Association Rules

1. Objectives

- Increase sales and reduce costs
- What products were often purchased together?
    - Beer and diapers?!
- What are the subsequent purchases after buying a PC?
- What kinds of DNA are sensitive to this new drug?
- Can we automatically classify web documents?
- Broad applications:
    - Basket data analysis, cross-marketing, catalog design, sale campaign analysis
    - Web log (click stream) analysis, DNA sequence analysis, etc.
- Example: Items frequently purchased together:
    - **Bread ➔ PeanutButter**
- Why associations:
    - Placement
    - Advertising
    - Sales
    - Coupons

2. Definitions

- Finding frequent patterns, associations, correlations, or causal structures among sets of items or objects in transaction databases, relational databases, and other information repositories.

- Frequent pattern: pattern (set of items, sequence, etc.) that occurs frequently in a database.

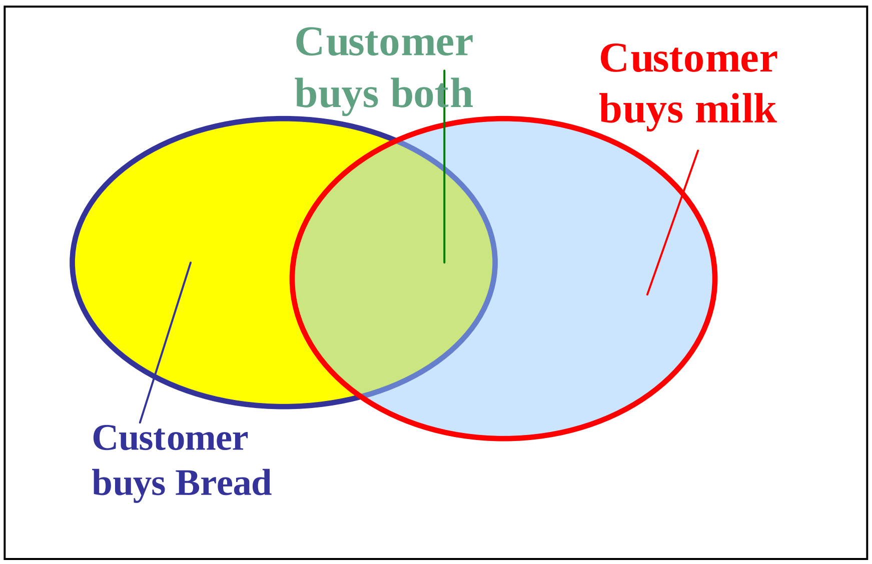
- Basic Concepts:

- A set of items: $I=\{x_1, \ldots, x_k\}$
- Transactions: $D=\{t_1,t_2, \ldots, t_n\}$, $t_j \subseteq I$
- A k-Itemset: $\{I_{i1},I_{i2}, \ldots, I_{ik}\} \subseteq I$
- Support of an itemset: Percentage of transactions that contain that itemset.
- Large (Frequent) itemset: Itemset whose number of occurrences is above a threshold.
- Example:

Transaction	**Items**
t_1	**Bread,Jelly,PeanutButter**
t_2	**Bread,PeanutButter**
t_3	**Bread,Milk,PeanutButter**
t_4	**Beer,Bread**
t_5	**Beer,Milk**

I = { Beer, Bread, Jelly, Milk, PeanutButter}
Support of {Bread,PeanutButter} = 3/5 = 60%

Transaction-id	Items bought
10	A, B, C
20	A, C
30	A, D
40	B, E, F



- Association Rules
  - Implication: X ➔ Y where X,Y ⊆ I and X ∩Y = ∅;
  - Support of AR (s) X ➔ Y:
    - Percentage of transactions that contain X ∪ Y
    - Probability that a transaction contains X∪Y.
  - Confidence of AR (a) X ➔ Y:

- Ratio of number of transactions that contain $X \cup Y$ to the number that contain X
- Conditional probability that a transaction having X also contains *Y*.

- Example:

Transaction-id	**Items bought**
10	A, B, C
20	A, C
30	A, D
40	B, E, F

Frequent pattern	**Support**
{A}	75%
{B}	50%
{C}	50%
{A, C}	50%

- For rule *A* ➔ *C*:

Support(A➔C) = P(A∪C) = support({*A*}∪{*C*}) = 50%
confidence (A➔ C) = P(C|A)

= support($\{A\} \cup \{C\}$)/support($\{A\}$) = 66.6%

- Another Example:

Transaction	**Items**
t_1	**Bread,Jelly,PeanutButter**
t_2	**Bread,PeanutButter**
t_3	**Bread,Milk,PeanutButter**
t_4	**Beer,Bread**
t_5	**Beer,Milk**

X➔ Y	Support	Confidence
Bread ➔ Peanutbutter	= 3/5 %= 60%	= (3/5)/(4/5)%=75%
Peanutbutter ➔ Bread	60%	= (3/5)/(3/5)%=100%
Jelly ➔ Milk	0%	0%
Jelly ➔ Peanutbutter	=1/5 % = 20%	= (1/5)/(1/5) % = 100%

- Association Rule Problem:
  - Given a set of items $I=\{I_1,I_2,\ldots,I_m\}$ and a database of transactions $D=\{t_1,t_2, \ldots, t_n\}$ where $t_i=\{I_{i1},I_{i2}, \ldots, I_{ik}\}$ and $I_{ij} \in I$, the Association Rule Problem is to identify all association rules X ➔ Y with a ***minimum support and confidence***.

  - **NOTE:** Support of X ➔ Y is same as support of $X \cup Y$.

- Association Rules techniques:
  - Find all frequent itemsets.
  - Generate strong association rules from the frequent itemsets: those rules must satisfy minimum support and minimum confidence.

3. Type of Association Rules

- Boolean AR:
  - It is a rule that checks whether an item is present or absent.
  - All the examples we have seen so far are Boolean AR.

