lambda_{max} \geq 0$.

Note that the boolean sum $BoolSum(\mathcal{X}(\mathcal{N}(u)))$ can cover a codeword $X(v)$ in the set $\mathcal{X} \setminus \mathcal{X}(\mathcal{N}(u))$ **iff** $X(v)$ has a value 0 at all the m rows $row_1, row_2, \cdots, row_m$. Therefore, the probability that the boolean sum of $\mathcal{X}(\mathcal{N}(u))$ covers an arbitrary codeword $X(v)$ in $X \setminus \mathcal{X}(\mathcal{N}(u))$ is

$$\begin{aligned} p_{cover|m>0} &= \prod_{i=1}^{m} \frac{|\mathcal{X}| - s' - \lambda_{row_i}}{|\mathcal{X}| - s'} \\ &= \prod_{i=1}^{m} (1 - \frac{\lambda_{row_i}}{|\mathcal{X}| - s'}) \end{aligned} \tag{5}$$

Thus the probability that the boolean sum of $\mathcal{X}(\mathcal{N}(u))$ does not cover any arbitrary codeword $X(v)$ in the set $\mathcal{X} \setminus \mathcal{X}(\mathcal{N}(u))$ is

$$\begin{aligned} p_{uncover|m>0} &= 1 - p_{cover|m>0} \\ &= 1 - \prod_{i=1}^{m} (1 - \frac{\lambda_{row_i}}{|\mathcal{X}| - s'}). \end{aligned} \tag{6}$$

Based on the above analysis, we conclude that a good superimposed code for our channel assignment should have a larger s and larger row weights λ since the higher the probability $p_{uncover}$, the less interference our channel assignment causes. Methods of constructing superimposed (s, L, N)-codes have been extensively studied in [21] [23] [25] [26] [27] [28] [29] [30]. Ref. [31] reports some optimal designs to construct an s-disjunct code with different N, s, t.

Let $p(m > 0|\mathcal{N}(u))$ denote the probability that there exists at least one row with a value 0 in $BoolSum(\mathcal{X}(\mathcal{N}(u)))$. Assuming that each codeword in $\mathcal{X}$ is independent, we have

$$\begin{aligned} p(m > 0|\mathcal{N}(u)) &= 1 - p(m = 0|\mathcal{N}(u)) \\ &= 1 - \prod_{i=1}^{N} (1 - \frac{\binom{t-\lambda_i}{s'}}{\binom{t}{s'}}) \end{aligned} \tag{7}$$

Therefore the probability that $CH_1(u) \neq \emptyset$ is

$$p(CH_1(u) \neq \emptyset) = p(m > 0|\mathcal{N}(u)) \cdot p_{uncover|m>0} \tag{8}$$

Now let's compute the probability that both $CH_1(u) = \emptyset$ and $CH_2(u) \neq \emptyset$ hold. Based on the definition of m, $CH_2(u) \neq \emptyset$ and $CH_1(u) = \emptyset$ hold **iff** the Boolean sum $BoolSum(\mathcal{X}(\mathcal{N}(u)))$

covers the codeword $X(u)$ and $m > 0$. According to Eq.(5), the probability that node u can find a secondary channel for communication is

$$p(CH_2(u) \neq \emptyset, CH_1(u) = \emptyset) = p(m > 0|\mathcal{N}(u)) \cdot p_{cover|m>0} \quad (9)$$

For completeness, we provide the probability that a channel from $CH_3(u)$ is picked. Note that both $CH_1(u) = \emptyset$ and $CH_2(u) = \emptyset$ hold **iff** the boolean sum $BoolSum(\mathcal{X}(\mathcal{N}(u)))$ covers the codeword $X(u)$ and $X(u)$ cannot have a value 0 at any row of the m rows, namely $m = 0$. According to Eq.(7), the probability that $CH_1(u) = \emptyset$ and $CH_2(u) = \emptyset$ is

$$\begin{aligned} p(CH_1(u) = \phi, CH_2(u) = \phi) &= p(m = 0|\mathcal{N}(u)) \\ &= \prod_{i=1}^{N} \left(1 - \frac{\binom{t-\lambda_i}{s'}}{\binom{t}{s'}}\right) \quad (10) \end{aligned}$$

The probability that P_2 holds and the probabilities that u picks up a channel from $CH_1(u)$, $CH_2(u)$, and $CH_3(u)$ with respect to s' for the superimposed $(3, 1, 13)$-code of size 13 (Fig. 1) are illustrated in Fig. 2. Notice that when $s' \leq s$, Algorithm 1 guarantees to choose a channel from $CH_1(u)$ is 1.

