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Sparse Target Counting and Localization in Sensor Networks Based on Compressive Sensing

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***Abstract*—In this paper, we propose a novel compressive sensing (CS) based approach for sparse target counting and positioning in wireless sensor networks. While this is not the first work on applying CS to count and localize targets, it is the first to rigorously justify the validity of the problem formulation. Moreover, we propose a novel greedy matching pursuit algorithm (GMP) that complements the well-known signal recovery algorithms in CS theory and prove that GMP can accurately recover a sparse signal with a high probability. We also propose a framework for counting and positioning targets from multiple categories, a novel problem that has never been addressed before. Finally, we perform a comprehensive set of simulations whose results demonstrate the superiority of our approach over the existing CS and non-CS based techniques.**

Keywords: sensor networks, target counting, target localization, compressive sensing

I. Introduction

Counting and positioning targets in a monitored area is of broad interests to many sensor network applications such as environmental monitoring, intrusion detection, and target tracking [1]–[4]. Nevertheless, the existing approaches yield poor performance on areas with overlapping target influences.

In this paper, we consider target locations as sparse signal and propose to reconstruct the signal using compressive sensing (CS) techniques [5]. Here we assume that targets are sparse compared with the number of grids utilized to represent the locations of the targets. This assumption can be easily satisfied in practice when point targets [6], [7] are randomly distributed in a large sensing area. We choose to employ CS because of the recent advances in sparse recovery for compressive sensing. CS is a newly developed sampling paradigm in data acquisition that can reconstruct a sparse signal from a small number of measurements within polynomial time [5] (More details can be found at the CS resource page *http://dsp.rice.edu/cs*).

CS has been applied to event/target counting and localization [7]–[9] . But none of the existing work provides a rigorous proof for the applicability of CS theory in this particular problem context (e.g., whether the necessary Restricted Isometry Property (RIP) [5] is properly satisfied or not). Moreover, the existing work assumes that each grid contains at most one event/target. In this paper, we provide a comprehensive analysis to justify the validity of our CS-based problem formulation. We also tackle the problem of counting and positioning targets from multiple categories. Since many existing sparse signal recovery algorithms assume the availability of signal sparsity level (i.e., the number of targets) which is unknown in the target counting scenario and must be recovered from the measured signal, we propose our own Greedy Matching Pursuit (GMP) algorithm which complements the existing family of sparse recovery algorithms. GMP is a greedy algorithm that iteratively identifies a grid which contributes the most to the observed measurements.

The main contributions of this paper are outlined as follows:

- We provide a CS based problem formulation for target counting and localization, and prove that the product of the measurement matrix and the target decay matrix obeys RIP with a high probability.
- We propose a novel GMP algorithm that can accurately count and localize targets from a small number of measurements.
- We conduct theoretical analysis to prove that GMP can recover sparse signal with a high probability.
- We develop a generic approach to counting and localizing targets from multiple categories.
- We perform an extensive simulation study to evaluate the performance of GMP with various parameter settings. The superiority of GMP compared with other popular target counting and sparse recovery algorithms is validated by the simulation results.

GMP is a greedy sparse recovery algorithm that can be applied to many CS based problems. Compared with well-known algorithms such as OMP [10] and Cosamp [11], GMP does not require prior knowledge of signal sparsity level and is lighter-weight from a computational perspective. Moreover, GMP is applicable to our framework for counting and positioning targets from different categories and achieves a significantly better performance than the other CS based sparse signal recovery algorithms. Finally, our simulation study indicates that GMP is as robust as ℓ_1-minimization while other greedy sparse recovery algorithms provide much weak robustness.

The rest of the paper is organized as follows: Section II presents the related work. The fundamentals of compressive sensing are introduced in Section III. The problem definition for CS based target counting and localization is discussed in Section IV. Section V is devoted to the development of our GMP algorithm and its performance analysis. Section VI reports our simulation results, followed by the conclusions in

Section VII.

II. Related work

In this section, we summarize the most relevant existing research on two problems: target counting/positioning and sparse signal recovery, as this paper contributes to not only target counting and positioning in sensor networks but also the generic theory of compressive sensing (through the development of GMP).

Non-CS approaches to target counting and positioning. Prior efforts on target counting/positioning in sensor networks were mainly focused on three directions: (1) Binary-sensing based approaches [12], [13] position targets by assuming that a sensor reports value $'1'$ if one or more targets are detected in its sensing range and $'0'$ otherwise. (2) Topological integration based approaches [14], [15] aim to obtain the *expected* target count in sensor networks. (3) Clustering based approaches [16], [17] are designed to identify multiple non-overlapping clusters, each of which contains one or more targets. The objective is to count the number of targets in each cluster.

Note that the binary sensing model and the topological integration model report bounds and expected values, respectively. On the other hand, the performance of clustering based algorithms relies heavily on the integration and partitioning of total target energy in the overlapping influence area, which results in coarse counting. None of the existing approaches has the ability to precisely count and localize targets from different categories.

Greedy sparse recovery. It is well-known that ℓ_1 minimization for sparse signal recovery produces highly accurate results if the measurement matrix satisfies the so-called Restricted Isometry Property (RIP) [5] (see Section III). However, the process of ℓ_1 minimization is computationally intensive, which limits its applications. On the other hand, greedy approaches such as Orthogonal Matching Pursuit (OMP) [10] provide fast solutions by iteratively selecting the optimal candidates, but fail to converge with a high probability. A popular recovery algorithm, Cosamp [11], performs component identification during each iteration to speed up the algorithm for various types of signals. More recent CS recovery algorithms also select multiple indices, including StOMP [18], ROMP [19], and I-ROMP [20]. Note that all the existing greedy algorithms for sparse recovery require the availability of the sparsity level, which is not directly applicable to our target counting and positioning problem since the sparsity level is one of the unknowns that should be estimated. Our proposed GMP algorithm estimates the sparsity level as well as the locations and values of the non-zeros in the sparse signal.