- Quantitative AR:
  - It describes associations between quantitative items or attributes.
  - Generally, quantitative values are partitioned into intervals.
  - Example:

Age(X,"30..39") $\wedge$ income(X,"80K..100K")

➔ buys(X, High Resolution TV)

- Single-Dimension AR:
    - It is a rule that references only one dimension.
    - Example:

      buys(X,"computer")

      ➔ buys(X,"financial_software")

      The single dimension is "buys"

    - The following rule is a multi-dimensional AR:

      Age(X,"30..39") ∧ income(X,"80K..100K")

      ➔ buys(X, High Resolution TV)

- Multi-level AR
    - It is a set of rules that reference different levels of abstraction.
    - Example:

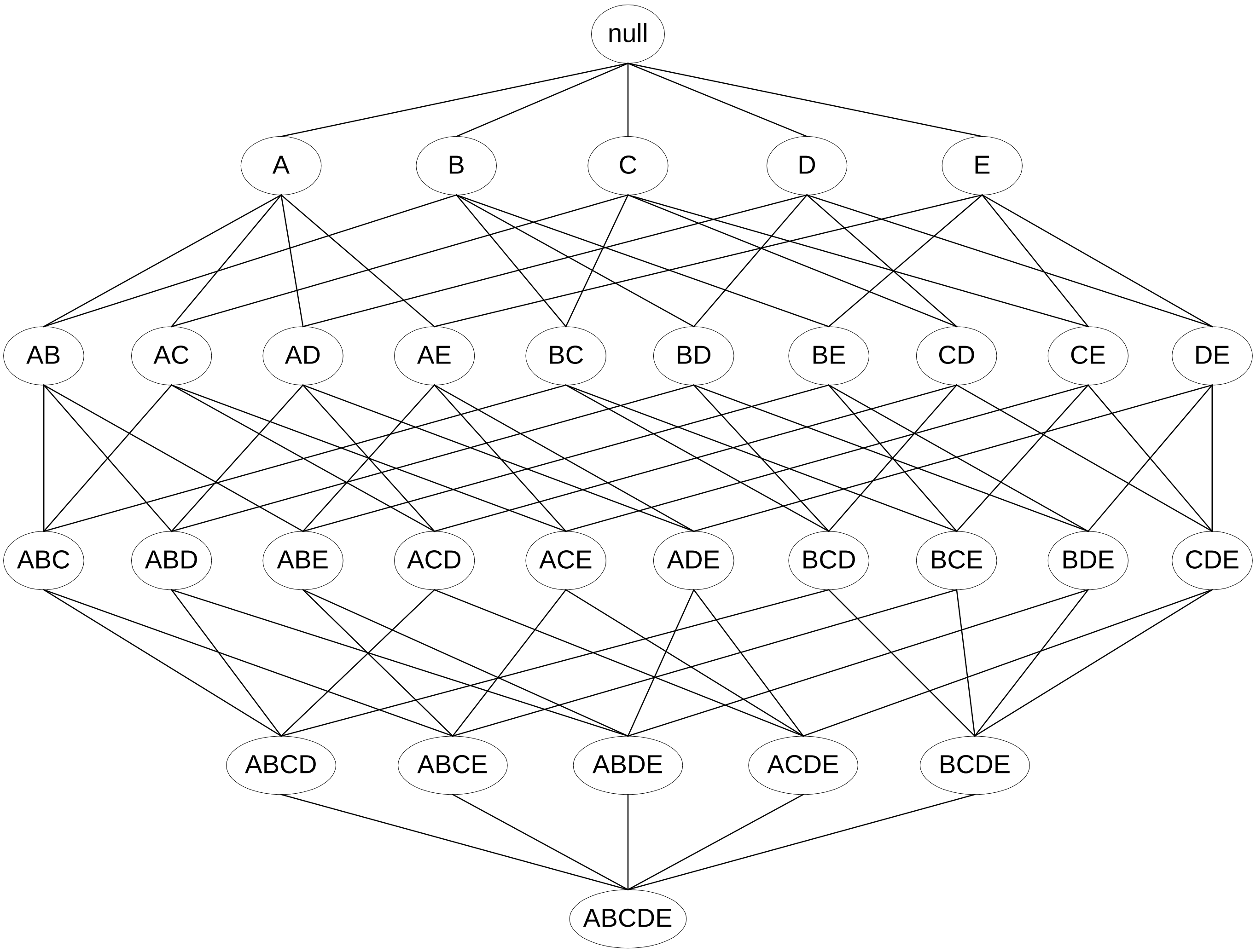
      Age(X,"30..39") ➔ buys(X, "desktop")

      Age(X,"20..29") ➔ buys(X, "laptop")

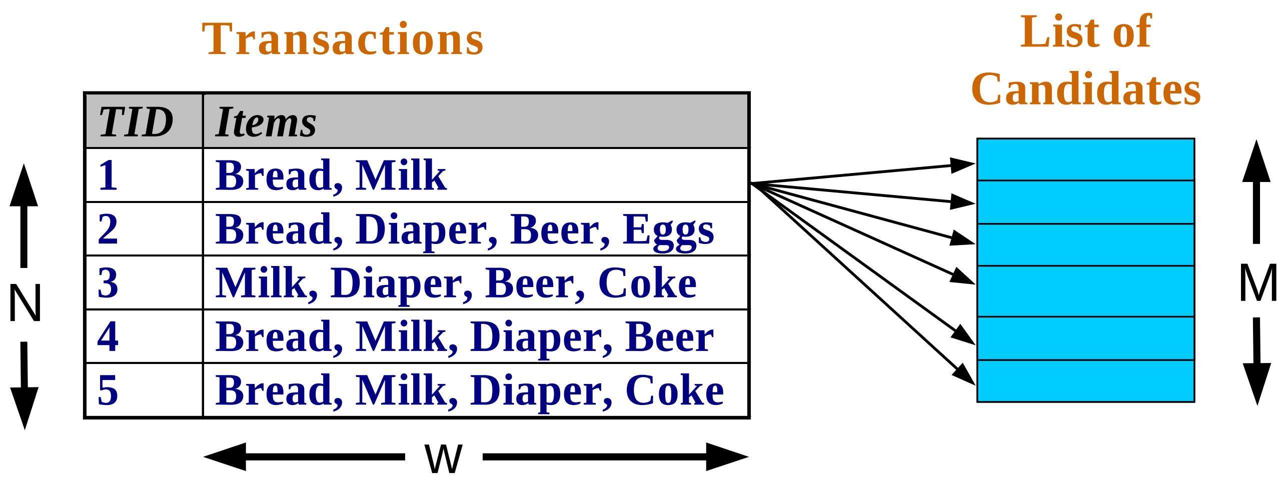
      Laptop → desktop → computer

4. Frequent Itemset generation

- Given d items, there are 2^d possible candidate itemsets



- Brute-force approach:
  - Each itemset in the lattice is a candidate frequent itemset
  - Count the support of each candidate by scanning the database