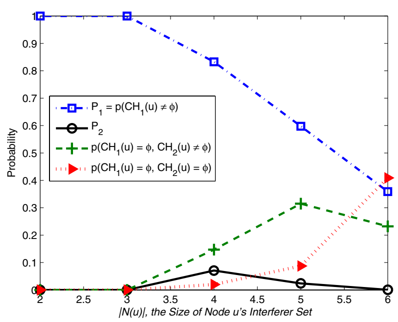

Figure 2: The probabilities that u picks up a channel from $CH_1(u)$, $CH_2(u)$, and $CH_3(u)$, respectively, and the probability that P_2 holds. Here $s = 3$, $t = N = 13$.

5.2 Channel Assignment for Broadcast Traffic

When a channel for broadcast is needed, we can apply Algorithm 1 directly. Let u be any node in a network $G(V, E)$. Let $\mathcal{N}(u)$ be the set of interferers of u. In the topology interference model, $\mathcal{N}(u)$ contains all two-hop neighbors of u, i. e. $\mathcal{N}(u) = N_2(u)$. Let $X(u)$ and $\mathcal{X}(N_2(u))$ be the codewords of u and its interferers. For broadcast channel assignment at node u the inputs to Algorithm 1 are $X(u)$ and $\mathcal{X}(N_2(u))$.

Note that Algorithm 1 does not care whether $\mathcal{N}(u)$ is a complete set of interferers or not. However, if $\mathcal{N}(u)$ is the complete set of interferers of u, and $|\mathcal{N}(u)| \leq s$ holds for $\forall u \in V$, broadcast does not cause any interference (see Theorem 5.2).

In reality, broadcast and unicast coexist. However, broadcast is inferior to unicast, as assumed by IEEE 802.11 standard. Therefore, when applying Algorithm 1 for broadcast channel assignment, u selects an unused channel in $CH_1(u) \neq \emptyset$ first. If fails, u picks up an unused channel in $CH_2(u) \neq \emptyset$. If no channels in $CH_1(u)$ and $CH_2(u)$ is available for u's broadcast, u picks up an unused primary channel from $CH_3(u)$.

5.3 Channel Assignment for Unicast Traffic

In this section, we consider the channel assignment for the unicast traffic from node u to node v, where u and v reside in each other's transmission range. In our consideration, it is u's responsibility to compute the channel for the link $(u \to v)$. For simplicity, we use $N(u)$ to denote $N_1(u)$, the one-hop immediate neighbor set of u. We have $u \in N(v)$ and $v \in N(u)$.

A simple idea would be to plug-in $X(u)$ and $\mathcal{X}(N(v)) \cup \{X(v)\}$ into Algorithm 1 to compute a channel for $(u \to v)$. However, since $\mathcal{X}(N(u))$ is available to u too, it is reasonable to use both $\mathcal{X}(N(u))$ and $\mathcal{X}(N(v))$ for $(u \to v)$ channel assignment. This is our motivation for designing Algorithm 2 for the unicast traffic from u to v. Note that in Algorithm 2 we consider $N(u)$ and $N(v)$ instead of $N_2(u)$ and $N_2(v)$ as the interferers for the unicast traffic from u to v. We will prove that the channel codewords from one-hop neighbors of both the sender and the receiver suffice for Algorithm 2 to achieve 100% throughput with a very simple scheduling algorithm.