Target counting based on compressive sensing. Compressive sensing has been applied to targeting counting/positioning in [7]–[9]. In [8], the unknown target positions are considered to form a sparse vector; and ℓ_1 minimization is applied directly to estimate the target locations. Meng et al. [7] considers a binary event model in wireless sensor networks and proposes a complex Bayesian counting and localization algorithm for estimating the locations of sparse events. These two techniques tacitly assume that their measurement matrices obey the RIP without providing a rigorous proof. In order to obtain an RIP-compliant matrix, [9] takes a computationally intensive preprocessing step: the measurement matrix is multiplied with its left inverse matrix and its orthogonal basis. In this paper, we provide a rigorous proof that our product of the measurement matrix and the target decay matrix satisfies RIP, and our GMP algorithm provides a high accuracy in target counting/positioning.

III. Fundamentals of Compressive Sensing

Conventional sampling theory mandates a sampling rate at least twice as large as the signal's maximum frequency (Nyquist rate) in order to guarantee accurate reconstruction. Nonetheless, for sparse signals, sampling at Nyquist rate could result in a significant waste of resources. Recent research shows that compressive sensing can reconstruct a sparse signal with a much lower sampling rate.

Let x be a $N \times 1$ column vector in R^N. Given an $N \times N$ orthogonal basis $\Psi = [\Psi(1), \Psi(2), \ldots, \Psi(N)]$ with each $\Psi(i)$ being a column vector, x can be expressed by Eq. (1),

$$x = \Psi s = \sum_{i=1}^{N} s_i \Psi(i), \tag{1}$$

where s is the coefficient sequence of x in the transform domain Ψ. The signal x is k-sparse if it is a linear combination of k basis vectors. That is, only k of the s_i coefficients are nonzero and the other $(N-k)$ ones are zero. If $k \ll N$, instead of acquiring N samples from x, compressive sensing (CS) aims to reconstruct x by taking only a small set of measurements:

$$y = \Phi x = \Phi \Psi s = As, \tag{2}$$

where y is a $M \times 1$ vector, $k < M \ll N$, Φ is a $M \times N$ measurement matrix, and A is a $M \times N$ matrix. For a $N \times 1$ vector s, it has been proved that if A holds the Restricted Isometry Property (RIP) [5], the solution obtained from the following ℓ_1-minimization

$$\min |s|_{\ell_1} \text{ subject to } y = As,$$

can be used to either (i) recover s exactly if s is k-sparse; or (ii) compute an approximate signal $\hat{s}$ that is at least as good as if it is computed when the values and locations of the k most significant coefficients of x are known.

The definition of RIP is given below: a matrix A obeys RIP with parameters (k, δ) for $\delta \in (0, 1)$ if

$$1 - \delta \leq \frac{\|Av\|_2^2}{\|v\|_2^2} \leq 1 + \delta, \tag{3}$$

holds for all k-sparse vector v.

If the measurement vector y is corrupted with noise, the measurement becomes

$$y = As + \mathcal{N}, \tag{4}$$

where $\mathcal{N}$ is an unknown error term (e.g. an additive white Gaussian noise (AWGN)). Then the ℓ_1-minimization with relaxed constraints for reconstruction is

$$\min|s|_{\ell_1} \text{ subject to } \|As - y\|_{\ell_2} < \epsilon \tag{5}$$

where ϵ bounds the amount of noise in the data. It has been proved [21] that the reconstruction error of s based on the value computed from Eq. (5) is bounded by $c_0\epsilon_0 + c_1\epsilon$, where c_0 and c_1 are small constants and ϵ_0 is the reconstruction error when y is noiseless.

While in theory solvable in polynomial time [5], ℓ_1-minimization is computationally expensive when N is large. RIP implies that any k columns of A are approximately orthogonal [10]. This property has been exploited to design greedy algorithms for recovering the signal by computing the largest/strongest coefficients of s iteratively. These algorithms include Matching Pursuit (MP) [22], Orthogonal Matching Pursuit (OMP) [10], and Cosamp [11], etc. In some applications prior information (e.g., sparsity) on s may be available, which can be utilized to design Bayesian algorithms [7].

IV. Problem Formulation

In this paper, we employ the CS theory to jointly consider target counting and positioning.

A. The Target Counting and Positioning Problem

Following the mainstream research in target counting [4], [14]–[17], [23], we consider point targets. Then the following target energy decay model [6], [24] can be adopted, which states that the signal energy at location j for a target at location i is roughly approximated by:

$$S_{ij} = \frac{P_0 G_{ij}}{d_{ij}^{\alpha}}, \tag{6}$$

where P_0 is the signal intensity at i, d_{ij} is the Euclidean distance between the target at i and the location j, G_{ij} captures the Raleigh fading of the target signal, and $\alpha \in [2.0, 5.0]$ is a decay factor determined by the environment [25]. The real and imaginary components of a Raleigh signal follow an independent and identical Gaussian distribution with zero mean and variance of σ_0^2 [**?**].

Consider a partition of the monitored area into N grids. Let s_i be the number of targets at grid i, where $s_i \in \{0, 1, 2, \ldots, m\}$ and m is a small integer representing the largest possible number of targets a grid can hold. Let $s = [s_1, s_2, \cdots, s_N]^T$ be a $N \times 1$ column vector. In our consideration, s is k-sparse, which means that s contains k non-zero values and $k \ll N$.

