Achievable Transmission Capacity of Cognitive Mesh Networks With Different Media Access Control

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***Abstract*—Spectrum sharing is an emerging mechanism to resolve the conflict between the spectrum scarcity and the growing demands for the wireless broadband access. In this paper we investigate the achievable transmission capacity of a wireless backhaul mesh network that shares the spectrums of the underutilized cellular uplink over the underlay spectrum sharing model with several commonly adopted medium access control protocols: slotted-ALOHA, CSMA/CA, and TDMA. By employing stochastic geometry, we derive the probabilities for a packet to be successfully transmitted in the primary cellular uplink and the secondary mesh networks. The achievable transmission capacity of the secondary network with outage probability constraints from both the primary and the secondary systems is obtained according to Shannon's Theory. The capacity region and the achievable capacity when the outage probabilities equal their corresponding threshold values are analyzed numerically and the results illustrate the effect of adjusting the mesh network parameters on the achievable transmission capacity under different MAC protocols.**

***Index Terms*—Achievable transmission capacity; secondary cognitive mesh networks; primary cellular network; media access control; outage probability constraint**

I. Introduction

Due to the rapid growth of wireless applications in recent years, the spectrums that can be used are becoming more and more scarce. Many measurements have shown that most of the allocated spectrums experience a low utilization, while heavy spectrum utilization often takes place in unlicensed bands. Cognitive radios (CRs), spectrum sensors, mesh networking, and other emerging technologies are believed to be able to facilitate new forms of spectrum sharing that can greatly improve the spectral efficiency, if policies are in place to support possible forms of sharing [1].

In cognitive networks, each node is equipped with a cognitive radio capable of sensing the available frequency bands that are not currently in use by any primary user. There is no doubt that with the introduction of CR to wireless networks, the design of algorithms and protocol implementations needs to be reconsidered. The achievable capacity of a cognitive radio network (CRN) is considered a fundamental key problem. On the other hand, wireless mesh networking (WMN) provides a practical technology for wireless Internet to offer both indoor and outdoor broadband wireless connectivity without costly wired network infrastructures. Therefore in this paper we aim to study the achievable transmission capacity of cognitive mesh networks in bits/hop/s/Hz/node according to Shannon's Theory with outage constraints from both the primary and the secondary networks under the physical interference model, in which a transmission is successful if and only if the signal-to-interference-and-noise-ratio (SINR) at the receiver is higher than a threshold.

Although the physical interference model is deemed to be more realistic than the protocol interferece model, a number of problems arise when analyzing the physical interference model mathematically. Because computing SINR involves a complex nonlinear function with multiple variables, many optimization problems under the SINR model are NP-hard. Moreover, since both the scheduling feasibility and the maximum allowable flow rate on each link are determined by SINR, computing an optimal solution to maximize capacity requires the joint consideration of the network, link, and physical layers. Due to these difficulties, theoretical results on CRNs over the physical interference model remain limited [2].

In this paper, we investigate the achievable transmission capacity of a cognitive mesh network over different MAC protocols under the physical interference model. As the utilization of the uplink in a cellular network is less efficient than that of the down-link [3], we assume an underlay spectrum sharing model, in which the mesh network is allowed to use the spectrums of the cellular uplink as long as it does not undermine the successful transmissions of the cellular users. Our main contributions are summarized as follows:

1) To our best knowledge, the only work that targets the capacity analysis of multi-hop cognitive networks is [2], which takes a cross-layer approach to maximize the network capacity by jointly considering power control, MAC scheduling, and routing. The achievable transmission capacity of multi-hop cognitive networks with different media access control has never been addressed.
2) By employing stochastic geometry, we obtain the successful transmission probabilities of the primary system[1]

[1]In this paper, "primary system" and "primary network" are used interchangeably. Similarly, "secondary system" and "secondary network" are used interchangeably.

and the secondary system under the physical interference model in which the cumulative effect of the interference from both systems are considered.

3) We derive the achievable transmission capacity of the secondary mesh network under the outage constraints of both the primary and the secondary systems. Three MAC schemes for the mesh network, namely slotted-ALOHA, carrier-sensing multiple-access with collision avoidance (CSMA/CA), and Time-Division Multiple Access (TDMA), are considered in analyzing the achievable transmission capacity.
4) The capacity region and the achievable transmission capacity when the outage probabilities of the primary and secondary systems are fixed to their corresponding threshold values, are analyzed numerically. The results indicate that the achievable transmission capacity of the secondary mesh network is affected by the power ratio, the transmission probability, and the receiver threshold.

The rest of the paper is organized as follows. In Section II, we review the most related work on cognitive network transmission capacity analysis. Section III presents our system model. In section IV we derive the achievable transmission capacity of the secondary network with outage constraints from both systems under three MAC protocols. The numerical results illustrating the capacity region and the transmission capacity when the outage probabilities of the primary and secondary systems are fixed to their corresponding threshold values are reported in Section V. The conclusion and future research are discussed in Section VI.

II. Related Work

It is a fundamental problem to understand whether a network can achieve the desirable transmission capacity in a cognitive context. Many research has been done to investigate the capacities of primary and secondary networks under a variety of wireless channel models and communication protocol assumptions. The scaling law of the cognitive network transport capacity is studied in [4]-[7], where [4] demonstrates that there is no performance loss for the secondary network. As reported in [5], two coexisting wireless networks that operate in the same geographic region can achieve the same throughput scaling law, which is equal to that established in [6] for a standalone wireless network. The results in [7] reveal that when the primary nodes are known to the secondary ones and the secondary nodes are denser than the primary ones, both networks can simultaneously achieve the same throughput scaling law as that of a standalone ad hoc network.

Since asymptotic analysis on the scaling law only seeks to answer how the total network capacity scales with the network size, the effect of many important system parameters is unintentionally ignored. Quantitative analysis on achievable transmission capacity of the secondary network while guaranteeing the outage probability constraints of the primary and the secondary systems is investigated in [8]-[12] under the physical interference model. These works define the achievable transmission capacity as the spatial density of the successful transmissions per unit area but they still neglect other system parameters. Due to the high computational complexity brought in by SINR in multi-hope context, all previous effort on the capacity analysis of cognitive networks is performed over single-hop networks, in which the distribution of the nodes is assumed to follow a Poisson process. To our knowledge, no MAC scheme is considered in any of the existing research. For multi-hop cognitive networks, [2] investigates the maximum capacity by jointly considering power control, scheduling, and routing.