Algorithm 2 Channel Assignment for unicast from u to v

Input: Codewords $\mathcal{X}(N(u))$, and $\mathcal{X}(N(v))$
Output: $CH(u \to v)$, a channel to the link from u to v.

function $CH(u \to v)$=**UnicastChannelSelect**($\mathcal{X}(N(u))$, $\mathcal{X}(N(v))$)
  $CH_1(u) \leftarrow SelectAChannel(BoolSum(\mathcal{X}(N(v) \cup \{v\})) \oplus BoolSum(\mathcal{X}(N(v) \cup \{v\} \setminus \{u\})))$ ▷ Find a primary channel that is secondary to all nodes in $N(v) \cup \{v\} \setminus \{u\}$.
  if $CH_1(u) \neq \emptyset$ **then**
    $CH(u \to v) \leftarrow CH_1(u)$
  else
    $CH_2(u) \leftarrow SelectAChannel(\overline{BoolSum(\mathcal{X}(N(u) \cup \{u\}))} \bigwedge BoolSum(\mathcal{X}(N(v))))$ ▷ Find a secondary channel that is secondary to all nodes in $N(u) \cup \{u\}$ but primary to at least one node in $N(v)$.
    if $CH_2 \neq \emptyset$ **then**
      $CH(u \to v) \leftarrow CH_2(u)$
    else
      $CH_3(u) \leftarrow SelectAChannel(X(u) \bigwedge \overline{X(v)})$ ▷ Select a channel that is primary to u and secondary to v.
      $CH(u \to v) \leftarrow CH_3(u)$
    end if
  end if
end function

The basic idea for Algorithm 2 is sketched below. Node u, the unicast source, first computes a channel that is primary to u but secondary to all nodes in $N(v) \cup \{v\} \setminus \{u\}$. In this case, the channel selected corresponds to a row with a value 1 in $X(u)$ and all 0's in $\mathcal{X}(N(v) \cup \{v\} \setminus \{u\})$. If this primary channel does not exist, u computes a channel that is secondary to all nodes in $N(u) \cup \{u\}$ but primary to at least one node in $N(v)$. If fails again, u picks up a primary channel that is secondary to v. As shown in Theorem 5.6, this channel selection criteria intends to minimize interference and accordingly maximize throughput.

The design motivation for Algorithm 2 is stated as follows. A node should utilize its primary channels if possible; Otherwise, it should choose a secondary channel that is secondary to all nodes in its closed neighborhood, but not secondary to all nodes in the receiver's neighborhood, since otherwise, the receiver may choose the same channel for its own unicast, causing interference.

Note that each node u runs a copy of Algorithm 2 to compute a channel k for the unicast link $(u \to v)$, where $v \in N(u)$. Therefore Algorithm 2 is a localized transmitter-oriented channel assignment algorithm.

5.3.1 Interference Analysis

An interesting problem is whether Algorithm 2 can compute an interference-free channel for u's transmission to v. Note that there are two different kinds of interferences for the unicast traffic: the direct interference caused by immediate neighbors and the indirect interference caused by the neighbors of the receiver. The first one results in the *exposed terminal problem* while the second one results in the *hidden terminal problem*.

The hidden and exposed terminal problems are well-known phenomenons in wireless networks due to the broadcast nature of the wireless media. For example, in Fig. 3, when node u is transmitting data to node v, the hidden terminal problem occurs when node x, which is unaware of the ongoing transmission, attempts to transmit, thus causing collision at node v. In Fig. 4, when node v is transmitting data to node u, the exposed terminal problem occurs when node x, which is aware of the ongoing transmission, refrains to communicate with y, thus causing degraded network throughput.

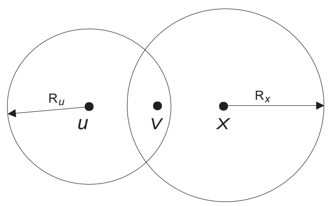

Figure 3: The hidden terminal problem in wireless networks.

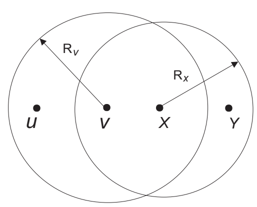

Figure 4: The exposed terminal problem in wireless networks.

In the following we prove that when the number of immediate neighbors of any node in the network is upper-bounded by s, the hidden/exposed problems can be solved and the network communication is free of interference. Note that in the following analysis, we assume that there is no broadcast traffic that can potentially interfere with the unicast traffic.