To count and localize the targets, a traditional approach is to place a large number of sensors at the monitored area and apply methods such as [16], [17]. For example, we could place one sensor at each grid and denote by a $N \times 1$ vector x the measurements at the N grids. Thus we have

$$x = \Psi s, \tag{7}$$

with Ψ being a $N \times N$ target energy decay matrix defined by Eq. (8).

$$\Psi = P_0 \begin{pmatrix} \frac{G_{11}}{d_{11}^{\alpha}} & \frac{G_{21}}{d_{21}^{\alpha}} & \cdots & \frac{G_{N1}}{d_{N1}^{\alpha}} \\ \frac{G_{12}}{d_{12}^{\alpha}} & \frac{G_{22}}{d_{22}^{\alpha}} & \cdots & \frac{G_{N2}}{d_{N2}^{\alpha}} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{G_{1N}}{d_{1N}^{\alpha}} & \frac{G_{2N}}{d_{2N}^{\alpha}} & \cdots & \frac{G_{NN}}{d_{NN}^{\alpha}} \end{pmatrix}. \tag{8}$$

Since s is k-sparse, compressive sensing theory can be applied to recover s with $k < M \ll N$ measurements. This means that we can randomly deploy M sensors, at most one per grid, and compute s from their measurements. Let y be a $M \times 1$ column vector recording the measurements of the M sensors. We have

$$y = \Phi x, \tag{9}$$

where

$$\Phi = [\Phi(1), \Phi(2), \ldots, \Phi(M)]^T. \tag{10}$$

Note that $\Phi(i)$ is an $1 \times N$ vector with all elements equal to zero except $\Phi(i, j) = 1$, where j is the index of the grid point at which the i_{th} sensor is located.

Combining Eq. (7) with Eq. (9) we obtain

$$y = \Phi\Psi s = As. \tag{11}$$

When measurement noise is considered, Eq. (11) should be expressed by Eq. (12).

$$y = As + \mathcal{N}, \tag{12}$$

where $\mathcal{N}$ is the additive Gaussian white noise.

Note that when deriving A, we utilize the actual position information of each sensor and assume that a target is located at a grid center. The location of a sensor can be computed from techniques such as those proposed in [**?**], [**?**].

B. Does A Obey RIP?

We shall show in this subsection that the target counting and localization model described above is solvable by CS. As stated in CS theory, a sufficient condition for the successful recovery of a signal by CS is that A obeys RIP. In our model, $A = \Phi\Psi$ can be written as:

$$A = P_0 \begin{pmatrix} \frac{G_{11'}}{d_{11'}^{\alpha}} & \frac{G_{21'}}{d_{21'}^{\alpha}} & \cdots & \frac{G_{N1'}}{d_{N1'}^{\alpha}} \\ \frac{G_{12'}}{d_{12'}^{\alpha}} & \frac{G_{22'}}{d_{22'}^{\alpha}} & \cdots & \frac{G_{N2'}}{d_{N2'}^{\alpha}} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{G_{1M'}}{d_{1M'}^{\alpha}} & \frac{G_{2M'}}{d_{2M'}^{\alpha}} & \cdots & \frac{G_{NM'}}{d_{NM'}^{\alpha}} \end{pmatrix}, \tag{13}$$

where $d_{ij'}$ is the distance from target i to the j_{th} sensor. Without loss of generality, we assume $P_0 = 1$. Since $G_{ij'}$ follows the Rayleigh fading as a complex Gaussian variable, $y = As$ can be expressed as:

$$y = (A_r s + i * A_i s), \tag{14}$$

where A_r and A_i are the corresponding real parts and imaginary parts of elements in A. If both A_r and A_i obey RIP, A must obey RIP. Moreover, since the real part and imaginary

part of $G_{ij'}$ are independently and identically distributed Gaussian variables, it is enough to prove A holds RIP when A_r holds. Thus we have the following theorem:

Theorem 4.1: When the number of sensors $M = O(k \cdot \log(N/k))$, the probability for A_r (after normalization) to satisfy

$$1 - \delta \leq \frac{\|A_r v\|_2^2}{\|v\|_2^2} \leq 1 + \delta \tag{15}$$

for all k-sparse vector v tends to 1.

Proof: Consider a row vector of A_r:

$$\langle A_r \rangle_{i'} = \eta \cdot \left\langle \frac{G_{1i'}}{d_{1i'}^{\alpha}}, \ldots, \frac{G_{Ni'}}{d_{Ni'}^{\alpha}} \right\rangle \tag{16}$$

where $G_{ij'}$ satisfies the Gaussian distribution with the mean of 0 and the variance of σ_0^2 and η is the normalization constant

$$\eta = \sqrt{\frac{N}{M} \cdot \frac{1}{\sum_{j=1}^{N} \frac{\sigma_0^2}{d_{ij}^{2\alpha}}}}. \tag{17}$$

Since all sensors are randomly distributed in the field, the product of $\langle A_r \rangle_{i'}$ and a k-sparse vector v, i.e., $\langle A_r v \rangle_{i'}$, follows Gaussian distribution with mean of 0 and variance of

$$\sigma^2 = \eta^2 \cdot \frac{1}{N} \cdot \left(\sum_{j=1}^{N} \frac{\sigma_0^2}{d_{ij}^{2\alpha}} \right) \cdot \sum_{h=1}^{k} v_h^2. \tag{18}$$

where v_h ($1 \leq h \leq k$) is the h_{th} non-zero element of v. As such, $\|A_r v\|_2^2$ satisfies χ^2-distribution (degree of freedom is M) with the mean $M\sigma^2$ and the variance $2M\sigma^4$. Since $M \gg 1$, $\|A_r v\|_2^2 / \|v\|_2^2$ can be approximated by the Gaussian distribution with the mean

$$\frac{M\sigma^2}{\sum_{h=1}^{k} v_h^2} = M \cdot \eta^2 \cdot \frac{1}{N} \cdot \sum_{j=1}^{N} \frac{\sigma_0^2}{d_{ij}^{2\alpha}} = 1. \tag{19}$$

and the variance $2/M$. According to the Chernoff bound, the probability for $|\|A_r v\|_2^2 / \|v\|_2^2 - 1| > \delta$ is at most

$$\Pr\left\{ \left| \frac{\|A_r v\|_2^2}{\|v\|_2^2} - 1 \right| > \delta \right\} \leq 2e^{-\frac{\delta^2 \cdot M}{8}} \tag{20}$$

