Utility-Based Cooperative Spectrum Sensing Scheduling in Cognitive Radio Networks

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***Abstract*—In this paper, we consider the problem of cooperative spectrum sensing scheduling (C3S) in a cognitive radio network when there exist multiple primary channels. Deviated from the existing research our work focuses on a scenario in which each secondary user has the freedom to decide whether or not to participate in cooperative spectrum sensing; if not, the SU becomes a free rider who can eavesdrop the decision about the channel status made by others. Such a mechanism can conserve the energy for spectrum sensing at a risk of scarifying the spectrum sensing performance. To overcome this problem, we address the following two questions: "which action (contributing to spectrum sensing or not) to take?" and "which channel to sense?" To answer the first question, we model our framework as an evolutionary game in which each SU makes its decision based on its utility history, and takes an action more frequently if it brings a relatively higher utility. We also develop an entropy based coalition formation algorithm to answer the second question, where each SU always chooses the coalition (channel) that brings the most information regarding the status of the corresponding channel. All the SUs selecting the same channel to sense form a coalition. Our simulation study indicates that the proposed scheme can guarantee the detection probability at a low false alarm rate.**

***Index Terms*—Cognitive radio networks; cooperative spectrum sensing; free rider; evolutionary game; coalition formation.**

I. INTRODUCTION

Spectrum sensing has become an essential function in cognitive radio networks for secondary users (SUs) to identify the temporarily unused/under-utilized licensed spectrum bands and to protect the transmissions of the primary users (PUs). Due to the uncertainty factors resulted from the channel randomness such as shadowing and fading, the detection performance of spectrum sensing may be significantly compromised. Fortunately, the uncertainty problems can be mitigated by allowing the spatially dispersed secondary users to cooperate and collaboratively make a decision regarding the status of the licensed bands [1]. This procedure is termed *cooperative spectrum sensing*, which has recently been actively studied in [2], [3], [4], [5], [6] due to its attractive performance.

Existing literature mainly focuses on a typical scenario where all the secondary users contribute to spectrum sensing. However in reality, it might be not necessary for each secondary user to perform spectrum sensing at every time slot as long as the sensing performance meets certain requirements. Spectrum sensing consumes a certain amount of energy that may alternatively be diverted to data transmissions. Moreover, secondary users in emerging mobile and ad hoc applications may tend to behave selfishly and take advantage of others to conserve energy for their own data transmissions. Therefore, it is of great importance to study the dynamic behaviors of selfish users in cooperative spectrum sensing.

We propose a novel cooperative framework, in which secondary users can decide whether to participate in spectrum sensing or do nothing to save their own energy. This framework is modeled as an evolutionary game [7], [8], which provides an excellent means to address the strategy uncertainty that a user/player may face when exploring different actions. For those SUs that do nothing, we take them as free riders that can eavesdrop the final decisions about the status of the primary users. By making different choices, SUs can get different utilities determined by their achieved revenue/throughput and energy consumption. Each SU selects its action based on its utility history, and a rational user should choose a strategy more frequently if that strategy brings a higher utility.

Since there exist multiple primary channels, each contributing secondary user needs to determine which channel to sense. To answer this question, we propose an "entropy" based coalition formation algorithm, where a SU chooses to join the coalition that brings the most information about the channel status distribution. As a result, all the SUs sensing the same channel form a coalition to collaboratively make the final decision regarding the status of the primary channel. Since entropy is a measure of the uncertainty of the channel status, each contributing secondary user joins the coalition that results in the largest entropy reduction. This algorithm ensures that the contributing SUs autonomously collaborate and self-organize into disjoint coalitions; and spectrum sensing of each channel is performed within the corresponding coalition independently.

We assess the performance of the proposed scheme in terms of detection probability and false alarm probability for each channel via simulation study. Our results demonstrate the effectiveness of the proposed scheme in detecting the presence of primary users, while maintaining a nice property of low false alarm probability.

The rest of the paper is organized as follows: Section II presents our system model, and Section III details the proposed C3S scheme. Our simulation results are reported in Section IV.

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We summarize our work and conclude the paper in Section V.