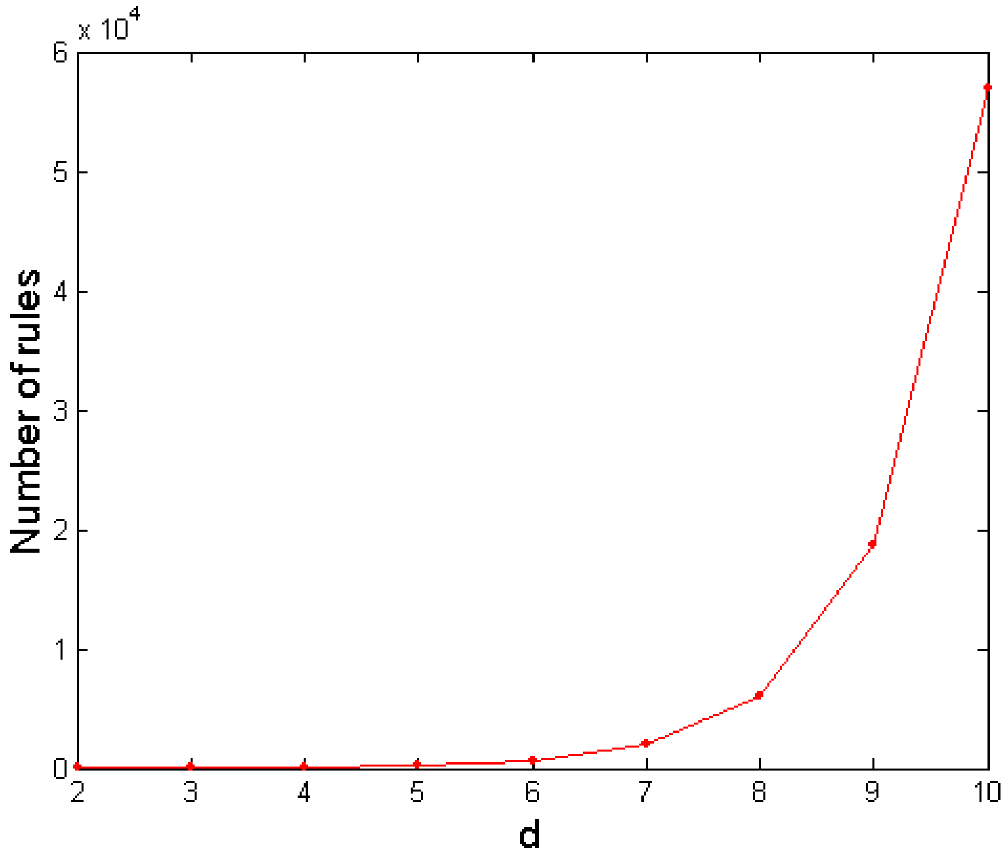
TID	*Items*
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

  - Match each transaction against every candidate
  - Complexity ~ O(NMw) => Expensive since M = $2^{\mathbf{d}}$!!!

- Complexiy:
  - Given d unique items:
  - Total number of itemsets = 2^d
  - Total number of possible association rules:

$$R = \sum_{k=1}^{d-1}\left[\binom{d}{k} \times \sum_{j=1}^{d-k}\binom{d-k}{j}\right] = 3^d - 2^{d+1} + 1$$

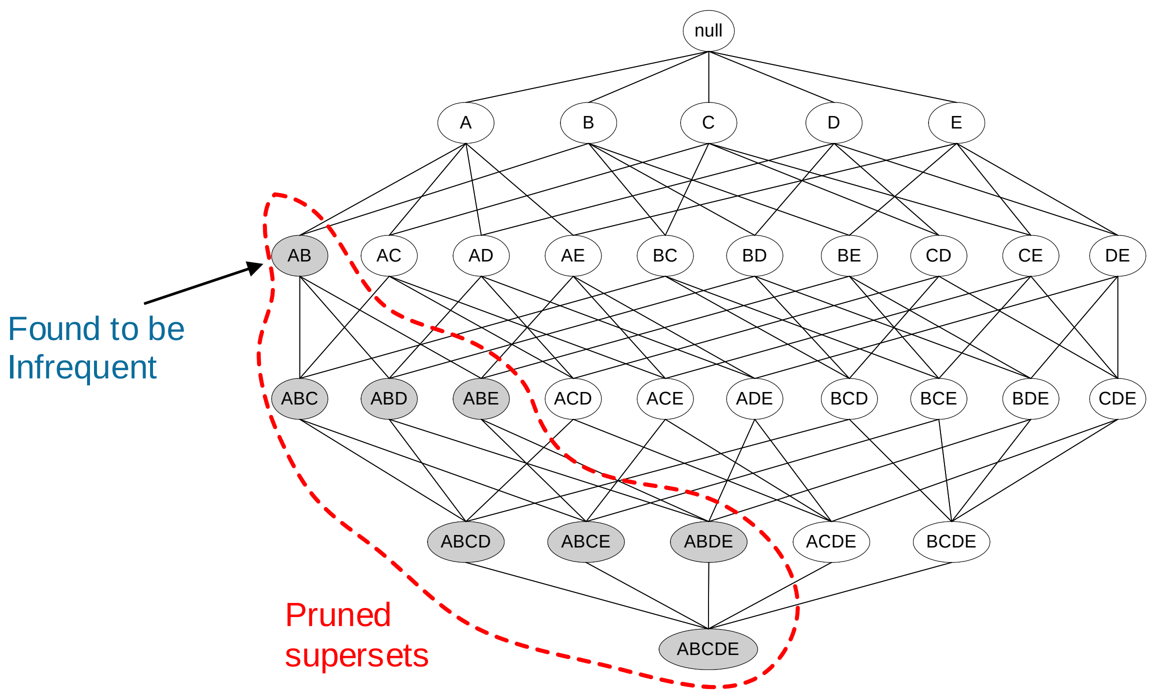
  - If d=6, R = 602 rules



- **Frequent Itemset Generation Strategies**
  - Reduce the number of candidates (M)
    - Complete search: $M=2^d$
    - Use pruning techniques to reduce M
  - Reduce the number of transactions (N)
    - Reduce size of N as the size of itemset increases
  - Reduce the number of comparisons (NM)
    - Use efficient data structures to store the candidates or transactions
    - No need to match every candidate against every transaction

5. Apriori Algorithm: Mining Single-Dimension Boolean AR

- It is used to mine Boolean, single-level, and single-dimension ARs.
- Apriori Principle