Since the total number of possible k-dimensional subspaces of A is

$$\binom{N}{k} \leq (eN/k)^k, \tag{21}$$

the probability that there exists a k-sparse vector v which satisfies $|\|A_r v\|_2^2 / \|v\|_2^2 - 1| > \delta$ is at most

$$\left(\frac{eN}{k} \right)^k \cdot 2e^{-\frac{\delta^2 M}{8}} = 2e^{-\frac{\delta^2 M}{8} + k \log\left(\frac{N}{k}\right) + 1}. \tag{22}$$

Note that when $M = O(k \cdot \log(N/k))$, (22) tends to 0. Thus, the probability for (15) to be satisfied tends to 1. ■

V. Greedy Matching Pursuit for Target Counting and Positioning

In this section, we propose our greedy matching pursuit algorithm (GMP) for target counting and positioning. We also explain how GMP can be used to count and localize the targets from multiple categories.

A. GMP – A Greedy Matching Pursuit Algorithm

Intuitively, the grid which has the most targets should exhibit the highest target energy. Since $s_i \in \{0, 1, \ldots, m\}$ (a finite set), one can enumerate all possible values of s_i for all grids and find the one that contributes the most to the observation vector y. The is the design motivation for our Greedy Matching Pursuit Algorithm (GMP). At each step, we identify the grid (denoted by i in the pseudocode) and the number of targets at the grid (denoted by z_i in the pseudocode) that can maximize Az, where z is a $N \times 1$ vector containing 0 at z_j for all $j \neq i$. Az is subtracted from y', the residual that captures the remaining observed target energy when the grids with more number of targets are removed from the previous steps. Initially y' is set to y. The algorithm terminates when no grid that contains at least one target are found. The pseudocode is given by Algorithm 1.

Algorithm 1 GMP (A, y)

Input:
- An $M \times N$ measurement matrix A.
- An M-dimensional signal measurement vector y.

Output:
- An N-dimensional reconstructed signal $\hat{s}$.

```
function GMP(A, y)
    ℵ ← {1, 2, ··· , N};
    y' ← y;
    z_i ← 0 ∀i ∈ {1, 2, ··· , N};   ▷ z = [z_1, z_2, ··· , z_N]^T is a N-dimensional column vector initialized to 0;
    while true do
        (i, z_i) ← argmin_{i∈ℵ, z_i∈{0,1,2,··· ,m}} ||y' − A[0, ··· , 0, z_i, 0, ··· , 0]^T||_2;   ▷ Find out i and z_i (the grid i that contains z_i number of targets) such that ||y'||_2 can be decreased to the maximum degree.
        if z_i = 0 then
            Break;
        end if
        ℵ ← ℵ \ {i};
        ŝ_i ← z_i;
        y' ← y' − Az;
        z_i ← 0;   ▷ Reset vector z to 0.
    end while
    return (ŝ);
end function
```

B. Counting and Positioning of Targets from Multiple Categories

We observe that by carefully designing the target energy decay matrix Ψ and the measurement matrix Φ, GMP can be used to count and localize targets from multiple categories

without referring to traditional classification methods. Here different categories of targets could have different energy decay models, based on which different target energy decay matrix Ψ can be formulated. For example, the category of targets following the energy decay model defined by Eq. (6) results in the matrix Ψ defined by Eq. (8). Even when the targets follow the same energy decay model as Eq. (6), P_0 and the path loss exponent α could be different, producing different categories of targets.

Assume there are t categories of targets, with each having its own matrix Ψ characterizing the category-specific target energy dissipation features. Denote these matrices by Ψ_i for $i = 1, 2, \cdots, t$. Then the matrix Ψ_{multi} for counting and positioning targets from multiple categories can be defined by

$$\Psi_{multi} = \begin{pmatrix} \Psi_1 & 0 & \cdots & 0 \\ 0 & \Psi_2 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & \Psi_t \end{pmatrix}. \tag{23}$$

Similarly we can obtain Φ_{multi}

$$\Phi_{multi} = \{\Phi_1, \Phi_2, \cdots, \Phi_t\}, \tag{24}$$

where $\Phi_i = \Phi$ for $i \in 1, 2, \cdots, t$, and Φ is the measurement matrix defined in Section IV-A, which contains exactly one 1 at each row and each column, with all other entries filled by 0's. The unknown vector containing the counting and positioning information is denoted by s_t^c. We have

$$s_t^c = \{s_{t1}^c, s_{t2}^c, \cdots, s_{tt}^c\}^T \tag{25}$$

where s_{ti}^c is a $N \times 1$ vector that denotes the location and number of targets of category i in the N grids. Let $A = \Phi_{multi}\Psi_{multi}$. Then GMP can be applied to count and localize targets from multiple categories for a given measurement vector y.

C. Performance Analysis

In the following, we prove that if all targets belong to the same category, GMP is capable of precisely positioning all targets when there is no measurement error. Since the positioning process for targets from multiple categories is essentially a concatenation of the positioning for each category, the correctness of GMP for multi-category target positioning follows in analogy.

Theorem 5.1: If A satisfies the RIP with a constant $\delta < 1/k$ for all $(k+1)$-sparse vectors, GMP always reconstructs s correctly - i.e., outputs $\hat{s} = s$.