II. System Model

We consider a cognitive radio network with M primary channels and N SUs, denoted by $\mathcal{M} = \{1, 2, ..., M\}$ and $\mathcal{N} = \{n_1, n_2, ..., n_N\}$, respectively. We assume that the system is time-slotted. At each time slot, M primary channels are sensed synchronously. In this paper, we design an evolutionary game to help each SU decide whether to participate in spectrum sensing or not, and partition all the contributing SUs into M coalitions, with each sensing one channel. The decision is made by the coalition head based on majority vote, and is broadcast to all members in the same coalition.

The problem of spectrum sensing can be formulated as a binary hypothesis testing [2]:

$$x(t) = \begin{cases} n(t), & H_0 \\ hs(t) + n(t), & H_1, \end{cases} \tag{1}$$

where $x(t)$ is the signal received by the secondary user, $s(t)$ is the primary users' transmitted signal, $n(t)$ is the additive white Gaussian noise (AWGN), and h is the amplitude gain of the channel. Here H_0 and H_1 denote the hypothesis of the absence and presence, respectively, of the primary user in the considered channel. According to [9], the received signal $x(t)$ will be transformed into a normalized output Y by energy detector. Then Y is compared to a detection threshold θ to decide whether the PU is present.

In a Rayleigh fading environment, the detection probability and false alarm probability of SU i detecting the status of primary user/channel j are, respectively, given by $P_{d,i,j}$ and $P_{f,i,j}$ as follows [2]:

$$\begin{aligned} P_{d,i,j} &= P\{Y_{i,j} > \theta_j | H_1\} \\ &= e^{-\frac{\theta_j}{2}} \sum_{n=0}^{m-2} \frac{1}{n!} (\frac{\theta_j}{2})^n + (\frac{1+\overline{\gamma}_{i,j}}{\overline{\gamma}_{i,j}})^{m-1} \\ &\quad \times [e^{-\frac{\theta_j}{2(1+\overline{\gamma}_{i,j})}} - e^{-\frac{\theta_j}{2}} \sum_{n=0}^{m-2} \frac{1}{n!} (\frac{\theta_j \overline{\gamma}_{i,j}}{2(1+\overline{\gamma}_{i,j})})^n] \quad (2) \\ P_{f,i,j} &= P\{Y_{i,j} > \theta_j | H_0\} = \frac{\Gamma(m, \frac{\theta_j}{2})}{\Gamma(m)} \quad (3) \end{aligned}$$

where $Y_{i,j}$ is the normalized output of SU i sensing the status of primary user j, θ_j is the detection threshold for primary user j, m is the time bandwidth product, $\overline{\gamma}_{i,j}$ denotes the average SNR of the received signal from the PU to SU, which is defined as $\overline{\gamma}_{i,j} = \frac{P_j h_{j,i}}{\sigma^2}$, with P_j being the transmit power of PU j, σ^2 being the Gaussian noise variance, and $h_{j,i} = \frac{\kappa}{d_{j,i}^{\nu}}$ being the path loss between PU j and SU i; here κ is the path loss constant, ν is the path loss exponent, and $d_{j,i}$ is the distance between PU j and SU i. $\Gamma(.,.)$ is the incomplete gamma function and $\Gamma(.)$ is the gamma function.

III. Utility-Based Cooperative Spectrum Sensing Scheduling

There are two major stages in our cooperative spectrum sensing scheduling scheme. First, each SU decides whether to be a contributor or a free rider based on their utility history. Second, each contributor makes a decision on which channel to sense, i.e., which coalition to join.

A. Which Action to Take?

In our model, each secondary user first makes its own decision about whether to contribute to sense or to do nothing as a free rider at each time slot.

We model this problem as an evolutionary game, which contains two kinds of players: the *contributors (denoted by $\mathcal{C}$)* that participate in spectrum sensing, and the *free riders (denoted by $\mathcal{F}$)* that only overhear the spectrum sensing decisions by others. Then the proposed cooperative sensing problem can be modeled by a game $(\mathcal{N}, U)$, with $\mathcal{N}$ being the set of players (the SUs) and U being the utility function or value of each player. Apparently, $\mathcal{C} \bigcup \mathcal{F} = \mathcal{N}$.