- Apriori algorithm:
    - Uses prior knowledge of frequent itemset properties.
    - It is an iterative algorithm known as level-wise search.
    - The search proceeds level-by-level as follows:
        - First determine the set of frequent 1-itemset; L1
        - Second determine the set of frequent 2-itemset using L1: L2
        - Etc.
    - The complexity of computing Li is O(n) where n is the number of transactions in the transaction database.
    - Reduction of search space:
        - In the worst case what is the number of itemsets in a level Li?
        - Apriori uses "**Apriori Property**":
    - **Apriori Property**:
        - It is an **anti-monotone** property: if a set cannot pass a test, all of its supersets will fail the same test as well.
        - It is called **anti-monotone** because the property is monotonic in the context of failing a test.
        - All nonempty subsets of a frequent itemset must also be frequent.
        - An itemset I is not frequent if it does not satisfy the minimum support threshold:

$$P(I) < min\_sup$$

- If an item A is added to the itemset I, then the resulting itemset $I \cup A$ cannot occur more frequently than I:

$$I \cup A \text{ is not frequent}$$

Therefore, $P(I \cup A) < min\_sup$

- How Apriori algorithm uses "Apriori property"?
  - In the computation of the itemsets in L_k using L_{k-1}
  - It is done in two steps:
    - Join
    - Prune

5.1. Join Step:

- The set of candidate k-itemsets (element of L_k), C_k, is generated by joining L_{k-1} with itself:

$$L_{k-1} \infty L_{k-1}$$

- Given l_1 and l_2 of L_{k-1}

$$L_i=l_{i1},l_{i2},l_{i3},\ldots,l_{i(k-2)},l_{i(k-1)}$$
$$L_j=l_{j1},l_{j2},l_{j3},\ldots,l_{j(k-2)},l_{j(k-1)}$$

Where L_i and L_j are sorted.

- L_i and L_j are joined if there are different (no duplicate generation). Assume the following:

$$l_{i1}=l_{j1},\ l_{i2}=l_{j1},\ \ldots,\ l_{i(k-2)}=l_{j(k-2)} \text{ and } l_{i(k-1)} < l_{j(k-1)}$$

- The resulting itemset is:

$l_{i1}, l_{i2}, l_{i3}, \ldots, l_{i(k-1)}, l_{j(k-1)}$

- Example of Candidate-generation:

L3={abc, abd, acd, ace, bcd}

Self-joining: L3 ∞ L3

abcd from ab**c** and ab**d**
acde from ac**d** and ac**e**

5.2. Prune step

- C_k is a superset of L_k ➔ some itemset in C_k may or may not be frequent.
- L_k: Test each generated itemset against the database:
  - Scan the database to determine the count of each generated itemset and include those that have a count no less than the minimum support count.
  - This may require intensive computation.

- Use Apriori property to reduce the search space:
  - Any (k-1)-itemset that is not frequent cannot be a subset of a frequent k-itemset.
  - Remove from C_k any k-itemset that has a (k-1)-subset not in L_{k-1} (itemsets that are not frequent)
  - Efficiently implemented: maintain a hash table of all frequent itemset.

- Example of Candidate-generation and Pruning:

L3={abc, abd, acd, ace, bcd}

Self-joining: L3 ∞ L3

abcd from ab**c** and ab**d**
acde from ac**d** and ac**e**

Pruning:
acde is removed because ade is not in L3

C4={abcd}

5.3. Example

Database TDB

Tid	Items
10	A, C, D
20	B, C, E
30	A, B, C, E
40	B, E

1st scan → C_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{D}	1
{E}	3

L_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{E}	3

C_2

Itemset
{A, B}
{A, C}
{A, E}
{B, C}
{B, E}
{C, E}

2nd scan → C_2

Itemset	sup
{A, B}	1
{A, C}	2
{A, E}	1
{B, C}	2
{B, E}	3
{C, E}	2

L_2

Itemset	sup
{A, C}	2
{B, C}	2
{B, E}	3
{C, E}	2

C_3

Itemset
{B, C, E}

3rd scan → L_3

Itemset	sup
{B, C, E}	2

5.4. Pseudo-code

C_k: Candidate itemset of size k
L_k : frequent itemset of size k
L_1 = {frequent items};
for (k = 1; L_k !=Æ; k++) **do**
    - C_{k+1} = candidates generated from L_k;
    - for each transaction t in database **do**
        increment the count of all candidates in C_{k+1}
        that are contained in t;
    endfor;
    - L_{k+1} = candidates in C_{k+1} with min_support
endfor;
return $\cup_k L_k$;

5.5. Challenges

- Multiple scans of transaction database
- Huge number of candidates
- Tedious workload of support counting for candidates
- Improving Apriori:
    - general ideas
    - Reduce passes of transaction database scans
    - Shrink number of candidates
    - Facilitate support counting of candidates
    - Easily parallelized

5.6. Improving the Efficiency of Apriori

- Several attempts have been introduced to improve the efficiency of Apriori:
  - Hash-based technique
    - Hashing itemset counts
    - Example:

      - Transaction DB:

TID	**List of Transactions**
T100	I1,I2,I5
T200	I2,I4
T300	I2,I3
T400	I1,I2,I4
T500	I1,I3
T600	I2,I3
T700	I1,I3
T800	I1,I2,I3,I5
T900	I1,I2,I3

- Create a hash table for candidate 2-itemsets:
  - Generate all 2-itemsets for each transaction in the transaction DB
  - H(x,y) = ((order of x) * 10 + (order of y)) mod 7

- A 2-itemset whose corresponding bucket count is below the support threshold cannot be frequent.

Bucket @	0	1	2	3	4	5	6
Bucket count	2	2	4	2	2	4	4
Content	{I1,I4}	{I1,I5}	{I2,I3}	{I2,I4}	{I2,I5}	{I1,I2}	{I1,I3}
	{I3,I5}	{I1,I5}	{I2,I3}	{I2,I4}	{I2,I5}	{I1,I2}	{I1,I3}
			{I2,I3}			{I1,I2}	{I1,I3}
			{I2,I3}			{I1,I2}	{I1,I3}

- Remember: support(x➔y) = percentage number of transactions that contain x and y. Therefore, if the minimum support is 3, then the itemsets in buckets 0, 1, 3, and 4 cannot be frequent and so they should not be included in C_2.

- Transaction reduction
  - Reduce the number of transactions scanned in future iterations.
  - A transaction that does not contain any frequent k-itemsets cannot contain any frequent (k+1)-itemsets: Do not include such transaction in subsequent scans.