Proof: We prove by induction. Thus, we first prove that GMP always generates the correct prediction at the first iteration. Note that if there are targets on Location i, there must exist $z_i > 0$ such that $\|s - z_i\|_2^2 \leq \|s\|_2^2 - 1$. Since A satisfies RIP with $\delta < 1/k$, we have

$$\|y - Az_i\|_2^2 = \|A(s - z_i)\|_2^2 \tag{26}$$
$$\leq (1+\delta) \cdot \|s - z_i\|_2^2 \tag{27}$$
$$\leq (1+\delta) \cdot (\|s\|_2^2 - 1). \tag{28}$$

Also note that if there is no target on Location j but GMP chooses $z_j > 0$, $\|s - z_j\|_2^2 \geq \|s\|_2^2 + 1$. As such,

$$\|y - Az_j\|^2 = \|A(s - z_j)\|_2^2 \tag{29}$$
$$\geq (1-\delta) \cdot \|s - z_j\|_2^2 \tag{30}$$
$$\geq (1-\delta) \cdot (\|s\|_2^2 + 1). \tag{31}$$

Since $\|s\|_2^2 \geq k$ and $\delta < 1/k$, we have

$$\frac{\|y - Az_i\|_2^2}{\|y - Az_j\|_2^2} \leq \frac{1+\delta}{1-\delta} \cdot \frac{\|s\|_2^2 - 1}{\|s\|_2^2 + 1} < 1. \tag{32}$$

Thus, there exists i with $s_i \geq 1$ such that $\|y - Az_i\|_2 < \|y - Az_j\|_2$ for all j with $s_j = 0$. This indicates that GMP always generates the correct prediction at the first iteration.

Now suppose that GMP generates the correct predictions for the first h iterations. To prove the correctness of the $(h+1)_{th}$ iteration, a key observation is that given $\delta < 1/k$, there is always $\delta < 1/(k-h)$ for all $h \geq 0$. At the beginning of the $(h+1)$-th iteration, define a $1 \times N$ vector s' such that $s'_i = s_i$ if there are targets on grid i and z_i has not yet been identified in the first h iterations, and $s'_i = 0$ otherwise. We have

$$\|s' - z_i\|_2^2 \leq \|s'\|_2^2 - 1 \tag{33}$$

iff $s'_i > 0$. If no targets is present on grid j, then $\|s' - z_j\|_2^2 \geq \|s'\|_2^2 + 1$. In analogy to the derivation in (28) and (31), we have

$$\frac{\|y' - Az_i\|_2^2}{\|y' - Az_j\|_2^2} \leq \frac{1+\delta}{1-\delta} \cdot \frac{\|s'\|_2^2 - 1}{\|s'\|_2^2 + 1} < 1. \tag{34}$$

Thus, GMP always generates the correct prediction at the $(h+1)$-th iteration. In summary, GMP predicts the positions of all targets without error. ■

VI. Simulation

A. Performance Parameters

We start by defining the parameters used for performance evaluation. For each grid i, let n_i and n'_i be the actual and estimated numbers of targets, respectively. The number of grids that contain at least one target is the sparsity level, which is denoted by k.

Definition 6.1: The **counting error**, denoted by **COE**, is defined to be the ratio of the difference between the estimated number and actual number *vs.* the actual number of targets:

$$COE = \frac{\sum_{i=1}^{N} |n_i - n'_i|}{\sum_{i=1}^{N} n_i}. \tag{35}$$

Assume there are in total t categories of targets. Let t_i and t'_i be the actual and estimated numbers of targets at category i, respectively. Then the categorical counting error is defined as follows:

Definition 6.2: The **categorical counting error (CCOE)** is defined as:

$$CCOE = \frac{\sum_{i=1}^{t} |t_i - t'_i|}{\sum_{i=1}^{t} t_i}. \tag{36}$$

Let n be the total number of targets with locations (x_1, y_1), $(x_2, y_2), \cdots, (x_n, y_n)$, respectively. Assume the corresponding estimated target locations are (x'_1, y'_1), (x'_2, y'_2), $\cdots$, $(x'_{\tilde{n}}, y'_{\tilde{n}})$, where $\tilde{n}$ is the number of estimated targets. In order to assign an estimated location to a target, we compute all pairs of distances between (x_i, y_i) and (x'_j, y'_j), and sort them in a non-decreasing order. Based on the sorted list, we assign a target to the first unused estimated location. Let $n_{\min} = \min\{n, \tilde{n}\}$.

Definition 6.3: The **localization error (LOCE)**, also known as **positioning error**, is defined by:

$$LOCE = \frac{\sum_{i=1}^{n_{\min}} \sqrt{(x_i - x'_i)^2 + (y_i - y'_i)^2}}{n_{\min} \cdot r}, \qquad (37)$$

where r is the grid size.

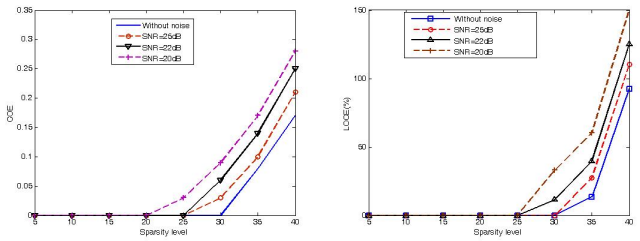
Note that in the definition of LOCE, we use the grid size r to normalize the localization error. If $LOCE < 100\%$, it indicates that the estimated location is close to the real location as their distance is shorter than the grid diameter. Also note that we do not consider the localization error of targets mis-identified by the algorithm as their localization error should be infinity based on our LOCE definition. Therefore it might occur that $LOCE = 0$ when $COE > 0$ in the simulation.