Different from previous works, we define the achievable transmission capacity of multi-hop cognitive mesh networks in bits/hop/s/Hz/node according to Shannon's Theory by considering the outage probability constraints of both the primary and the secondary systems over different MAC schemes, and investigate how the network capacity is affected by the parameters of the secondary system rather than the spatial density.

III. System Model

A. Spectrum Sharing Model

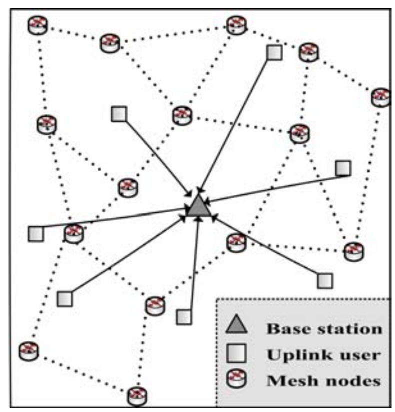


Fig. 1: The coexisting cellular and mesh networks

In this paper, we consider an underlay spectrum sharing model shown in Fig. 1, in which the primary system constitutes the uplink communications of a cellular network and the secondary system is a mesh network that are collocated in the same area. Secondary users are allowed to use the typically underutilized uplink primary spectrums as long as they do not deteriorate the primary communications [13]. As we know, the call of the users in cellular networks follows a Poisson process [14]; therefore we can assume that the distribution of the transmitting nodes in the primary system follows a stationary Poisson point process with a density λ in the finite two-dimensional plane.

We assume that nodes in the secondary mesh network are relatively stationary. Since the achievable transmission capacity in bits-hop/s/Hz/node is highly dependent on the MAC mechanism [15], the effect of three commonly adopted MAC schemes, namely slotted ALOHA, CSMA/CA, and TDMA, on the achievable transmission capacity of the secondary mesh network is investigated in section IV.

B. Interference Model

There exist two interference models: the *protocol interference model* and the *physical interference model* [5], to define the conditions for a successful transmission. Though the analysis on the protocol interference model is much simpler than that on the physical interference model, the protocol interference model does not consider the cumulative effect of the interfering transmissions. Therefore in this paper, we adopt the physical interference model when evaluating the SINR at the receiver side, which is deemed to be more accurate.

To make our elaboration easier, we consider a typical receiver and the corresponding desired transmitter for each of the primary and secondary systems. For a propagation channel model with Rayleigh fading, the received power at the typical receiver from a transmitter i is $P_k \delta_{ki} |d_{ki}|^{-\alpha}$, where P_k is the transmission power of the system k, with $k = p$ denoting the primary system and $k = s$ the secondary system, α is the path loss exponent, d_{ki} is the distance between the transmitting node i in system k and the typical receiver, and δ_{ki} is the fading factor on the power transmitted from the node i in system k to the typical receiver. Considering the cumulative interference from the transmitters of both the primary system and the secondary system, the SINR at the typical receiver of system k is:

$$SINR_k = \frac{P_k \delta_{k0} d_{k0}^{-\alpha}}{I_p + I_s + N_0} \tag{1}$$

where δ_{k0} is the fading factor on the power transmitted from the desired transmitter to the receiver, N_0 is the thermal noise power, d_{k0} is the distance between the desired transmitter and the typical receiver of system k, and $I_p = \sum_{i \in PU} P_p \delta_{pi} |d_{pi}|^{-\alpha}$ and $I_s = \sum_{i \in SU} P_s \delta_{si} |d_{si}|^{-\alpha} a_i$ are the cumulative interference powers from the transmitting nodes of the primary and the secondary system to the typical receiver of system k, respectively, with a_i being a binary random variable denoting whether or not the node is in the transmitting mode ($Prob\{a_i = 1\} = p$ denotes the node is in transmitting mode and $Prob\{a_i = 0\} = 1 - p$ denotes the node is not in transmitting mode). Note that PU and SU are respectively the sets of primary and secondary users that do not include the desired transmitter and the typical receiver. As spectrum sharing systems are interference-limited [8], the thermal noise is negligible. Hence for simplicity, SIR is used instead of SINR.

$$SIR_k = \frac{P_k \delta_{k0} d_{k0}^{-\alpha}}{I_p + I_s} \tag{2}$$

With a Rayleigh fading model, the probability density function of δ_{ki} is given by an exponential function with a unit mean:

$$f_{\delta_{ki}}(x) = exp(-x) \tag{3}$$

The signal can be correctly decoded at the typical receiver of system k if the corresponding SIR is greater than a predefined threshold η_k. Therefore the probability of a successful transmission can be defined as

$$\begin{aligned} P(SIR_k \geq \eta_k) &= P(\frac{P_k \delta_{k0} d_{k0}^{-\alpha}}{I_p + I_s} \geq \eta_k) \\ &= P\{\delta_{k0} \geq \frac{\eta_k d_{k0}^{\alpha}}{P_k}(I_p + I_s)\} \\ &= E_{\{\delta_{ki}\}}\{\int_{\frac{\eta_k d_{k0}^{\alpha}}{P_k}(I_p+I_s)}^{+\infty} e^{-x} dx\} \\ &= E_{\{\delta_{ki}\}}\{exp(-\frac{\eta_k d_{k0}^{\alpha}}{P_k}(I_p + I_s))\} \end{aligned} \tag{4}$$

where $E_{\{\delta_{ki}\}}$ denotes the expectation with respect to the set of random variables δ_{ki}. Since all nodes from the primary and secondary systems transmit independently, I_p and I_s can be assumed independent. Hence we have

$$\begin{aligned} & E_{\{\delta_{ki}\}}\{exp\{-\frac{\eta_k d_{k0}^{\alpha}}{P_k}(I_p + I_s)\}\} \\ = \ & E_{\{\delta_{pi}\}}\{exp(-\frac{\eta_k d_{k0}^{\alpha}}{P_k} I_p)\} \\ & \times E_{\{a_i\}} E_{\{\delta_{si}\}}\{exp(-\frac{\eta_k d_{k0}^{\alpha}}{P_k} I_s)\}. \end{aligned} \tag{5}$$