- Other techniques include:
  - Partitioning (partition the data to find candidate itemsets)
  - Sampling (Mining on a subset of the given data)
  - Dynamic itemset counting (Adding candidate itemsets at different points during a scan)

6. Mining Frequent Itemsets without Candidate Generation

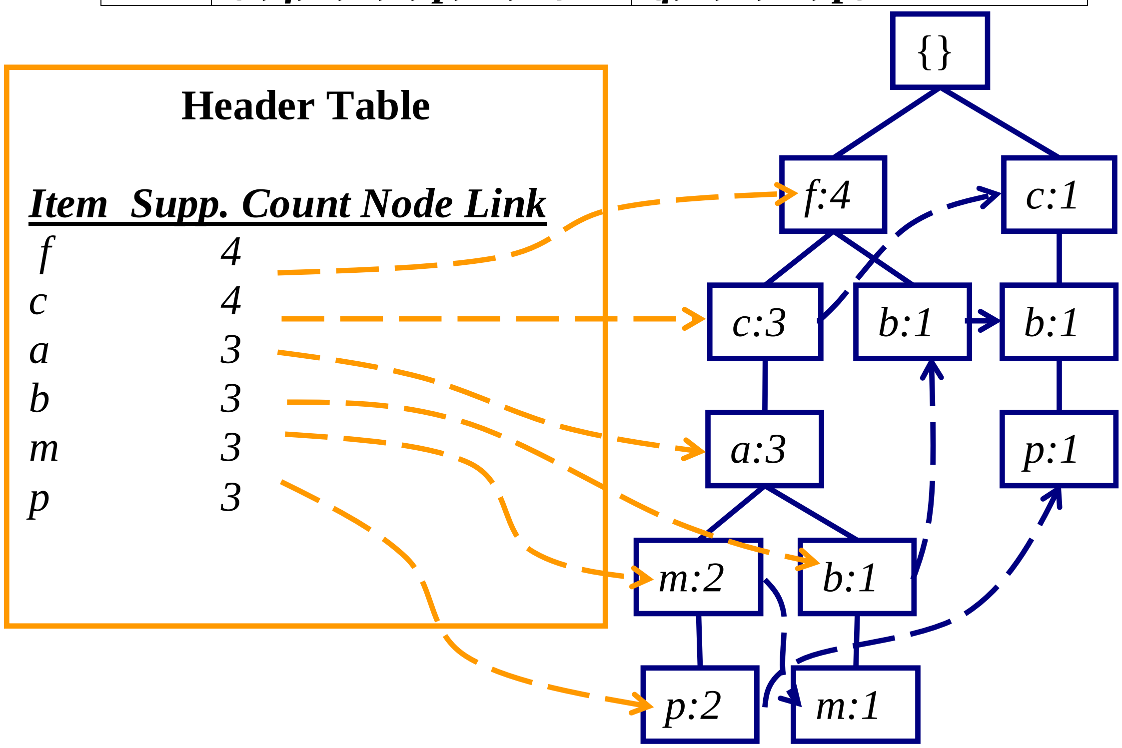
- Objectives:
  - The bottleneck of *Apriori*: candidate generation
  - Huge candidate sets:
    - For 10^4 frequent 1-itemset, Apriori will generate 10^7 candidate 2-itemsets.
    - To discover a frequent pattern of size 100, e.g., $\{a_1, a_2, \ldots, a_{100}\}$, one needs to generate $2^{100} \approx 10^{30}$ candidates.
  - Multiple scans of database:
    - Needs (*n +1*) scans, *n* is the length of the longest pattern.

- Principal
  - Compress a large database into a compact, Frequent-Pattern tree (FP-tree) structure
    - Highly condensed, but complete for frequent pattern mining
    - Avoid costly database scans
  - Develop an efficient, FP-tree-based frequent pattern mining method
    - A divide-and-conquer methodology: decompose mining tasks into smaller ones
    - Avoid candidate generation: sub-database test only!

- Algorithm:
  1. Scan DB once, find frequent 1-itemset (single item pattern)
  2. Order frequent items in frequency descending order, called ***L order***: (in the example below: F(4), c(4), a(3), etc.)
  3. Scan DB again and construct FP-tree
     a. Create the root of the tree and label it null or {}
     b. The items in each transaction are processed in the L order (sorted according to descending support count).
     c. Create a branch for each transaction
     d. Branches share common prefixes

- Example: ***min_support = 0.5***

TID	***Items bought***	***(Ordered) frequent items***
100	***{f, a, c, d, g, i, m, p}***	***{f, c, a, m, p}***
200	***{a, b, c, f, l, m, o}***	***{f, c, a, b, m}***
300	***{b, f, h, j, o}***	***{f, b}***
400	***{b, c, k, s, p}***	***{c, b, p}***
500	***{a, f, c, e, l, p, m, n}***	***{f, c, a, m, p}***



- Node Structure:

Item	count	node pointer	child pointers	parent pointer

6.1. Mining Frequent patterns using FP-Tree

- General idea (divide-and-conquer)
  - Recursively grow frequent pattern path using the FP-tree
- Method
  - For each item, construct its conditional pattern-base, and then its conditional FP-tree
  - Recursion: Repeat the process on each newly created conditional FP-tree
  - Until the resulting FP-tree is empty, or it contains only one path (single path will generate all the combinations of its sub-paths, each of which is a frequent pattern)