B. Simulation Set-up

MATLAB is used to perform all simulations. We randomly deploy M sensors at an area of N grids, where $M \ll N$. To place targets, we select $k < M$ grids randomly, and put n_i targets at each selected grid, where n_i is chosen uniformly at random from $\{1, 2, 3\}$. The target energy decay model follows Eq. (6) with $P_0 = 1$ and $\alpha = 2$. Both real and imaginary parts of Rayleigh fading follow an independent and identical Gaussian distribution with the mean of 0 and the variance of 0.5 as in [26], [27]. If our algorithm reports a target at a grid, the center of the grid is used as the estimated location of the reported target.

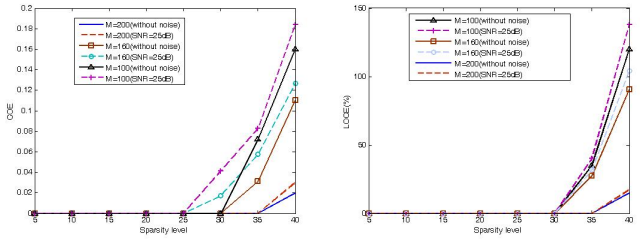
In order to test the robustness of our algorithm, we intentionally add Gaussian white noise $\mathcal{N}(0, \sigma^2)$ to the observation vector y. We use SNR to quantify the signal to noise ratio. In our simulation, we vary M, N, k and SNR to test different sparsity level, sensor density, target density, and noise. Each presented result is the average of 50 runs.

We also implement several state-of-the-art algorithms for performance comparison. To verify the strength of our CS algorithm in target counting and positioning, we compare it with the algorithm for binary proximity sensors [4], the median-based fault-tolerant target detection algorithm [6], and the clustering based algorithm [16]. They are denoted by **Binary**, **FTTD**, and **Cluster**, respectively. We also test the ability of our algorithm in sparse recovery by comparing with well-known CS recovery algorithms including Orthogonal Matching Pursuit (OMP) [10], Compressive Sampling Matching Pursuit (Cosamp) [11], and the ℓ_1-magic [28]. Note that both OMP and Cosamp require the availability of the sparsity level k, which is a parameter to be estimated in our algorithm. Therefore we tailor OMP by changing its exit condition from "iterating k times" to "iterating until the residual is smaller than a threshold ($< 10^{-6}$)". For Cosamp, since k is used to establish a matrix, we decide to provide the actual value of k and compare LOCE only. All these algorithms have been discussed in Section II.



(a) (b)

Fig. 1. Performance of GMP for single category of targets without noise (w-o) and under different levels of noise: $SNR = 25dB, 22dB, 20dB$ when $N = 900, M = 160$. (a) Counting error *vs.* sparsity level; (b) Localization error *vs.* sparsity level.



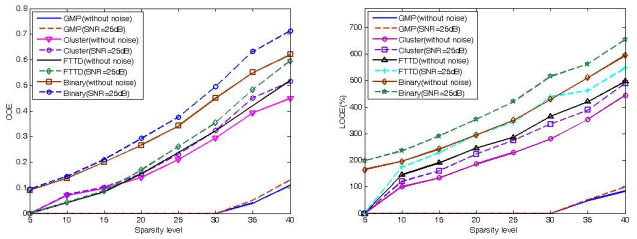
(a) (b)

Fig. 2. Performance of GMP for single category of targets when M varies and $N = 900, SNR = 25dB$. (a) Counting error *vs.* sparsity level; (b) Localization error *vs.* sparsity level.

C. Single Category of Targets

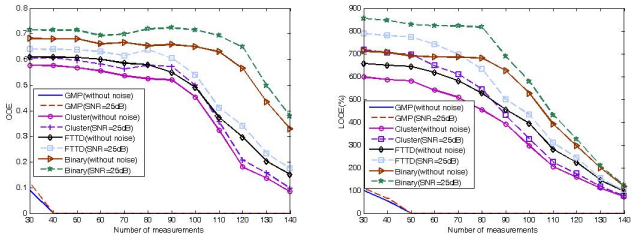
Fig. 1(a) reports the COE of GMP when the sparsity level k varies from 5 to 40 at a step size of 5 under different measurement noise levels. We plot 4 curves for the cases of no error, $SNR = 25dB, 22dB$ and $20dB$, respectively, when $N = 900$ and $M = 160$. We notice that if $k \leq 30$, we can precisely count all targets when there is no measurement noise. The counting error increases as k increases and decreases with an increasing SNR. This phenomena is consistent with the CS theory as the recovery error is proportional to the input noise. The same observation is obtained in Fig. 1(b), which reports the LOCE of GMP. Note that when k is a little more than 20 and $SNR = 20dB$, the localization error is 0 while counting error exists. This is because we did not consider the positioning error of miss-counted or wrongly-inserted targets.

In Fig. 2(a) and Fig. 2(b), we plot the performance of GMP when $M = 100, 160$, and 200. As in the previous scenario, the sparsity level varies from 5 to 40 at a step size of 5. We consider cases where there is no measurement error and $SNR = 25dB$. It is observed that COE and LOCE increase as k increases or M decreases. This is reasonable as higher sampling rate leads to a higher accuracy in sparse recovery. When $M = 200$, GMP can estimate all targets with 100%



(a) (b)

Fig. 3. Comparison of GMP and three classical target counting algorithms when $N = 900, M = 160$, and $SNR = 25dB$. (a) Counting error *vs.* sparsity level k; (b) Localization error *vs.* sparsity level k.



(a) (b)

Fig. 4. Comparison of GMP and three classical target counting algorithms when $N = 900$, $k = 10$, and $SNR = 25dB$. (a) Counting error *vs.* M; (b) Localization error *vs.* M.