Since the distribution of the transmitting nodes in the primary system follows a stationary Poisson point process, we can assume that the aggregate interference power I_p is a shot noise process [10]. From [16], we have

$$\begin{aligned} & E_{\{\delta_{pi}\}}\{exp(-\frac{\eta_k d_{k0}^{\alpha}}{P_k} I_p)\} \\ = \ & E_{\{\delta_{pi}\}}\{exp(-\frac{\eta_k d_{k0}^{\alpha}}{P_k} \sum_{i \in PU} P_p \delta_{pi} |d_{pi}|^{-\alpha})\} \\ = \ & exp\{-\lambda \int_{-\infty}^{\infty} 1 - E[e^{\frac{P_p}{P_k} \eta_k d_{k0}^{\alpha} \delta_{pi} |x|^{-\alpha}}] dx\} \\ = \ & exp(-C_\alpha d_{k0}^2 \eta_k^{2/\alpha} \gamma_{kp}^{2/\alpha} \lambda). \end{aligned} \tag{6}$$

where $\gamma_{kp} = P_p / P_k$ is the *power ratio* between the primary system and the system k, and $C_\alpha = (2\pi/\alpha)\Gamma(2/\alpha)\Gamma(1-2/\alpha)$.

Given a regular mesh network, assume that the distances from the SU transmitters to a receiver of the system k is known. According to (3), $E[e^{-\delta_{si} y}] = \frac{1}{1+y}$. We have

$$\begin{aligned} & E_{\{a_i\}} E_{\{\delta_{si}\}}\{exp(-\frac{\eta_k d_{k0}^{\alpha}}{P_k} I_s)\} \\ = & E_{\{a_i\}} E_{\{\delta_{si}\}}\{exp(-\frac{\eta_k d_{k0}^{\alpha}}{P_k} \sum_{i \in SU} P_s \delta_{si} |d_{si}|^{-\alpha} a_i)\} \\ = & \prod_{i \in (su_k)} [1 - p + \frac{p}{1 + \eta_k (\frac{d_{k0}}{d_{si}})^{\alpha} \frac{P_s}{P_k}}] \end{aligned} \tag{7}$$

where (su_k) is the set of the interfering nodes from the secondary network to the system k. From (6) and (7),we obtain the probability for a packet to be successfully received by the BS in the primary system and a mesh node in the secondary system as follows:

$$P(SIR_p \geq \eta_p) = \exp\{-C_\alpha d_{p0}^2 \eta_p^{2/\alpha}\lambda\} \times \prod_{i\in(su_p)} [1-p+\frac{p}{1+\eta_p(x_{pi})^\alpha/\gamma_{sp}}] \quad (8)$$

$$P(SIR_s \geq \eta_s) = \exp\{-C_\alpha d_{s0}^2 \eta_s^{2/\alpha}\lambda\gamma_{sp}^{2/\alpha}\} \times \prod_{i\in(su_s)} [1-p+\frac{p}{1+\eta_s(x_{si})^\alpha)}] \quad (9)$$

where $x_{pi} = d_{p0}/d_{si}$, and $x_{si} = d_{s0}/d_{si}$.

IV. Achievable Transmission Capacity of The Secondary Network

In this section, we investigate the achievable transmission capacity of the cognitive mesh network with outage constraints from the primary system as well as the secondary system. Because the network capacity is highly affected by the employed MAC scheme, three MAC protocols are considered: slotted-ALOHA, CSMA/CA, and TDMA. In a cognitive network with a underlay model, the primary system operates as if no secondary network is present. Therefore we assume that the parameters of the primary network are fixed; but we can change the parameters such as the receiver threshold η_s, the power ratio γ_{sp}, the transmission probability, or the number of slots in the secondary system to achieve the maximum capacity of the secondary network while guaranteeing the target outage probabilities of the primary and secondary system. Since the capacity in packets/s/node does not take into account the spectral efficiency of each packet, we define the transmission capacity in bits/hop/s/Hz/node, which measures the number of bits each node can transmit to one of its neighboring nodes per second per Hertz, according to Shannon's Theory in our work.

A. Achievable Transmission Capacity Over Slotted-ALOHA

As claimed in [17], idle states do not exist in a saturated network with slotted-ALOHA. During each time slot, each node in the network transmits a packet with a probability P_A provided that the node has a data packet to transmit, or is ready to receive a packet with the probability $1 - P_A$. Thus the probability for an arbitrary mesh node receiving a packet from its neighbors is $P_A(1 - P_A)P(SIR_s \geq \eta_s)$. According to Shannon's Theory, a packet can carry $\log_2(1 + \eta_s)$ bits/s/Hz information. Hence the transmission capacity in bits/hop/s/Hz/node of the secondary mesh network can be defined as:

$$C_{ALOHA} = P_A(1-P_A)\log(1+\eta_s)P(SIR_s \geq \eta_s) \quad (10)$$

where $P(SINR_s \geq \eta_s)$ is the successful transmission probability given in section III-B. From (8) and (9), we obtain the outage probability constraints with respect to P_A, η_s, and γ_{sp} as follows:

$$1-\exp\{-C_\alpha d_{p0}^2 \eta_p^{2/\alpha}\lambda\} \prod_{i\in(su_p)} [1-P_A+\frac{P_A}{1+\eta_p x_{pi}^\alpha/\gamma_{sp}}] \leq \theta_p \quad (11)$$

$$1-\exp\{-C_\alpha d_{s0}^2 \eta_s^{2/\alpha}\lambda\gamma_{sp}^{2/\alpha}\} \prod_{i\in(su_s)} [1-P_A+\frac{P_A}{1+\eta_s x_{si}^\alpha)}] \leq \theta_s \quad (12)$$

where θ_p, θ_s are the outage probabilities of the primary system and the secondary system, respectively. Taking logarithm, the problem of computing the maximum achievable transmission capacity can be formulated as:

$$\max f(P_A,\eta_s,\gamma_{sp}) = \ln P_A + \ln(1-P_A) + \ln(\log(1+\eta_s)) - \beta_s\eta_s^{2/\alpha}\gamma_{sp}^{2/\alpha} + \sum_{i\in(su_s)} \ln[1-P_A+\frac{P_A}{1+\eta_s x_{si}^\alpha}] \quad (13)$$