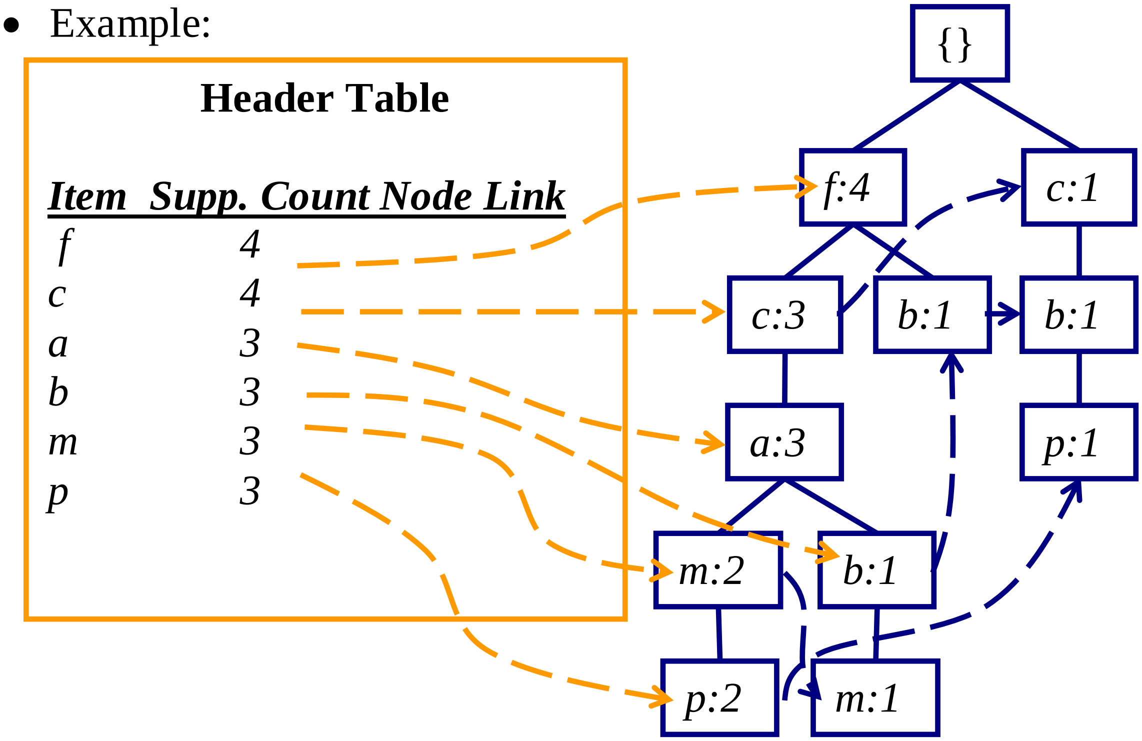
6.2. Major steps to mine FP-trees

- Main Steps:
  1. Construct conditional pattern base for each node in the FP-tree
  2. Construct conditional FP-tree from each conditional pattern-base
  3. Recursively mine conditional FP-trees and grow frequent patterns obtained so far If the conditional FP-tree contains a single path, simply enumerate all the patterns

- Step 1: From FP-tree to Conditional Pattern Base

  - Starting at the frequent header table in the FP-tree
  - Traverse the FP-tree by following the link of each frequent item, starting by the item with the highest frequency.
  - Accumulate all of transformed ***prefix paths*** of that item to form a conditional pattern base

- Example:



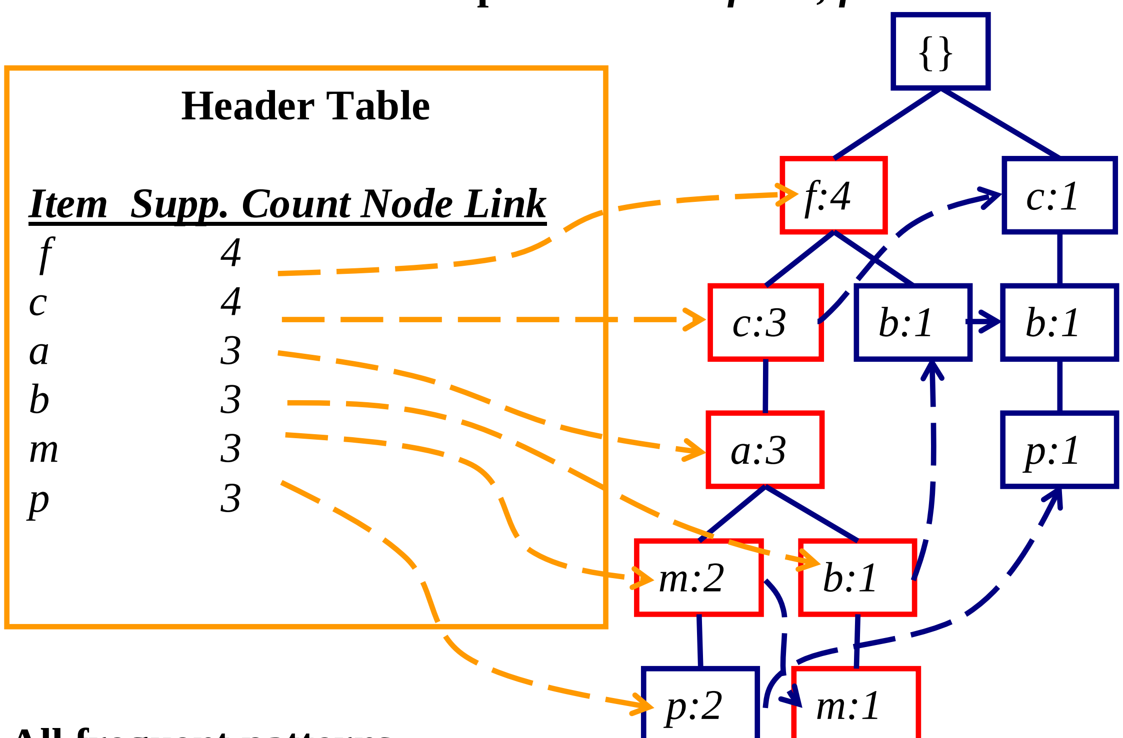
Conditional pattern bases	
Item	**Conditional pattern base**
c	f:3
a	fc:3
b	fca:1, f:1, c:1
m	fca:2, fcab:1
p	fcam:2, cb:1

- Properties of FP-tree for Conditional Pattern Base Construction:
    - Node-link property
        - For any frequent item a_i, all the possible frequent patterns that contain a_i can be obtained by following a_i's node-links, starting from a_i's head in the FP-tree header.
    - Prefix path property

- To calculate the frequent patterns for a node a_i in a path P, only the prefix sub-path of a_i in P need to be accumulated and its ***frequency count should carry the same count as node a_i***.