THEOREM 5.3. *Let u and v be any pair of immediate neighbors in the network $G(V, E)$. If $|N(w)| \leq s$ holds for $\forall w \in V$, Algorithm 2 yields* **hidden** *terminal interference-free channel assignment for the unicast traffic from u to v.*

PROOF. Let x be any hidden terminal, as shown in Fig. 3. We have $x \in N(v)$. Since $|N(v)| \leq s$, $|N(v) \cup \{v\} \setminus \{u\}| \leq s$. Therefore the Boolean sum of all codewords owned by $N(v) \cup \{v\} \setminus \{u\}$ does not cover the codeword of u due to the s-disjunct property of the superimposed code $\mathcal{X}$ used for channel assignment. Thus $CH_1(u) \neq \emptyset$ holds in Algorithm 2 and u can choose one of its primary channels that are secondary to all nodes in $N(v) \cup \{v\} \setminus \{u\}$. Let k be the channel selected by u for the unicast from u to v.

We claim that it is impossible for any node $x \in N(v) \cup \{v\} \setminus \{u\}$ to choose k for unicast based on Algorithm 2. Assume x needs a channel to unicast to y. Since $|N(y)| \leq s$, $CH_1(x) \neq \emptyset$. Therefore x will choose one of its primary channels that are secondary to all nodes in $N(y) \cup \{y\} \setminus \{x\}$ based on Algorithm 2. However, k is secondary to x since $x \in N(v)$. Therefore the unicasts from u to v and from x to y do not interfere since they use different channels.

Note that any node w in $N(u)$ but not in $N(v)$ may choose the same channel as that of u for unicast. But this unicast does not cause interference at v since v is out of w's transmission range. $\square$

THEOREM 5.4. *Let v and u be any pair of immediate neighbors in the network $G(V, E)$. If $|N(w)| \leq s$ holds for $\forall w \in V$, Algorithm 2 yields* **exposed** *terminal interference-free channel assignment for the unicast traffic from v to u.*

PROOF. Let x be any exposed terminal to the unicast from v to u, as shown in Fig. 4. Let y be the destination of the unicast traffic from x. We have $x \in N(v)$, $x \notin N(u)$, and $y \notin N(v) \cup N(u)$. Thus the ACK from y to x does not reach v. For the same reason, the ACK from u to v does not reach x. Therefore, no matter which channels the links $(u \to v)$ and $(y \to x)$ receive from Algorithm 2, the two ACKs do not collide at v and x.

Since v and y are hidden with respect to x, based on Theorem 5.3,v and y choose different channels when $|N(w)| \leq s$ holds for $\forall w \in V$ in the network. Therefore, the ACK from y to x and the data from v to u do not collide at x. For the same reason, the ACK from u to v and the data from x to y do not collide at v.

Based on this analysis, Algorithm 2 yields **exposed** terminal-free channel assignment. $\square$

Note that Theorems 5.3 and 5.4 hold when $|N(w)| \leq s$ for $\forall w \in V$ for a network $G(V, E)$. Assuming no interference caused by broadcast traffic (see Subsection 5.2), these two theorems indicate that Algorithm 2 yields interference-free communications in the network G when the maximum node degree (the number of one-hop neighbors) is $\leq s$.

THEOREM 5.5. *If $|N(w)| \leq s$ for $\forall w \in V$ holds for a network $G(V, E)$, Algorithm 2 yields interference-free communications in G.*

PROOF. Proof follows from Theorems 5.3 and 5.4. $\square$

5.3.2 Throughput Analysis

It is interesting to observe that the induced graph of the edges being assigned the same channel via Algorithm 2 is a forest. Recent research [32, 33] indicates that with a simple scheduling algorithm (maximal weight independent set scheduling), a tree graph can achieve 100% throughput under the primary interference constraints. This result can be applied to analyze the achievable throughput via Algorithm 2.