Subject to:

$$\varepsilon_p \leq -\beta_p\eta_p^{2/\alpha} + \sum_{i\in(su_p)} \ln[1-P_A+\frac{P_A}{1+\eta_p x_{pi}^\alpha/\gamma_{sp}}] \quad (14)$$

$$\varepsilon_s \leq -\beta_s\eta_s^{2/\alpha}\gamma_{sp}^{2/\alpha} + \sum_{i\in(su_s)} \ln[1-P_A+\frac{P_A}{1+\eta_s x_{si}^\alpha}] \quad (15)$$

where $\varepsilon_k = \ln(1-\theta_k)$ and $\beta_k = C_\alpha d_{k0}^2\lambda$ for system k. By maximizing the value f, we obtain the maximum achievable transmission capacity $C_{ALONA} = e^f$. With Taylor series expansion, we have $\ln(1-x) = -x + \Theta(x^2) \approx -x(-1 \leq x < 1)$. Therefore,

$$\ln[1-P_A+\frac{P_A}{1+\eta_p x_{pi}^\alpha/\gamma_{sp}}] = \ln[1-P_A(1-\frac{1}{1+\eta_p x_{pi}^\alpha/\gamma_{sp}})] \approx -P_A(1-\frac{1}{1+\eta_p x_{pi}^\alpha/\gamma_{sp}}) \quad (16)$$

$$\ln[1-P_A+\frac{P_A}{1+\eta_s x_{si}^\alpha}] = \ln[1-P_A(1-\frac{1}{1+\eta_s x_{si}^\alpha/\gamma_{sp}})] \approx -P_A(1-\frac{1}{1+\eta_s x_{si}^\alpha}) \quad (17)$$

Hence, our problem can be reformulated as:

$$\max f(P_A,\eta_s,\gamma_{sp}) = \ln P_A + \ln(1-P_A) + \ln(\log(1+\eta_s)) - \beta_s\eta_s^{2/\alpha}\gamma_{sp}^{2/\alpha} - P_A f_2(\eta_s) \quad (18)$$

subject to

$$\varepsilon_p \leq -\beta_p\eta_p^{2/\alpha} - P_A f_1(\gamma_{sp}) \quad (19)$$

$$\varepsilon_s \leq -\beta_s\eta_s^{2/\alpha}\gamma_{sp}^{2/\alpha} - P_A f_2(\eta_s) \quad (20)$$

where $f_1(\gamma_{sp}) = \sum_{i\in(su_p)}(1-\frac{1}{1+\eta_p x_{pi}^\alpha/\gamma_{sp}})$ and $f_2(\eta_s) = \sum_{i\in(su_s)}(1-\frac{1}{1+\eta_s x_{si}^\alpha})$. From the objective function, we have

$$\frac{\partial f}{\partial P_A} = \frac{1}{P_A} - \frac{1}{1-P_A} - f_2(\eta_s) \quad (21)$$

where $0 \leq P_A \leq 1$. Let partial differential $\frac{\partial f}{\partial P_A} = 0$, we obtain $P_A^0 = \frac{2+f_2-\sqrt{4+f_2^2}}{2f_2}$. When $0 \leq P_A < P_A^0$, $\frac{\partial f}{\partial P_A} > 0$, and when $P_A^0 < P_A \leq 1$, $\frac{\partial f}{\partial P_A} < 0$. Thus $f(P_A,\eta_s,\gamma_{sp})$ increases with P_A when $0 \leq P_A < P_A^0$, but it starts to decrease when $P_A > P_A^0$. From the constraints (19) and (20), we have:

$$P_A \leq \frac{-c}{f_1(\gamma_{sp})} \quad (22)$$

$$P_A \leq \frac{-\varepsilon_s - \beta_s \eta_s^{2/\alpha} \gamma_{sp}^{2/\alpha}}{f_2(\eta_s)} \quad (23)$$

where $c = \varepsilon_p + \beta_p \eta_p^{2/\alpha}$. Then the optimal transmission probability P_A^* in slotted-ALOHA can be defined by:

$$\min\left\{\frac{2 + f_2(\eta_s) - \sqrt{4 + f_2^2(\eta_s)}}{2f_2(\eta_s)}, \frac{-c}{f_1(\gamma_{sp})}, \frac{-\varepsilon_s - \beta_s \eta_s^{2/\alpha} \gamma_{sp}^{2/\alpha}}{f_2(\eta_s)}\right\} \quad (24)$$

By plugging (24) into the objective function (18), our problem can be represented by $\max f_{P_A^*}(\eta_s, \gamma_{sp})$. Using partial differential and numerical analysis, we can obtain the optimal $\gamma_{sp}^* = g(\eta_s)$. Then the problem is translated to $\max f_{(P_A^*, \gamma_{sp}^*)}(\eta_s)$, which is a one-dimensional nonlinear programming without constraints. Now we can resolve it using methods such as the one-dimensional Newton iteration [18] to obtain the maximum achievable transmission capacity $C^* = e^{f(P_A^*, \gamma_{sp}^*, \eta_s^*)}$.

In order to give intuitions on how the parameters of P_A, η_s, and γ_{sp} impact on the achievable transmission capacity of slotted-ALOHA based cognitive mesh networks, we consider a specific case where the outage probabilities of the primary system and the secondary system equal their corresponding threshold values θ_p and θ_s, respectively. Thus (13)-(15) can be simplified as follows:

$$\max C_{ALOHA} = P_A(1 - P_A) \log(1 + \eta_s)(1 - \theta_s) \quad (25)$$

s.t

$$\varepsilon_p = -\beta_p \eta_p^{2/\alpha} - P_A \sum_{i \in (su_p)} (1 - \frac{1}{1 + \eta_p x_{pi}^\alpha / \gamma_{sp}}) \quad (26)$$

$$\varepsilon_s = -\beta_s \eta_s^{2/\alpha} \gamma_{sp}^{2/\alpha} - P_A \sum_{i \in (su_s)} (1 - \frac{1}{1 + \eta_s x_{si}^\alpha}) \quad (27)$$