The utility function for a contributor C ($C \in \mathcal{C}$) is given by

$$U(C) \quad = \quad R(C) - E(C) \tag{4}$$

where $R(C)$ is the revenue received by C, and $E(C)$ is the cost in terms of energy consumed for spectrum sensing per time slot. Similarly, the utility function for a free rider F ($F \in \mathcal{F}$) is defined as

$$U(F) \quad = \quad E(F) - H(F) \tag{5}$$

where $E(F)$ is the return in terms of saved energy for not participating in spectrum sensing and $H(F)$ is the punishment for not contributing. The values of $R(C)$ and $H(F)$ are related to the spectrum sensing performance. We will introduce specific utility equations in the next subsection.

Assume that all the secondary users are rational and selfish, and they are all interested in maximizing their own utilities. To decide which action to take, the SUs perform the following update algorithm:

1) Initially, each SU (each player) has two choices (C-contributor, or F-free rider), and selects each choice with a probability of 50%.
2) At each time slot t:
    - each player n_i selects the action $e \in \{C, F\}$ with probability $p_{n_i}(e, t)$;
    - each player computes the utility $U_{n_i}(e, t)$ for the selection of action e at time slot t.
3) Each user n_i approximates the average utility for the action e within the past T time slots (including the slot t), which can be expressed as $\overline{U}_{n_i}(e)$; each user n_i also approximates the average utility of the mixed actions (all the actions) $\overline{U}_{n_i}$ in the past T slots. Note that if there are less than $T-1$ slots in the past, all slots need to be considered.
4) The probability of user n_i selecting the action $e \in \{C, F\}$ for the next time slot can be computed by:

$$p_{n_i}(e, (t+1)) = p_{n_i}(e, t) + \eta_{n_i}[\overline{U}_{n_i}(e) - \overline{U}_{n_i}] p_{n_i}(e, t) \tag{6}$$

with η_{n_i} being the step size of adjustment determined by n_i.

If strategy $e \in \{C, F\}$ results in a higher utility compared to the average utility, the probability of e being adopted in the next slot should grow. And the *growth rate* is expressed by:

$$\dot{p}_e = \frac{p(e,t+1) - p(e,t)}{p(e,t)} = \eta[\overline{U}(e) - \overline{U}]$$

Apparently, the growth rate is proportional to the difference between strategy e's current utility and the current average utility achieved by the mixed strategies selected in the past T slots.

B. Which Channel to Sense?

In this subsection, we answer the question "which channel to sense" by developing an "entropy" based coalition formation algorithm. The basic idea lies in that each contributor selects to join a coalition that brings the most information about the corresponding channel's status distribution.

Entropy is a measure of the uncertainty associated with a random value. One important feature about entropy is that if the value of a random variable is highly predictable, its entropy should be low. The formal definition of entropy for a discrete random variable is as follows:

For a discrete random variable X with possible values $x_1, ..., x_n$ and probability mass function p(X), the entropy can be explicitly written as [10]:

$$H(X) = \sum_{i=1}^{n} p(x_i) \log_b \frac{1}{p(x_i)} = -\sum_{i=1}^{n} p(x_i) \log_b p(x_i) \quad (7)$$

The common values of b are 2, Euler's number e, and 10. In this paper, we take b=2.

In this paper, the random values are the statuses of each channel, denoted by H_1 and H_0, respectively. We denote this discrete random variable by X^i for channel i, whose possible values are drawn from $\{x_1^i, x_2^i\}$, where x_1^i=1 and $x_2^i = 0$ indicate that channel i is busy and idle, respectively. We aim to predict the value of X^i. To accurately predict the channel status distribution, we calculate two probabilities $p_1^i = p(x_1^i = 1|H_1)$ and $p_2^i = p(x_2^i = 0|H_0)$. Apparently, the higher the p_1^i and p_2^i, the more accurate our estimation is. Thus we borrow the entropy concept here. The goal of our method is to reduce the entropy for each channel as much as possible when deciding which channel to sense for each player.

All the contributors in $\mathcal{C}$ need to be dispersed into the M channels. The secondary users contributing to cooperative sensing for the channel i form a coalition, denoted by S_c^i. Since each secondary user can only sense one channel at each time slot, the collection of the coalitions satisfies the following conditions:

- $\forall i, j \in \{1, ..., M\}, S_c^i \cap S_c^j = \emptyset$
- $S_c^1 \cup S_c^2 \cup ... \cup S_c^M = \mathcal{C}$

The collection of the coalitions is called a partition of $\mathcal{C}$.