- Step 2: Construct Conditional FP-tree
  - For each pattern-base
    - Accumulate the count for each item in the base
    - Construct the FP-tree for the frequent items of the pattern base
  - Example:

    m-conditional **pattern base: *fca:2, fcab:1***



All frequent patterns concerning *m:*

m,
fm, cm, am,
fcm, fam, cam,
fcam

- Mining Frequent Patterns by Creating Conditional Pattern-Bases:

Item	**Conditional pattern-base**	**Conditional FP-tree**
p	{(fcam:2), (cb:1)}	{(c:3)}|p
m	{(fca:2), (fcab:1)}	{(f:3, c:3, a:3)}|m
b	{(fca:1), (f:1), (c:1)}	Empty
a	{(fc:3)}	{(f:3, c:3)}|a
c	{(f:3)}	{(f:3)}|c
f	Empty	Empty

- Step 3: Recursively mine the conditional FP-tree

{}
|
f:3
|
c:3
|
a:3

m-conditional **FP-tree**

Cond. pattern base of "am": (fc:3)

{}
|
f:3
|
c:3

am-conditional **FP-tree**

Cond. pattern base of "cm": (fa:3)

{}
|
f:3

cm-conditional **FP-tree**

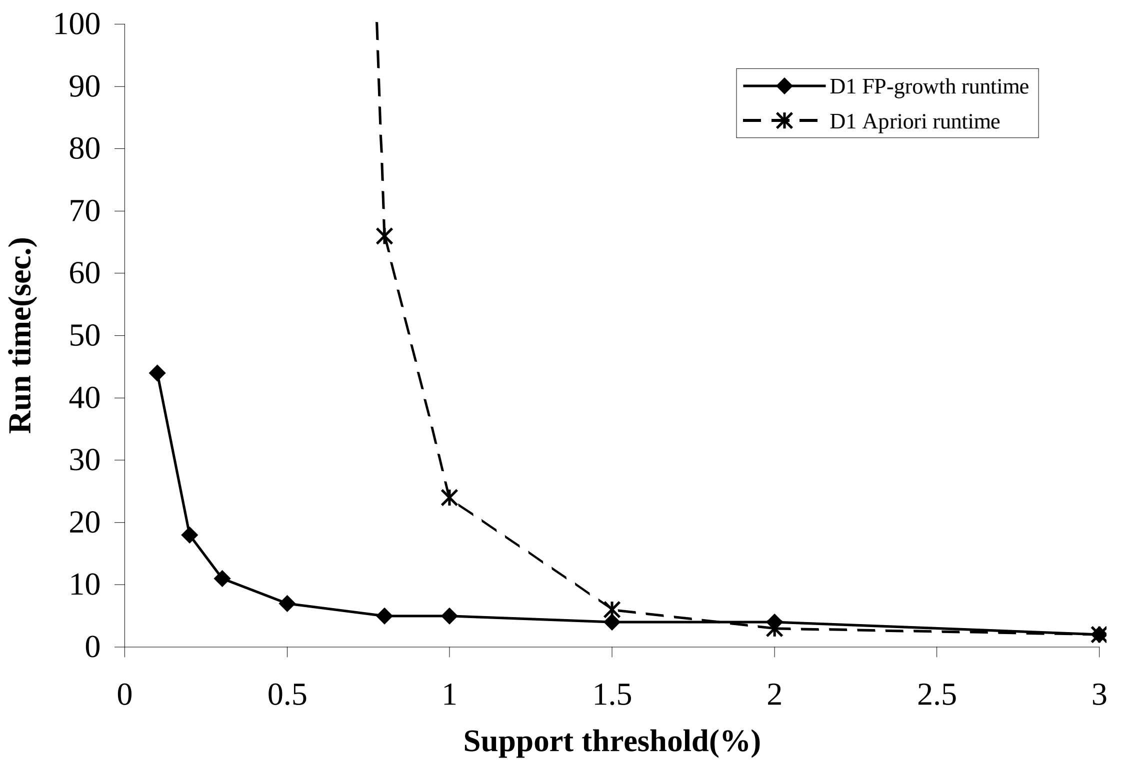
Cond. pattern base of "cam": (f:3)

{}
|
f:3

cam-conditional **FP-tree**

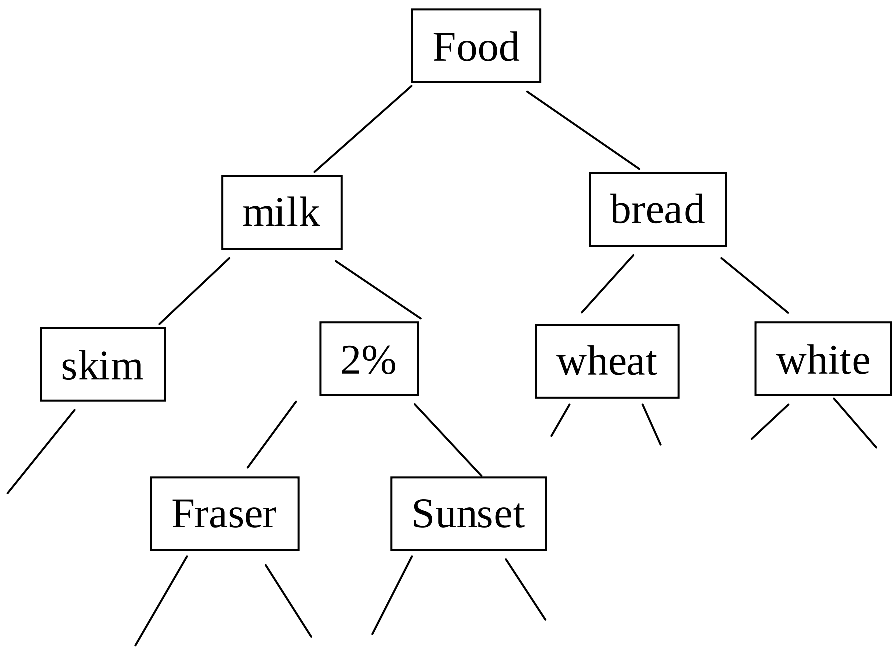
- Why is FP-Tree mining fast?
    - The performance study shows FP-growth is an order of magnitude faster than Apriori
    - Reasoning:
        - No candidate generation, no candidate test
        - Use compact data structure
        - Eliminate repeated database scan
        - Basic operation is counting and FP-tree building

- FP-Growth vs. Apriori: Scalability with the support Threshold [Jiawei Han and Micheline Kamber]



7. Multiple-Level Association Rules

- Items often form hierarchy.
- Items at the lower level are expected to have lower support.
- Rules regarding itemsets at appropriate levels could be quite useful.
- Transaction database can be encoded based on dimensions and levels
- We can explore shared multi-level mining



7.1. Approach

- A top-down, progressive deepening approach:
  - First find high-level strong rules:

milk ➔ bread [20%, 60%].

- Then find their lower-level "***weaker***" rules:

2% milk ➔ wheat bread [6%, 50%].