From (26), we can see that the probability P_A denoting a mesh node having a packet to transmit is affected by the power ratio γ_{sp} between the primary system and the secondary system when the parameters of the primary system are given, i.e.:

$$P_A = \frac{-c}{\sum\limits_{i \in (su_p)} (1 - \frac{1}{1 + \eta_p x_{pi}^\alpha / \gamma_{sp}})} = f(\gamma_{sp}) \quad (28)$$

where $c = \varepsilon_p + \beta_p \eta_p^{2/\alpha}$, which is the same as (22). From (27) and (28), we have

$$(\varepsilon_s + \beta_s \eta_s^{2/\alpha} \gamma_{sp}^{2/\alpha}) = f(\gamma_{sp}) \sum_{i \in (su_s)} (1 - \frac{1}{1 + \eta_s x_{si}^\alpha}) \quad (29)$$

By solving the nonlinear equation (29) with the Newton Iteration method, we obtain $\gamma_{sp} = G_1(\eta_s)$. Hence P_A is determined by η_s, i.e., $P_A = f(\gamma_{sp}) = G_2(\eta_s)$. Then the achievable transmission capacity of our cognitive mesh network with outage constraints over slotted-ALOHA can be restated as:

$$\begin{aligned} C_{ALOHA} &= G_2(\eta_s)(1 - G_2(\eta_s)) log(1 + \eta_s)(1 - \theta_s) \\ &= f(\gamma_{sp})(1 - f(\gamma_{sp}) log(1 + G_1^{-1}(\gamma_{sp}))(1 - \theta_s) \\ &= P_A(1 - P_A) log(1 + G_2^{-1}(P_A))(1 - \theta_s) \quad (30) \end{aligned}$$

This indicates that the achievable transmission capacity of the secondary network is only affected by the receiver threshold η_s, or P_A, or γ_{sp} of the secondary network. By changing the values of η_s, or P_A, or γ_{sp}, we can obtain the maximum achievable transmission capacity with outage probability constraints.

B. Achievable Transmission Capacity Over CSMA/CA

When the number of concurrent co-channel transmissions becomes large, slotted-ALOHA usually yields a low throughput. In such a case, CSMA/CA can be adopted to limit the effect of interference. Nevertheless, a transmitter still faces the hidden and exposed terminal problems resulted from the interference out of its carrier-sensing range [19]. In this study, we assume that the carrier-sensing range equals the transmission range. Thus a transmitter can only transmit when all its neighbors do not transmit, or it has to wait for a random backoff time before transmission. Additionally, we assume that the network is saturated. At the steady state, each node has a packet to transmit in a generic slot time with a probability P_C, which depends on the conditional collision probability and the "backoff scheme". The derivation of P_C has been addressed in [20]. Here we assume that P_C is a variable that affects the optimal achievable transmission capacity. Given P_C, the transmission probability for a node can be expressed by

$$P_s = P_C(1 - P_C)^{n-1} \quad (31)$$

where n is the number of contending nodes within the carrier-sensing range of the transmitter. Hence the successful transmission probabilities defined in (8) and (9) can be restated as follows:

$$P(SIR_p \geq \eta_p) = \exp\{-C_\alpha d_{p0}^2 \eta_p^{2/\alpha} \lambda\} \times \prod_{i \in (\widetilde{su}_p)} [1- P_C(1 - P_C)^{n-1} + \frac{P_C(1 - P_C)^{n-1}}{1 + \eta_p x_{pi}^\alpha / \gamma_{sp}}] \quad (32)$$

$$P(SIR_s \geq \eta_s) = \exp\{-C_\alpha d_{s0}^2 \eta_s^{2/\alpha} \lambda \gamma_{sp}^{2/\alpha}\} \times \prod_{i \in (\widetilde{su}_s)} [1- P_C(1 - P_C)^{n-1} + \frac{P_C(1 - P_C)^{n-1}}{1 + \eta_s x_{si}^\alpha}] \quad (33)$$

Then, the capacity in bits/hop/s/Hz/node of the secondary mesh network over CSMA/CA with outage probability constraints can be defined as:

$$\max C_{CSMA/CA} = P_C(1-P_C)^{n-1} \log(1+\eta_s) P(SIR_s \geq \eta_s) \quad (34)$$

subject to

$$1-\exp\{-C_\alpha d_{p0}^2\eta_p^{2/\alpha}\lambda\}\times\prod_{i\in(\widetilde{su}_p)}[1-P_C(1-P_C)^{n-1} + \frac{P_C(1-P_C)^{n-1}}{1+\eta_p x_{pi}^\alpha/\gamma_{sp}}]\le\theta_p \tag{35}$$

$$1-\exp\{-C_\alpha d_{s0}^2\eta_s^{2/\alpha}\lambda\gamma_{sp}^{2/\alpha}\}\times\prod_{i\in(\widetilde{su}_s)}[1-P_C(1-P_C)^{n-1} + \frac{P_C(1-P_C)^{n-1}}{1+\eta_s x_{si}^\alpha)}]\le\theta_s \tag{36}$$

Considering the special case where the outage probabilities of the primary and secondary systems equal θ_p and θ_s, respectively. The capacity of the CSMA/CA based secondary network in bits/hop/s/Hz/node can be defined as:

$$\max C_{CSMA/CA}=P_C(1-P_C)^{n-1}(1-\theta_s)\log(1+\eta_s) \tag{37}$$

subject to

$$\varepsilon_p=-\beta_p\eta_p^{2/\alpha}-P_C(1-P_C)^{n-1}\sum_{i\in(\widetilde{su}_p)}(1-\frac{1}{1+\eta_p x_{pi}^\alpha/\gamma_{sp}}) \tag{38}$$

$$\varepsilon_s=-\beta_s\eta_s^{2/\alpha}\gamma_{sp}^{2/\alpha}-P_C(1-P_C)^{n-1}\sum_{i\in(\widetilde{su}_s)}(1-\frac{1}{1+\eta_s x_{si}^\alpha}) \tag{39}$$

From (38) and (39), we obtain $P_C=\widetilde{f}(\gamma_{sp})=\widetilde{G_2}(\eta_s)$ and $\gamma_{sp}=\widetilde{G_1}(\eta_s)$. The achievable transmission capacity over CSMA/CA with respect to different parameters is given below:

$$\begin{aligned} C_{CSMA/CA} &= \widetilde{G_2}(\eta_s)(1-\widetilde{G_2}(\eta_s))\log(1+\eta_s)(1-\theta_s) && (40)\\ &= \widetilde{f}(\gamma_{sp})(1-\widetilde{f}(\gamma_{sp}))\log(1+\widetilde{G_1}^{-1}(\gamma_{sp}))(1-\theta_s) && (41)\\ &= P_C(1-P_C)^{n-1}\log(1+\widetilde{G_2}^{-1}(P_C))(1-\theta_s) && (42)\end{aligned}$$

From these relationships we observe that the achievable transmission capacity of the CSMA/CA based secondary network, $C_{CSMA/CA}$, is affected by the parameters P_C, γ_{sp}, and η_s. Moreover, the maximum achievable transmission capacity can be derived from (40), or (41), or (42).