Let's study Algorithm 2 again. It has the following nice feature:

LEMMA 5.3. *Let $(w \to u)$ and $(u \to v)$ be two adjacent edges in $G(V, E)$. Assume k_1 is the channel assigned to $(w \to u)$ and k_2 is the channel to $(u \to v)$ by Algorithm 2. We have $k_1 \neq k_2$.*

PROOF. Channels k_1 and k_2 are computed by w and u respectively. If $CH_1(w) \neq \emptyset$, $k_1 \in CH_1(w)$. Therefore k_1 is primary to w but secondary to $N(u) \cup \{u\} \setminus \{w\}$. In this case, since k_1 is secondary to u, $k_1 \notin CH_1(u)$ and $k_1 \notin CH_3(u)$. Also because k_1 is primary to w, k_1 can not be in $CH_2(u)$ since $w \in N(u)$ and all channels in $CH_2(u)$ are secondary to $N(u) \cup \{u\}$. Thus channel k_1 can not be selected by u for the edge $(u \to v)$ if $k_1 \in CH_1(w)$.

If $CH_1(w) = \emptyset$ and $CH_2(w) \neq \emptyset$, k_1 is selected from $CH_2(w)$ by w, which means that k_1 is secondary to all nodes in $N(w) \cup \{w\}$ but primary to at least one node in $N(u)$. Therefore k_1 can not be in $CH_2(u)$ since it contains channels secondary to all nodes in $N(u) \cup \{u\}$. $k_1 \notin CH_1(u)$ and $k_1 \notin CH_3(u)$ hold too since k_1

is secondary to u as $u \in N(w)$. Therefore channel k_1 can not be selected for the edge $(u \rightarrow v)$ if $k_1 \in CH_2(w)$.

If k_1 is selected from $CH_3(w)$, k_1 is primary to w and secondary to u, therefore $k_1 \notin CH_1(u)$ and $k_1 \notin CH_3(u)$. We claim that $k_1 \notin CH_2(u)$ too since otherwise k_1 would be secondary to w because $w \in N(u)$ and all channels in $CH_2(u)$ are secondary to the nodes in $N(u) \cup \{u\}$.

Therefor the channel k_1 assigned to the link $(w \rightarrow u)$ by Algorithm 2 could not be assigned to the link $(u \rightarrow v)$. We have $k_1 \neq k_2$. □

Note that the proof of Lemma 5.3 utilizes the fact that CH_3 is always non-empty. This is guaranteed by the following requirement on the channel codewords: for any two channel codewords $X(u)$ and $X(v)$, there exists two channels k_1 and k_2 such that k_1 is primary to u and secondary to v, and k_2 is primary to v and secondary to u.

COROLLARY 5.1. *Let k_1 and k_2 be the channels assigned to the edges $(u \rightarrow v)$ and $(v \rightarrow u)$, respectively, by Algorithm 2. Then $k_1 \neq k_2$.*

PROOF. Claim follows from Lemma 5.3. □

Corollary 5.1 indicates that the channels used for DATA and for ACK are always different. Lemma 5.3 indicates that two adjacent links can transmit DATA or ACK concurrently. Therefore, a multihop path can achieve maximum throughput in MR-MC networks since all nodes can transmit simultaneously without causing any collision.

Let $G_k(V, E_k)$ be the induced graph containing all edges receiving channel k based on Algorithm 2. We have

LEMMA 5.4. *For $\forall k \in C$, where C is the set of orthogonal channels, G_k is a forest.*

PROOF. For contradiction we assume that G_k is not a forest. In other words, G_k contains a circle $\mathcal{O}$. Consider any two adjacent edges $(w \rightarrow u)$ and $(u \rightarrow v)$ in $\mathcal{O}$. Based on Lemma 5.3, the channels assigned to $(w \rightarrow u)$ and $(u \rightarrow v)$ must be different. Therefore only one of them can appear in G_k. A contradiction to the assumption that $(w \rightarrow u)$ and $(u \rightarrow v)$ both appear in G_k. Thus no circle $\mathcal{O}$ exists in G_k. □

Lemma 5.3 indicates that each tree in G_k has a star-shaped topology[4], and the number of concurrent transmissions supported equals the total number of stars in all G_k.