First, we derive the values of p_1^i and p_2^i. Since we employ majority vote as our fusion rule, we have

$$\begin{aligned} p_1^i &= p(x_1^i = 1|H_1) \\ &= \text{Pr (more than half nodes in } S_c^i \text{ report } H_1|H_1) \end{aligned}$$

Equivalently, we define

$$\begin{aligned} p_1^i &= p(x_1^i = 1|H_1) \\ &= \sum_{k=\lceil \frac{1+|S_c^i|}{2} \rceil}^{|S_c^i|} \text{Pr } (k \text{ SUs in } S_c^i \text{ report } H_1|H_1) \end{aligned}$$

When there are k SUs from S_c^i that detect the presence of a PU and report H_1, we say *the k SUs form* S_d^k. With different k members in S_d^k, we have $K = \binom{|S_c^i|}{k}$ different S_d^k, which is denoted by $S_{d,j}^k$, with $\{j = 1, 2..., K\}$. Formally,

$$\text{Pr } (k \text{ SUs in } S_c^i \text{ report } H_1|H_1) = \sum_{w=1}^{K} \prod_{\substack{\forall m, S_m \in S_{d,w}^k \\ \forall n, S_n \in S_c^i \& S_n \notin S_{d,w}^k}} P_{d,S_m,i}(1 - P_{d,S_n,i}) \quad (8)$$

Thus

$$p_1^i = p(x_1^i = 1|H_1) = \sum_{k=\lceil \frac{1+|S_c^i|}{2} \rceil}^{|S_c^i|} \sum_{w=1}^{K} \left\{ \prod_{\substack{\forall m, S_m \in S_{d,w}^k \\ \forall n, S_n \in S_c^i \& S_n \notin S_{d,w}^k}} P_{d,S_m,i}(1 - P_{d,S_n,i}) \right\} \quad (9)$$

where $P_{d,S_m,i}$ and $P_{d,S_n,i}$ denote the detection probabilities of the coalition member S_m and S_n for channel i, whose values can be determined by (2).

Next we derive the probability that the channel i is idle. Let P_F^i denote the probability of the false alarm rate for channel i. We have

$$\begin{aligned} p_2^i &= p(x_2^i = 0|H_0) = 1 - P_F^i \\ &= 1 - \text{Pr (more than half nodes in } S_c^i \text{ raise false alarm)} \end{aligned}$$

Similarly,

$$P_F^i = \sum_{k=\lceil \frac{1+|S_c^i|}{2} \rceil}^{|S_c^i|} \text{Pr } (k \text{ SUs in } S_c^i \text{ raise false alarm}) \quad (10)$$

For simplicity, we assume that the local false alarm probabilities computed by the SUs within the coalition for channel i are the same, which is denoted by $P_{f,i}$. Therefore, after coalition fusion the false alarm probability for channel i can be expressed as:

$$P_F^i = \sum_{k=\lceil \frac{1+|S_c^i|}{2} \rceil}^{|S_c^i|} (P_{f,i})^k (1 - P_{f,i})^{|S_c^i|-k} \quad (11)$$

Consequently,

$$\begin{aligned} p_2^i &= p(x_2^i = 0|H_0) = 1 - P_F^i \\ &= 1 - \sum_{k=\lceil \frac{1+|S_c^i|}{2} \rceil}^{|S_c^i|} (P_{f,i})^k (1 - P_{f,i})^{|S_c^i|-k} \end{aligned} \quad (12)$$

Finally, we derive the entropy for our prediction about the channel status distribution as follows:

$$H(X^i) = \sum_{z=1}^{2} p_z^i \log_b \frac{1}{p_z^i} = -\sum_{z=1}^{2} p_z^i \log_b p_z^i \qquad (13)$$

A new contributor should always choose the channel whose entropy can be reduced the most because of its participation. Let $H(X^i)$ be the entropy of channel i with its current coalition members. If a new contributor n_V joins, the new entropy $H(X^i, n_V)$ can be obtained by (13). Formally, the entropy based coalition selection algorithm can be elaborated as follows:

Step 1) Compute the entropy difference $\Delta H(n_V) = H(X^i) - H(X^i, n_V)$ for the set $\mathcal{M}$ of candidate channels.