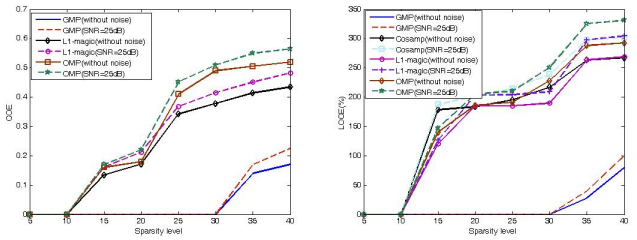
accuracy even at $k = 35$ and produce no positioning error. We conclude that GMP is an effective sparsity recovery algorithm that yields high accuracy.

1) GMP vs. *popular target counting and localization methods:* Fig. 3(a) and 3(b) compare GMP with Cluster [17], FTTD [6], and Binary [23]. Fig. 3(a) depicts the COE *vs.* the sparsity level k when $N = 900$,$M = 160$, and $SNR = 25dB$. It can be observed that GMP brings much lower error than all other algorithms. When $k = 30$, both COE and LOCE of GMP are 0 if there is no measurement noise; while other algorithms produce a counting error of 30% or more and a positioning error of more than 200% under the same scenario. Even when k is as small as 10, the positioning error of Cluster, FTTD, and Binary is still above 100%, though their counting errors are relatively small (0.05 or higher). When $k = 40$, GMP has a counting accuracy of about 80% and a positioning error of less than 100% (inside a grid) while other algorithms have a much lower accuracy.

In Fig. 4(a) and Fig. 4(b), we study the dependence of COE and LOCE on the number of measurements M when $N = 900$, $k = 10$ and $SNR = 25dB$. It is noticed that as an overall trend, the larger the M is, the smaller the error for all algorithms except Binary will be. The exception, i.e., Binary, produces a higher COE when M is around 100 and $SNR = 25dB$. We attribute this exception to the algorithmic design of Binary as it relies on the overlapping areas of neighboring sensors to estimate the number of targets. Note that both COE and LOCE of GMP are zero when $M \geq 40$, while other algorithms yield much higher errors when $M < 100$. This clearly indicates that GMP as a compressive sensing based algorithm does not need a large number of measurements to precisely estimate the number and location of the targets.

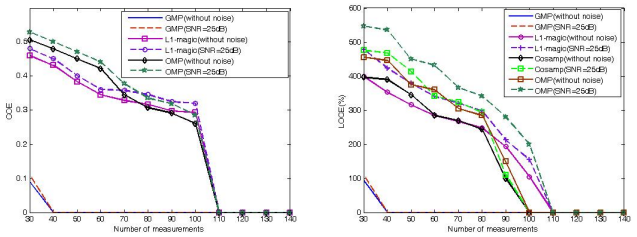
2) GMP vs. *popular compressive sensing algorithms:* We now report our comparative study of GMP *vs.* OMP [10], Cosamp [11], and ℓ_1-magic [28], the three popular CS based sparse recovery algorithms. As noted earlier, Cosamp relies on k to build a matrix as a core part of its algorithm. Therefore we will provide the actual k value to Cosamp and compare against it the localization error only.

From Fig. 5(a) and 5(b), we observe the superiority of GMP in terms of COE and LOCE over the other three CS-based algorithms. None of OMP, Cosamp, and ℓ_1-magic achieves a



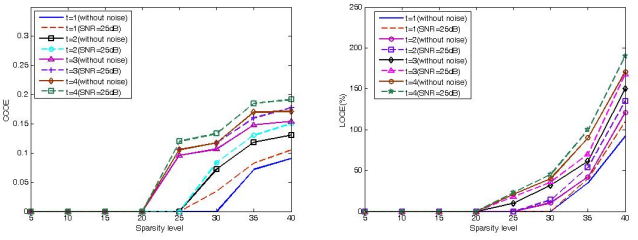
(a) (b)

Fig. 5. Comparison of GMP and three CS sparse recovery algorithms when $N = 900$, $M = 160$, and $SNR = 25dB$. (a) Counting error *vs.* the sparsity level k; (b) Localization error *vs.* the sparsity level k.



(a) (b)

Fig. 6. Comparison of GMP and three CS sparse recovery algorithms when $N = 900$, $k = 10$, and $SNR = 25dB$. (a) Counting error *vs.* M; (b) Localization error *vs.* M.



(a) (b)

Fig. 7. Performance of GMP to count targets from multiple categories when the number of target categories varies from 1 to 4 at $N = 900$,$M = 160$ and $SNR = 25dB$. (a) Categorical Counting error *vs.* sparsity level; (b) Localization error *vs.* sparsity level.

counting accuracy of 100% when k is ≥ 10 while GMP can accurately count all targets when k is less than 30. On the other hand, the estimated target locations via GMP are close to their actual values when $30 \leq k \leq 40$ as LOCE is close to 100% while the other three yields much higher localization errors.

Fig. 6(a) and 6(b) again prove the superiority of GMP over OMP, Cosamp, and ℓ_1-magic. We observe that for $k = 10$, GMP requires 40 measurements to precisely recover all targets while the other three algorithms produce high COE and LOCE even when M is as high as more than 100. This indicates that GMP can reconstruct the signal with 100% accuracy when only 5% ($\frac{M}{N} = 5\%$ when $M = 40$ and $N = 900$) of the signals are collected.