COROLLARY 5.2. *Each tree in G_k is a star.*

PROOF. Proof follows from that of Lemma 5.3. □

COROLLARY 5.3. *The number of concurrent transmissions supported by the network equals the total number of stars in all G_k for all $k \in C$.*

PROOF. Since each star topology can support only one unicast at any time, claim follows. □

Brzezinski, Zussman, and Modiano [32] has proved the following lemma:

LEMMA 5.5. *A maximal weight independent set scheduling algorithm achieves 100% throughput for a tree network.*

[4]Since we consider directed links, this topology actually is a star-shaped DAG (Directed Acyclic Graph).

Therefore we have

THEOREM 5.6. *There exists a simple scheduling algorithm such that Algorithm 2 yields 100% throughput.*

PROOF. Proof follows from Lemma 5.4 and Lemma 5.5. □

Brzezinski, Zussman, and Modiano [32] presents multiple algorithms based on matroid intersection to partition the network into subnetworks with large capacity regions to maximize the throughput of each of the subnetwork. Algorithm 2, which is much simpler, maximizes the throughput if each node has a unique channel codewords satisfying the condition elaborated in Section 3.1.

5.3.3 Simulation Study

In this subsection, we conduct simulation to evaluate Algorithm 2 in terms of channel utilization and usage fairness. Our goal is to investigate: 1. the number of concurrent transmissions; 2. the channel usage fairness.

In the simulation we have considered an area of a 100×100 square units with 13 randomly deployed nodes. The simulation settings are listed as follows:

- All simulation results are averaged over 100 different topologies.
- The number of available channels in the network is set to $N = 13$.
- The superimposed $(3, 1, 13)$-code $\mathcal{X}$, as shown in Fig. 1, is applied in the simulation.
- Each node randomly picks a unique codeword from $\mathcal{X}$ as its channel codeword.
- The average node degree is denoted by d, where d varies from 2 to 6.
- The number of radios equipped by each node is denoted by Q, where $Q \in \{2, 4, 6, 8, 10, 12\}$. Q varies under different topologies.

Note that the number of channels utilized by a node can be measured by the number of concurrent transmissions supported by that node. Therefore for an arbitrary node u, we denote its channel utilization by the number of supported concurrent transmissions.

Fig. 5 describes the relationship among the number of concurrent transmissions supported by each node, the average node degree d, and the number of radios Q. For each settings of d and Q, the results are averaged on all the nodes in the network over 100 different topologies. As shown in Fig. 5, when the number of radios is fixed in the network, the smaller the average node degree, the larger the number of concurrent transmissions supported by each node. This is because the smaller the average node degree, the less number of interferers a node may have, namely the more number of channels available for concurrent transmissions.

When the average node degree is fixed, the larger the number of radios, the more the number of concurrent transmissions supported by each node. This result is intuitive since the number of concurrent transmissions is bounded by the number of radios in the network. Comparing the six curves in Fig. 5, we find that the smaller the number of radios, the smaller the number of concurrent transmissions supported by each node. We also find that when $d \leq s$ and Q is fixed, the number of concurrent transmissions supported by each node reaches its maximum, that is Q.

Fairness in channel usage is another important issue in wireless networks. Note that in our simulation study, the channel assignment matrix $\mathcal{X}$ has a constant column weight, which means that

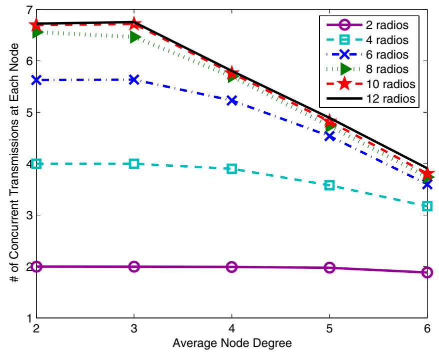

Figure 5: The average number of concurrent transmissions supported by each node.

each node in the network has the same numbers of primary channels and secondary channels. Since the channel codeword is picked randomly and independently for each node, intuitively the channel usage should be fair. This has been validated by our simulation result reported in Fig. 6.

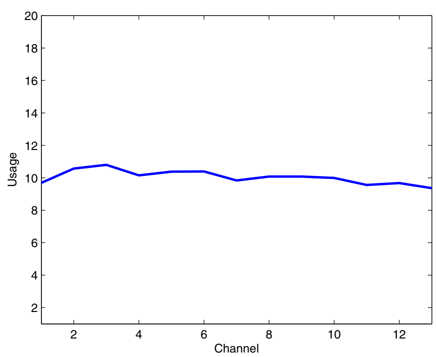

Figure 6: The channel usage of each channel when average node degree is 3.