C. Achievable Transmission Capacity Over TDMA

TDMA is known to be optimal under high traffic load for its ability to guarantee that every node can eventually transmit its own data [21]. In TDMA, each node is assigned a slot beforehand and can transmit packets only in the slot assigned to it; thus reducing the interference from others on a large scale. For a saturated mesh network, we adopt the static spatial TDMA here, where the network is partitioned into subnets according to the spatial locations of the nodes: a node is placed in a subnet with its spatially nearest neighbors, and the number of nodes in one subnet equals the number of time slots. Only one node is allowed to transmit in a subnet within a slot but nodes in different subnets might transmit simultaneously. An example static spatial TDMA scheme with four slots is shown in Fig. 2.

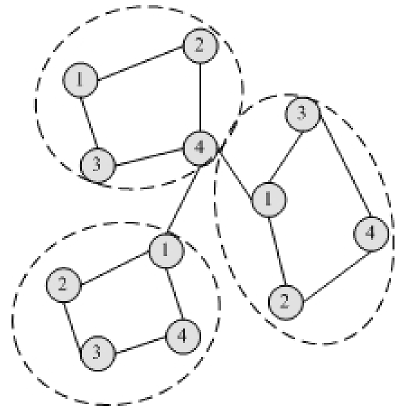


Fig. 2: An example of TDMA subnets

The probabilities for a packet to be successfully received by the BS in the primary network and a mesh node in the TDMA based secondary network can be given as:

$$P(SIR_p\ge\eta_p)=\exp\{-C_\alpha d_{p0}^2\eta_p^{2/\alpha}\lambda\}\times\prod_{i\in(\widehat{su}_p)}(\frac{1}{1+\eta_p x_{pi}^\alpha/\gamma_{sp}})$$

$$P(SIR_s\ge\eta_s)=\exp\{-C_\alpha d_{s0}^2\eta_s^{2/\alpha}\lambda\gamma_{sp}^{2/\alpha}\}\times\prod_{i\in(\widehat{su}_s)}(\frac{1}{1+\eta_s x_{si}^\alpha})$$

Then, the problem of obtaining the maximum achievable transmission capacity over TDMA can be given as follows:

$$\max C_{TDMA}=\frac{1}{V}\log(1+\eta_s)P(SIR_s\ge\theta_s) \tag{43}$$

subject to

$$1-\exp\{-C_\alpha d_{p0}^2\eta_p^{2/\alpha}\lambda\}\times\prod_{i\in(\widehat{su}_p)}(\frac{1}{1+\eta_p x_{pi}^\alpha/\gamma_{sp}})\le\theta_p \tag{44}$$

$$1-\exp\{-C_\alpha d_{s0}^2\eta_s^{2/\alpha}\lambda\gamma_{sp}^{2/\alpha}\}\times\prod_{i\in(\widehat{su}_s)}(\frac{1}{1+\eta_s x_{si}^\alpha})\le\theta_s \tag{45}$$

Consider the special case where the outage probabilities of the primary system and the TDMA based secondary system equal θ_p and θ_s, respectively. The transmission capacity in bits/hop/s/Hz/node of the secondary network can be defined as:

$$\max C_{TDMA}=\frac{1}{V}(1-\theta_s)\log(1+\eta_s) \tag{46}$$

subject to

$$\begin{aligned}\varepsilon_p &= -\beta_p\eta_p^{2/\alpha}-\sum_{i\in(\widehat{su}_p)}\ln(1+\eta_p x_{pi}^\alpha/\gamma_{sp}) && (47)\\ \varepsilon_s &= -\beta_s\eta_s^{2/\alpha}\gamma_{sp}^{2/\alpha}-\sum_{i\in(\widehat{su}_s)}\ln(1+\eta_s x_{si}^\alpha) && (48)\end{aligned}$$

where V is the number of time slots needed for each node in the network to transmit once to its neighbor. Under typical network settings, the ratios $x_{pi}=d_{p0}/d_{si}$ and $x_{si}=d_{s0}/d_{si}$ are affected by V. Let $x_{pi}=f_1(V)$ and $x_{si}=f_2(V)$. Then we obtain $V=\widehat{f}(\gamma_{sp})=\widehat{G_2}(\eta_s)$ and $\gamma_{sp}=\widehat{G_1}(\eta_s)$ from (47) and (48). Therefore, the achievable transmission capacity over TDMA can be written as:

$$C_{TDMA} = \frac{1}{\widehat{G_2}(\eta_s)} log(1+\eta_s)(1-\theta_s) \quad (49)$$

$$= \frac{1}{\widehat{f}(\gamma_{sp})} log(1+\widehat{G_1}^{-1}(\gamma_{sp}))(1-\theta_s) \quad (50)$$

$$= \frac{1}{V} log(1+\widehat{G_2}^{-1}(V))(1-\theta_s) \quad (51)$$

This indicates that the achievable transmission capacity of the TDMA secondary network is affected by V, γ_{sp}, or η_s, and the maximum achievable transmission capacity can be derived based on (49), or (50), or (51).

V. Numerical Analysis

In this section, we present the numerical results on the achievable transmission capacity of an underlay cognitive mesh network based on our analysis. We employ the parameter settings listed in Table I, which are the same as those adopted by [8].