D. Multiple Categories of Targets

We now report our simulation results when there are multiple categories of targets. Here we assume that all targets follow the same energy decay model defined by Eq. (6) but with different P_0 and α values. Let t be the total number of target categories. In our simulation, t varies from 1 to 4. The parameter settings for the four categories of targets are specified as follows.

$$\text{Category 1:} \quad P_0 = 1.0, \quad \alpha = 2.0; \tag{38}$$

$$\text{Category 2:} \quad P_0 = 0.5, \quad \alpha = 2.0; \tag{39}$$

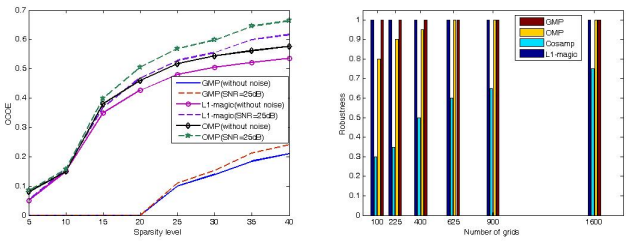
$$\text{Category 3:} \quad P_0 = 1.5, \quad \alpha = 3.0; \tag{40}$$

$$\text{Category 4:} \quad P_0 = 1.0, \quad \alpha = 3.0. \tag{41}$$

The simulation setup is similar to that of single category target counting except that we associate with each target a category number randomly selected from 1 to t. For example, if there are $t = 3$ categories of targets, we randomly pick up an integer from $\{1, 2, 3\}$ as the category for each target. We first study the performance of GMP then compare it with other popular CS algorithms.

Fig. 7(a) and Fig. 7(b) present the performance of GMP *vs.* the sparsity level k at $N = 900$, $M = 160$, and $SNR = 25dB$ when t varies from 1 to 4. It can be observed that the categorical counting error CCOE and the localization error LOCE both increase when k or t increases. This is attributed to the fact that t actually increases the sparsity level from k to roughly tk. As indicated in Section V-B, when the targets in a grid belong to t different categories, they actually occupy t entries in the vector s_t^c and therefore effectively increases the sparsity level. This is the reason why GMP can precisely count and localize smaller number of targets when t increases.

We also compare the CCOE of GMP with those of OMP, Cosamp, and ℓ_1-magic for multiple categories of data and report the results in Fig. 8(a). Notice that GMP achieves a much better performance in terms of counting errors. In fact, none of OMP, Cosamp, and ℓ_1-magic can produce zero counting error even when only 5 grids have targets present while GMP can precisely count the number of targets when $k \leq 20$. This clearly indicates that GMP has a strong ability of target classification that significantly outperforms the other three CS sparse recovery algorithms.



(a) (b)

Fig. 8. (a) Categorical Counting errors of GMP, OMP, and ℓ_1-magic *vs.* the sparsity level k when $N = 900$, $M = 160$, $t = 3$ and $SNR = 25dB$; (b) Stability of GMP, OMP, Cosamp, and ℓ_1-magic.

E. Robustness of CS Algorithms

During our simulation, we found that most compressive sensing algorithms are very "sensitive" to the sparse signals. A change in the position of a non-zero value or a change in the value itself can make the algorithm fail to converge. In other words, when we increase or decrease the number of targets in a grid with at least one target, or when we move targets from one grid to another, the algorithm may fail. This phenomenon indicates the lack of resiliency of certain CS algorithms.

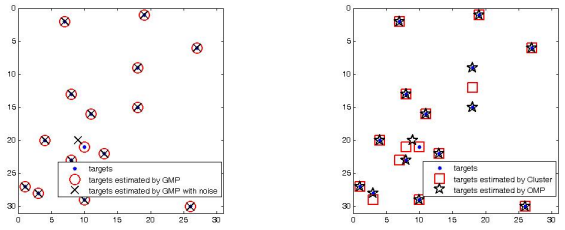
We quantify the resiliency of an algorithm by the *rate* that the algorithm can successfully converge. In this study, the number of grids N is set to 100, 225, 400, 625, 900, or 1600. We also fix $M/N = 0.2$ and $k/N = 0.02$. The probability for an algorithm to successfully converge in 100 runs is reported in Fig.8(b). We observe that GMP and ℓ_1-magic are robust for all simulation settings while Cosamp is the weakest in robustness. As N increases, the robustness of both OMP and Cosamp increases with OMP performing much better.

F. An Example

Figs. 9(a) and 9(b) illustrate an example of target counting and localization. An area of $60m \times 60m$ is divided into 900 grids, and 15 targets are randomly deployed in this area. We set SNR to $25dB$ when measurement noise is considered. Set $M = 90$ to randomly collect 90 measurements. Fig.9(a) presents the targets estimated by GMP. We notice that GMP can precisely recover the number (count) and location of all 15 targets when there is no measurement noise. If noise is considered, GMP correctly estimate the locations of all but one target when $SNR = 25dB$. For comparison purpose, Fig.9(b) reports the targets estimated by the Cluster algorithm [17] and OMP [10]. This example clearly indicates that GMP provides high accuracies in both target counting and target localization. Similar results have been obtained when applying FTTD, Binary ℓ_1-magic, and Cosamp to the example.

VII. Conclusion

This paper investigates the problem of sparse target counting and positioning in wireless sensor networks based on compressive sensing. We first prove that the product of measurement



(a) (b)

Fig. 9. An example to illustrate target counting and localization. (a) Targets estimated by GMP with and without noise; (b)Targets estimated by OMP and Cluster.

and target energy decay matrices obeys RIP, which validates our CS-based problem formulation. Since ℓ_1-minimization incurs high computational overhead and the existing greedy sparse recovery algorithms require the availability of sparsity level, which needs to be estimated in our study, we propose a novel greedy algorithm GMP and prove that GMP can reconstruct the original signal at high accuracy with overwhelming probability when the number of measurements is sufficiently large. Additionally, we investigate the applicability of GMP for counting and positioning targets from multiple categories. Our simulation results validate the superiority of GMP over existing algorithms. In particular, the results show that GMP significantly reduces the number of measurements while achieving a high detection accuracy.

ACKNOWLEDGMENTS

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