6. DISCUSSION

6.1 Strength of Algorithms 1 and 2

Note that Algorithms 1 and 2 are both localized. They require the availability of the channel codewords from one or two-hop neighborhood, which results in low communication overhead since the binary codewords are short. In addition, both algorithms have low computation overhead since only simple Boolean algebraic is involved.

Algorithm 1 is generic. It is suitable for both unicast and broadcast traffic. As long as the codewords of the set of interferers are available, an interference-aware channel can be computed. Under certain conditions, this channel causes no interference.

The underlying design principle for unicast channel assignment (Algorithm 2) is the same as that of Algorithm 1: a node always selects a channel that causes the least interference to its neighborhood based on its current knowledge. With a simple scheduling algorithm, Algorithm 2 can achieve 100% throughput.

Neither of the two algorithms relies on the s-disjunct superimposed code, which is introduced to identify the scenarios when interference free communications are possible. However, if the channel codewords form an s-disjunct code, Algorithms 1 and 2 can compute a channel for better interference mitigation. In addition, the larger the s, the better the performance.

Both algorithms can be uploaded to the same node for broadcast and unicast channel computation. However, broadcast may be inferior to unicast, as in IEEE 802.11 standard. In this case, a channel has a higher priority to be assigned for unicast. If the probability of a channel being primary or secondary is the same for all nodes, the channel usage is fair.

Note that even though we assume the frequency channels in our discussion, both algorithms work with any kind of orthogonal channels: time slots, orthogonal codes, etc., as long as the channels can be labelled by a binary string indicating their primary and secondary roles to each node.

6.2 Superimposed Codes

The s-disjunct property elaborated in Lemma 4.1 plays a significant role in interference-free channel assignment. It is clear that the strength s should be strong and the size t should be large for a superimposed code $\mathcal{X}$ of length N to be applicable to a MR-MC network with N available orthogonal channels. Given N, computing a satisfiable superimposed s-disjunct code is non-trivial. As reported by D'yachkov and Rykov in [31], the following relationship of N, t, s, and λ_{max} holds.

LEMMA 6.1. *Let $t > \lambda_{max} > s \geq 1$ and $N > 1$ be integers.*

1. *For any superimposed $(s, 1, N)$-code of length N, size t, and maximum row weight λ_{max}:*

$$N \quad \geq \quad \lceil \frac{(s+1)t}{\lambda_{max}} \rceil \tag{11}$$

2. *If $\lambda_{max} \geq s+2$, $(s+1)t = \lambda_{max} N$, and there exists a superimposed $(s, 1, N)$-code $\mathcal{X}$ with size t and maximum row weight λ_{max}, then*

   - *Code $\mathcal{X}$ has a constant column weight $w = s + 1$, and a constant row weight $\lambda = \lambda_{max}$, and the maximal dot product of any two codewords in $\mathcal{X}$ is 1.*
   - *The following inequality holds true:*

$$\lambda^2 - \frac{\lambda(\lambda-1)}{s+1} \quad \leq \quad t \tag{12}$$

Note that for a superimposed $(s, 1, N)$-code, the upper bound of s is limited by N. Therefore s cannot be a large number if the number of available channels N in the network is small. However, this should not be a restriction on the application of superimposed codes in IEEE 802.16e based stationary MR-MC wireless mesh networks. The OFDMA technique in IEEE 802.16e [34] [35] allows bandwidth to be divided into many lower-speed sub-channels to increase resistance to multi-path interference. Typically a large number of non-overlapping orthogonal sub-channels are available for simultaneous transmissions. Therefore in this case, s can be large since N is large.

However, the non-overlapping channels in 802.11 standards are limited (3 non-overlapping channels in IEEE 802.11b/g; 12 non-overlapping channels in original IEEE 802.11a). Therefore s in 802.11-based wireless mesh networks is limited to some small number, which may affect the effectiveness of channel assignment.