TABLE I: Network Parameter Settings

symbols	interpretations	value
α	pass loss exponent	4
η_p	threshold of the primary cellular network	10
θ_p	outage probability in the primary network	0.05
θ_s	outage probability in the secondary mesh network	0.08
d_{p0}	the distance between a typical user and the base station in the primary network	10
d_{s0}	the distance between the desired transmitting node and the receiver in secondary network	10
N	the number of nodes in the mesh network	400
λ	the spatial density of the primary network	10^{-6}

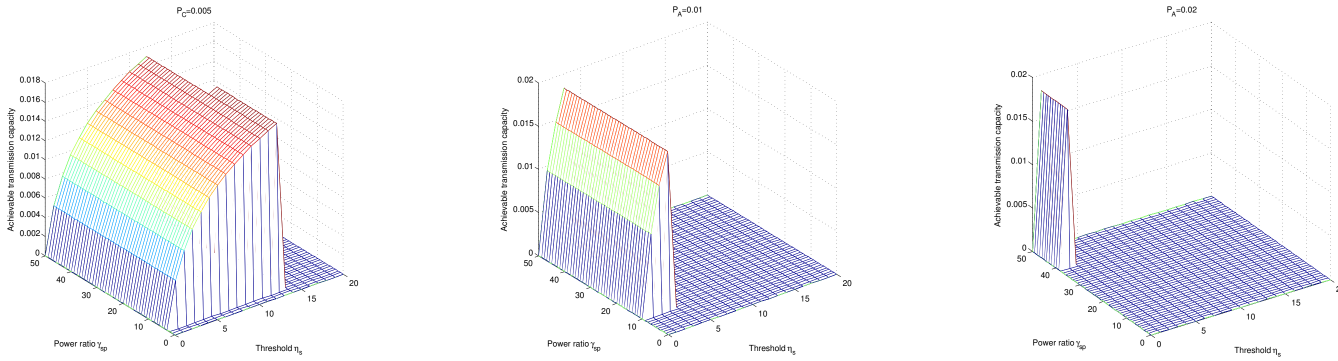
We focus on the worst-case scenario when study the achievable transmission capacity, which provides a lower bound of the per node network capacity. Therefore we assume that the typical receiver of the secondary network is the closest to the BS as it is the most interfered. For simplicity, only a square grid topology of the mesh network is considered. In CSMA/CA, we assume the carrier-sensing range equals transmission range, thus the number of contending nodes within the sensing range is 4. In the spatial TDMA scheme, the topology of the subnet is square too, accordingly the minimum distance between two nodes that can transmit simultaneously is $\sqrt{V}d$, where d is the one-hop distance and V is the number of slots. Given these parameters, our numerical results on the achievable transmission capacity of the secondary mesh network with outage constraints from both the primary system and the secondary system over different MAC protocols are reported in Figs. 3, 4, and 5.

Fig. 3 reports the capacity regions over slotted-ALOHA by setting the transmission probability P_A to be 0.005, 0.01, or 0.02. The results indicate that the capacity of the secondary network increases with the increase of the receiver threshold, while roughly remains unchanged when the power ratio increases. To satisfy the outage probability constraints, the power ratio should be higher than 2 and the threshold should be lower than 13 when $P_A = 0.005$; it should increase to more than 30 and the threshold decrease to lower than 1 when P_A increases to 0.02. This is because more interference arises when P_A increases; thus the secondary network should reduce its power to limit its interference to the primary network to guarantee its normal communications in our underlay model. For the same reason the threshold should be set lower enough to guarantee the normal communications of the secondary network.

Similarly, Fig. 4 illustrates the capacity regions over CSMA/CA when the transmission probability $P_C = 0.005, 0.01, 0.02$. To satisfy the outage probability constraints, the power ratio should be higher than 2 and the threshold should be lower than 19 when P_C is 0.005; it should increase to higher than 20 and the threshold should be lower than 3 when P_C increases to 0.02. Compared to the case of slotted-ALOHA, the power ratio can be lower and the threshold can be higher for the same P_C without violating the outage probability constraints; and a higher capacity is achieved when all parameters are the same. This is because CSMA/CA reduces the interference in the carrier-sensing region.

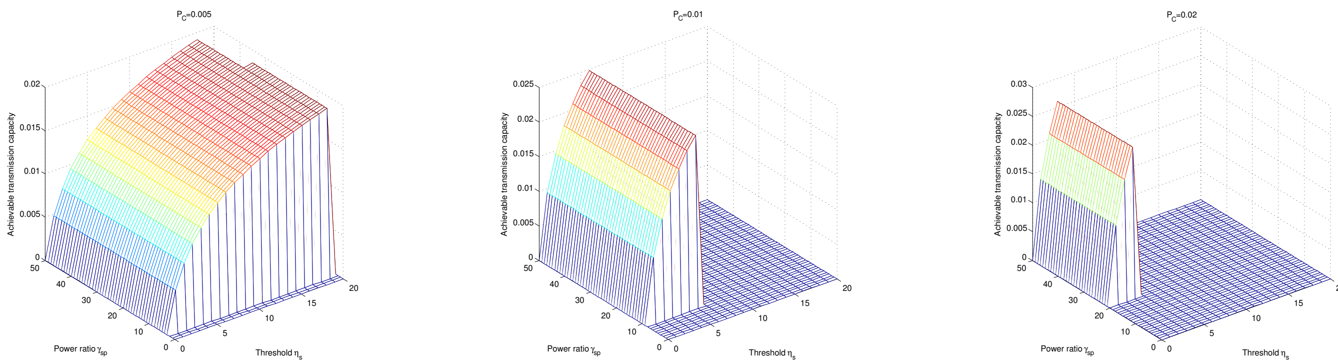
Fig. 5 reports the capacity regions over TDMA by setting the number of slots V to be 10, 15, or 20. From Fig. 5 we observe that when $V = 10$, the feasible region of the power ratio is higher than 40, and the threshold is lower than 1. When the number of slots increases to 20, the lower bound of the power ratio decreases to 10 and the higher bound of the threshold increases to 5. The reason is that when the number of slots is increased, the number of nodes that can transmit simultaneously becomes smaller, thus reducing the interference to both the primary and the secondary network.