Step 2) Select channel $\widehat{i}$ such that

$$\widehat{i} = \arg\max_{i \in \mathcal{M}} (H(X^i) - H(X^i, n_V)).$$

After the contributor n_V joins the right coalition, it receives its revenue $R(n_V)$. However, it also consumes a certain amount of energy for spectrum sensing. Hence we adopt the following utility function for contributor n_V:

$$U(n_V) = R(n_V) - E(n_V) = \mu \Delta H(n_V) - \omega \xi \qquad (14)$$

where $\Delta H(n_V)$ is the entropy reduction, μ is a predetermined parameter defining the value of the revenue, ξ is the energy consumption for spectrum sensing per time slot, and ω is used to transfer per unit energy consumption into equivalent expenditure.

For free riders, we assume that the utilities for all the free riders are the same, which are defined as follows:

$$U(F) = \omega\xi - \lambda min\{1, -logS(H_{max})\} \qquad (15)$$

where $S(H_{max})$ is a measurement of the degree of the satisfaction with the detection performance, which can be modeled as a sigmoid function of the maximum entropy. In our consideration, we take the largest entropy of the M channels H_{max} as the measurement of the detection performance. In our consideration, the range of $S(H_{max})$ is between [0,1]. Here λ is a predetermined parameter defining the harshness of the penalty. We can see that when the detection performance is highly satisfied, the value of $S(Hmax)$ is close to 1; thus the penalty is close to 0. On the other hand, the value of the penalty is high to encourage the SUs to participate in spectrum sensing. Similar to (15), ξ is the energy saved from spectrum sensing, and ω is used to transfer per unit energy into equivalent revenue.

The sigmoid function for the satisfaction degree of the detection performance is calculated by:

$$S(H_{max}) = \frac{1}{1 + e^{-a(\widetilde{H} - H_{max})}} \qquad (16)$$

where $\widetilde{H}$ is the predefined requirement for the entropy, and a decides the steepness of the satisfactory curve.

We summarize our proposed algorithm in Algorithm I.

Algorithm I: Cooperative Spectrum Sensing Scheduling

```
1.Initialization:
  t=1
  ◇ $\forall n_i \in \mathcal{N}$ selects a proper step size $\eta_{n_i}$;
  ◇ $\forall n_i \in \mathcal{N}$, $e \in \{C, F\}$, $p_{n_i}(e,t) = 50\%$.
2. $\forall n_i \in \mathcal{N}$ selects an action $e$ with probability $p_{n_i}(e,t)$.
   For each contributor $S_i \in \mathcal{C}$
   ◇ Calculates the entropy for each channel $j$;
   ◇ Selects channel $\hat{j}$ that brings in the largest entropy reduction;
   ◇ Receives the utility determined by (14).
3. After each contributor joins a coalition, each free rider
   ◇ Gets the largest entropy of the $M$ channels $H_{max}$;
   ◇ Receives the utility determined by (15).
4. Each user updates the probability of each action for the next time slot
   by (6)
5. t=t+1, go to Step 2
```

IV. SIMULATION EVALUATION

A. Simulation Setup

In our simulation study, we consider a network that consists of two PUs deployed in a $3km \times 3km$ square area with SUs surrounding the PUs. We set the parameters following the simulation setup in [11], which are listed in Table I.