- Variations at mining multiple-level association rules.
  - Level-crossed association rules:

2% *milk* ➔ *Wonder wheat bread*

- Association rules with multiple, alternative hierarchies:

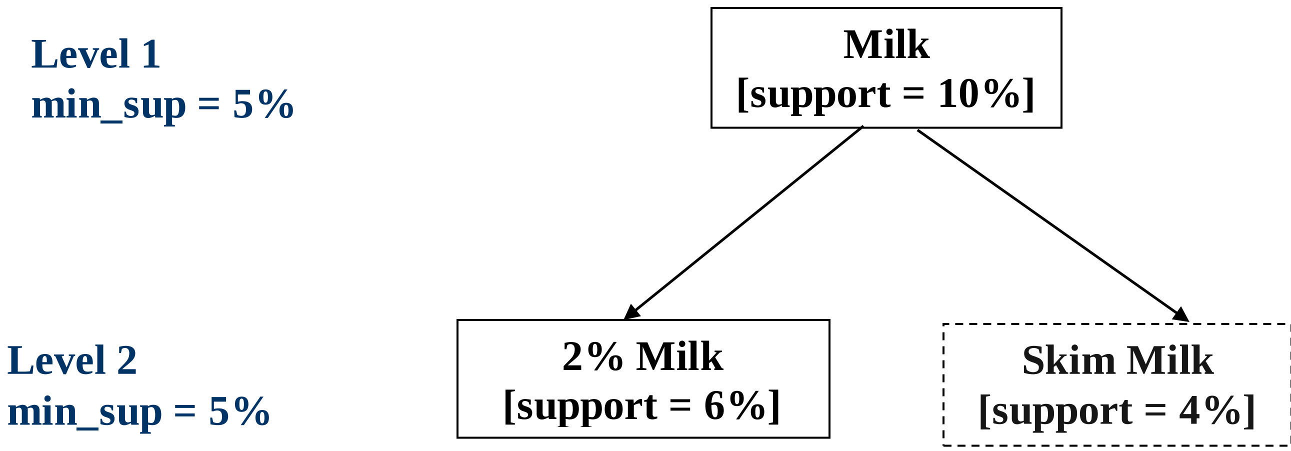
2% *milk* ➔ *Wonder bread*

- Two multiple-level mining associations strategies:
  - Uniform Support
  - Reduced support

- Uniform Support: the same minimum support for all levels

  - One minimum support threshold.
  - No need to examine itemsets containing any item whose ancestors do not have minimum support.
  - Drawback:
    - Lower level items do not occur as frequently. If support threshold

too high ➔ miss low level associations
too low ➔ generate too many high level assoc.



- Reduced Support: reduced minimum support at lower levels
    - There are 4 search strategies:
        - Level-by-level independent
        - Level-cross filtering by k-itemset
        - Level-cross filtering by single item
        - Controlled level-cross filtering by single item

    - **Level-by-Level independent:**
        - Full-breadth search
        - No background knowledge is used.
        - Each node is examined regardless the frequency of its parent.

    - **Level-cross filtering by single item:**
        - An item at the ith level is examined if and only if its parent node at the (i-1)th level is frequent.
    - **Level-cross filtering by k-itemset:**
        - A k-itemset at the ith level is examined if and only if its corresponding parent k-itemset at the (i-1)th level is frequent.

- This restriction is stronger than the one in level-cross filtering by single item
- They are not usually many k-itemsets that, when combined, are also frequent:

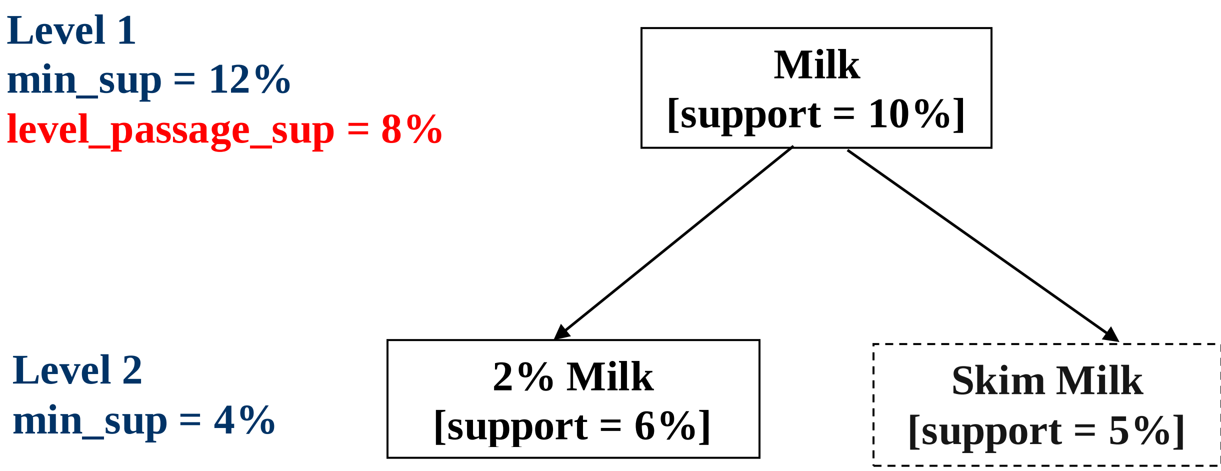
➔ Many valuable patterns can be mined

- **Controlled level-cross filtering by single item:**
  - A variation of the level-cross filtering by single item: Relax the constraint in this approach
  - Allow the children of items that do not satisfy the minimum support threshold to be examined if these items satisfy the level passage threshold:

    level_passage_supp

  - ***level_passage_sup*** Value: It is typically set between the min_sup value of the given level and the min_sup of the next level.

- Example:


Level 1
min_sup = 12%
level_passage_sup = 8%
Milk
[support = 10%]
Level 2
min_sup = 4%
2% Milk
[support = 6%]
Skim Milk
[support = 5%]

7.2. Redundancy Filtering

- Some rules may be redundant due to “ancestor” relationships between items.

- Definition: A rule R1 is an ancestor of a rule, R2, if R1 can be obtained by replacing the items in R2 by their ancestors in a concept hierarchy.

- Example

R1: milk ➔ wheat bread [support = 8%, confidence = 70%]
R2: 2% milk ➔ wheat bread [support = 2%, confidence = 72%]

Milk in R1 is an ancestor of 2% milk in R2.

- We say the first rule is an ancestor of the second rule.
- A rule is redundant if its support is close to the “expected” value, based on the rule’s ancestor:

    - R2 is redundant since its confidence is close to the confidence of R1 (kind of expected) and its support is around 2% = (8% * ¼)
    - R2 does not add any additional information.