In Fig. 6 we report the achievable transmission capacity of the secondary mesh network when the outage probabilities of the primary and secondary systems are fixed to their corresponding threshold (upper-bound) values. In such a case, the achievable transmission capacity is a function of only one variable as any two of the three parameters (the power ratio, the receiver threshold, the transmission probability for slotted-ALOHA and CSMA/CA or the number of slots for TDMA) can be uniquely determined by the third one (Section IV). Therefore the four subfigures in Fig. 6 illustrates the same thing from a different angle. We notice from Fig. 6(a) that the achievable transmission capacity of the secondary network over different MAC increases with the increase of the threshold η_s, but it starts to decrease after η_s increases to a certain value. The optimal thresholds such that the transmission capacity is maximized are around 2, 2, and 4 for slotted-ALOHA, CSMA/CA, and TDMA, respectively. Similarly (Fig. 6(b)), the optimal power ratios are about 24 for slotted-ALOHA and CSMA/CA, and 13 for TDMA. Fig. 6(c) indicates that the network capacity over the slotted-ALOHA and CSMA/CA increases with the increase of the transmission probability, but decreases when the probability exceeds a certain value, with the optimal values achieved at about 0.013 for slotted-ALOHA and 0.017 for CSMA/CA. For TDMA (Fig. 6(d)), too small number of slots results in too much interference, thus affecting the successful transmission probability. On the other hand, too



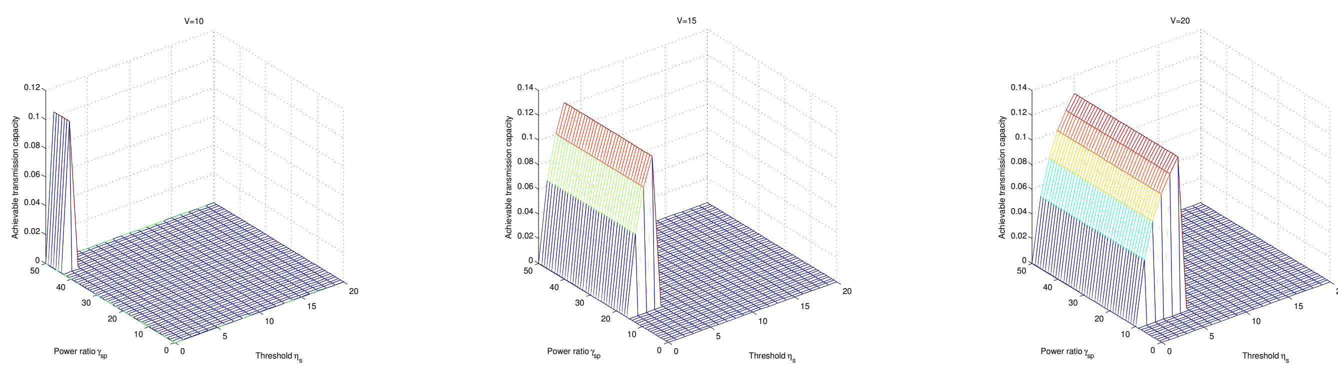
(a) $P_A = 0.005$ (b) $P_A = 0.01$ (c) $P_A = 0.02$

Fig. 3: Achievable transmission capacity of slotted-ALOHA based secondary mesh network.



(a) $P_C = 0.005$ (b) $P_C = 0.01$ (c) $P_C = 0.02$

Fig. 4: Achievable transmission capacity of CSMA/CA based secondary mesh network.

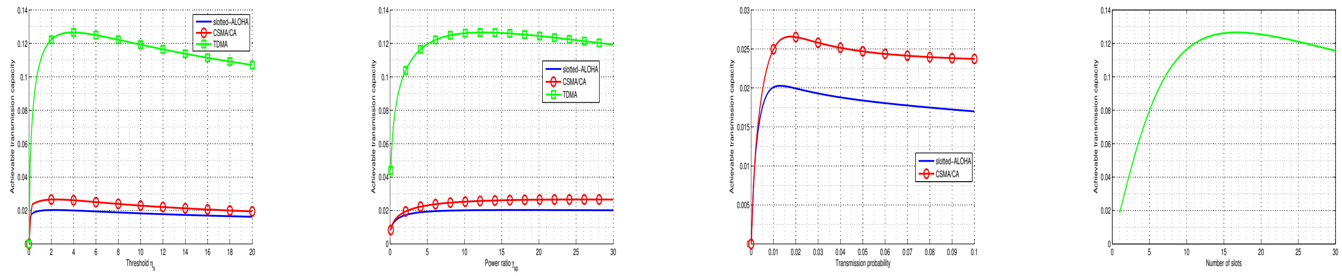


(a) $V = 10$ (b) $V = 15$ (c) $V = 20$

Fig. 5: Achievable transmission capacity of TDMA based secondary mesh network.

many slots waste spectrum resources, causing the decrease of the transmission capacity. The optimal number is 17, which yields the highest transmission capacity.

Fig. 6 also reveils that the maximum achievable transmission capacity is about 0.02 for slotted-ALOHA, 0.027 for CSMA/CA, and 0.126 for TDMA. This indicates that the cognitive mesh network over TDMA achieves the highest capacity while it experiences the lowest capacity if slotted-ALOHA is adopted. The reason behind this phenomenon is that in a saturated network, contention-based MAC protocols



(a) Threshold η_s (b) Power ratio γ_{sp} (c) Transmission probability P (d) Number of slots V

Fig. 6: Achievable transmission capacity with different parameters.

such as slotted-ALOHA and CSMA/CA suffer from too many collisions that leads to lower throughput and higher latencies, while TDMA can reserve slots for each node to transmit and thus does not cause collisions or introduces very low interference. Because carrier sense can reduce the interference by restraining nodes from transmitting simultaneously, the network with CSMA/CA yields a higher capacity than one with slotted-ALOHA.

VI. Conclusions and Future Research

In this paper, we investigate the achievable transmission capacity of a secondary cognitive mesh network sharing the uplink spectrums with cellular users under the outage probability constraints of both the primary and secondary systems. The probabilities of successful transmissions in the primary and secondary networks are derived based on stochastic geometry under the physical interference model. The achievable transmission capacities of the cognitive mesh network with different media access control (slotted-ALOHA, CDMA/CA, and TDMA) in terms of bits/hop/s/Hz/node are obtained based on Shannon's Theory. The capacity regions and the transmission capacities when the outage probabilities equal their corresponding threshold values are analyzed numerically and the results reveal that the transmission probability (number of slots in TDMA), the receiver threshold, and the power ratio, significantly impact on the achievable transmission capacity of the secondary network. For further research, we will consider more efficient spectrum policies in our capacity analysis.

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