TABLE I: System Parameters

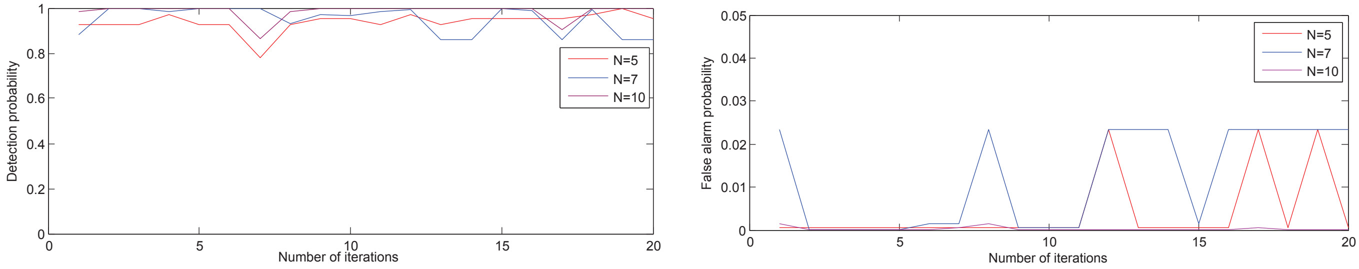
Parameter	Semantic Meaning	Value
m	time bandwidth product	5
ν	path loss exponent	3
κ	path loss constant	1
ξ	energy consumption for spectrum sensing per slot	1
ω	equivalent revenue per unit energy	10
λ	the parameter to determine the value of penalty	10
μ	the parameter to determine the value of revenue	10
η	adjustment step size	0.06
$\widetilde{H}$	entropy threshold	0.3
σ^2	Gaussian noise variance	$-90dBm$
P_{PU}	PU transmit power	$100mW$

Since all the information needed to make a decision for each SU is its utility history, our algorithm is pure localized and distributed; thus it scales well to large networks. Therefore there is no need to simulate a network that contains many PUs/channels. Note that the results reported in this section are averaged over 20 runs.

B. Simulation Results

Since our algorithm allows some of the SUs to be free riders, apparently, the energy for spectrum sensing can be conserved. However, we also need to guarantee the detection performance for each channel. Figures 1a and 1b illustrate the detection probability and false alarm probability for channel 1, respectively. Similarly, the detection performance for channel 2 is shown in Figures 2a and 2b.

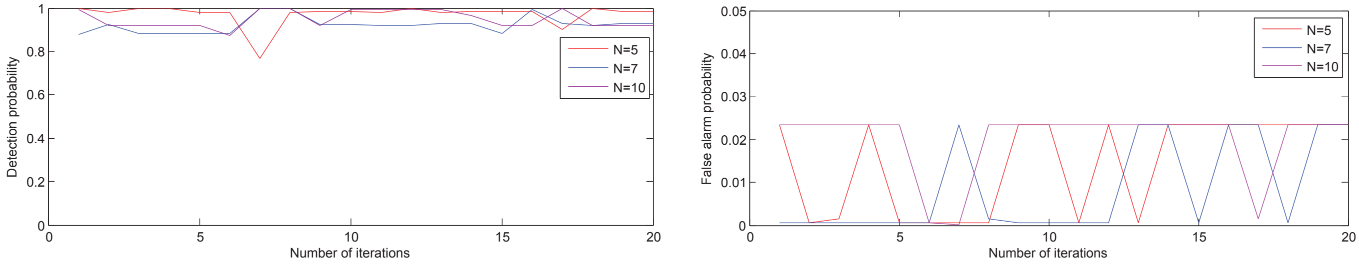
As depicted in Figure 1a and Figure 2a, our algorithm achieves high detection probabilities for both channels with different network scales. We also observe that a larger network results in a better detection probability. This improvement mainly comes from the fact that the increase in the network



(a) Detection probability

(b) False alarm probability

Fig. 1: Detection performance for channel 1.



(a) Detection probability

(b) False alarm probability

Fig. 2: Detection performance for channel 2.

size implies more information could be used to estimate the channel status. Another nice feature of our algorithm is that the false alarm probabilities for both channels are effectively restrained. From Figures 1b and 2b, we can see that the false alarm probabilities are always below 0.025 for both channels.

V. CONCLUSION

In this paper, we propose a novel idea of cooperative spectrum sensing scheduling when there exist M primary channels and N secondary users. Different from existing research focusing on cooperative sensing, the SUs in our consideration have the freedom to choose whether or not to contribute to spectrum sensing. Such a mechanism can help to reduce the overall energy consumption for spectrum sensing. We also introduce the concept of entropy to estimate the channel status distribution. The SUs make decisions about which channel to sense based on the entropy of each channel, and each contributor always selects to sense the channel that brings the most information of the status distribution. This method effectively reduces the uncertainty of the channel status. According to the extensive simulation study, our scheme is proved to be effective and flexible. It achieves a high detection probability and a low false alarm probability.

In our future research we intend to jointly consider the two problems of which action to take and which channel to sense as a secondary user may want to decide which action to take based on the channel it needs to sense.

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