A good news is that it is very likely that we still have disjunct property with more than s codewords. Let's introduce the definition for α-almost s-disjunct code proposed in [29] [36]: *A binary matrix* *is α-almost s-disjunct if for any randomly selected set of s columns, the probability that they cover no other column is at least α.* In [29], authors proposed a study on a 3-disjunct superimposed code of size 30, where the number of codewords is much larger than s. The results indicate that this superimposed code is 0.95-almost 15 disjunct, and 0.6-almost 30 disjunct. This study tells us that a less powerful s-disjunct superimposed code could work well in our channel assignment.

6.3 Scalability Considerations

In superimposed codes, although t increases superlinearly compared to N [31], it is still a bounded number. Therefore, when applying a superimposed code in a MR-MC network, the network size is restricted because a superimposed code can only accommodate at most t nodes. To overcome this problem, we propose the following scalability enhancement.

As shown in Fig. 7, we map the network by cellular grids (regular hexagonal grids). The side length of each grid is R_{max}, where R_{max} is the maximum interference range a node can have in the network. Since the chromatic number of face coloring of such a graph is 3, the cellular grids of the network can be easily classified into 3 categories denoted by A, B, and C.

Given a superimposed $(s, 1, N)$-code $\mathcal{X}$, we evenly divide $\mathcal{X}$ into 3 subsets: t_A, t_B and t_C. Each subset exclusively contains about $1/3$ codewords of $\mathcal{X}$, representing a possible channel assignment for a grid category. For example, nodes belonging to the grids of category A are assigned channels based on t_A; nodes belonging to grids of category B are assigned channels based on t_B; and nodes belonging to grids of category C are assigned channels based on t_C, as shown in Fig. 7.

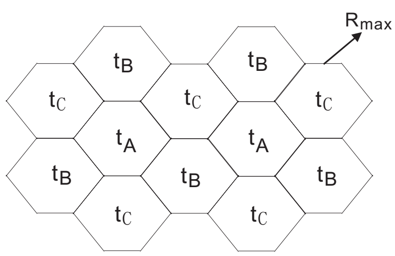

Figure 7: Channel assignment in a scalable network under a cellular grid topology.

Facilitated with a cellular grid topology, the network can scale to infinite size, though the superimposed $(s, 1, N)$-code has a bounded size t.

6.4 Applications to Mobile Mesh Networks

Since both algorithms are localized, and the communication overhead for a node to obtain the channel codewords from its neighborhood is low, channel assignment for mobile MR-MC wireless mesh networks can be easily supported. We will quantitatively study the performance of our algorithms in a mobile mesh network and test their support to popular mobile routing protocols in our future research.

6.5 Future Research

This paper presents our exploratory work toward capacity improvement in MR-MC mesh networks. We will study the performance of our algorithms in an mobile environment and test their capability of simultaneously supporting both unicast and broadcast. Additionally, we will design a MAC protocol based on these two algorithms to efficiently utilize the network resource for throughput maximization. Furthermore, we will explore the impact of channel codeword on the performance of channel assignment based on our algorithms.

7. CONCLUSION

In this paper, we have designed two localized channel assignment algorithms based on s-disjunct superimposed codes for multi-radio multi-channel wireless mesh networks. Our algorithms can effectively support channel allocation for both unicast and local broadcast since channels are pertained to transmitters instead of links even though the interferers at the destination affects channel selection. The selected channels are expected to cause low overall switching delay and low interference to the local neighborhood. In addition, we have identified the conditions when interference-free channel assignment can be achieved and when hidden/exposed terminal problems can be avoided. For unicast, our algorithm results in 100% network throughput with a simple scheduling algorithm. Since we do not make any assumptions on the underlying network settings such as traffic patterns and MAC/routing protocols, our channel assignment algorithms are applicable to a wide range of MR-MC mesh networks.

8. ACKNOWLEDGMENTS

The research of Xing, Cheng, and Ma is supported in part by the NSF CAREER award CNS-0347674. The research of Liang is supported in part by ONR under grant N00014-07-1-0395 and N00014-07-1-1024.

We are grateful to Dr. Lili Qiu, our shepherd, for her help in revising this paper. We are also grateful to the anonymous reviewers for their invaluable and constructive comments and suggestions.

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