Arrays, Vectors, Matrices

- Goal:
  - Scientific Eng.
  - Numerical computations
  - Array are very efficient way of organizing data since accessing array elements requires O(1).

- Characteristics of an array:
  - Arrays are one the main data organization
  - Arrays group values and permit fast access by numeric index
  - Characteristics:
    - **Base @:** Address of the first element of the array
    - **Size**: Size of each element of the array
    - **Type**: The type of each element
    - **Dimensions**:
      - For each dimension of the array:
        - ✓ **Upper bound** of the index range
        - ✓ **Lower bound** of the index range
  - Internal representation of an array
    - Memory cells have to be reserved for the array:

==> is achieved by declaration statements

- Requires a mapping or accessing function to be decoded
- uses the characteristics of the array

- In Java:
  - An array of size n: int [10] myarray;
  - the lower bound is 0 and the upper bound is 9

- **Single Dimensional Arrays: Vectors**
  - Example: Let MT be a one dimensional array: int MT [n];

MT [0]	MT [1]	…	MT [i]	…	MT [n-1]

- Where is the location of MT [i] ?
  - **Mapping:**
    - Let **size_of_element** be the size of each element (Type of the array)
    - The mapping function is:

      @ of MT [0] is at Base.

      @ of MT [2] is at Base+ **size_of_element**

      . . .

      @ of MT [i] is at Base+(i-lower_bound)* **size_of_element**

- In Java, C, C++ where the lower_bound is 0:

  The @ of MT[i] = base address + i * size_of_element

- **Two-dimensional arrays: Matrices**
  - Two-dimensional arrays are also called table.
  - **Requires two dimensions:**
    - Rows
    - Columns
  - **Internal Representation**:
    - How is a 2-dimensional array stored in the sequential memory?
    - Representation:
      - Abstraction: User's View

10	20	30
-10	-20	-30
5	10	15

      - Implementation: System's View
        - ✓ **Two common schemes**:
          i. **Row major order**: rows are placed one after another in memory. Examples: Java, C, C++, Pascal, etc.
          ii. **Column major order**: columns are placed one after another in memory. Example: Fortran.

- **Example:**
  - ✓ Row Major:

10	20	30
-10	-20	-30
5	10	15

10	20	30	-10	-20	-30	5	10	15

  - ✓ Column Major:

10	20	30
-10	-20	-30
5	10	15

10	-10	5	20	-20	10	30	-30	15

- **Abstraction Layout**:

Base

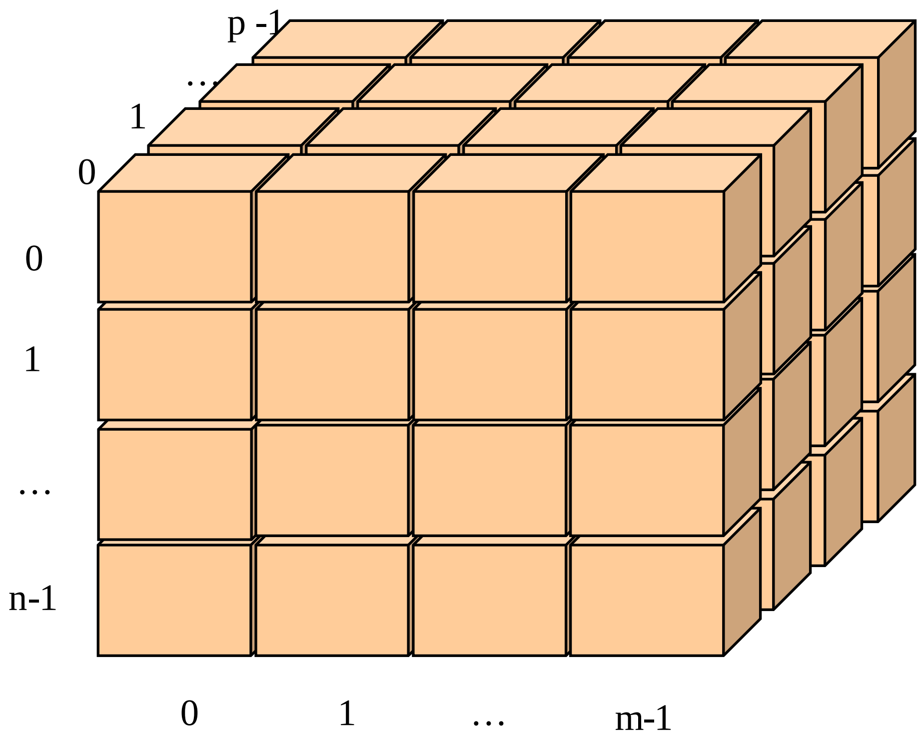
M[0][0]	…	M[0][j]	…	M[0][n-1]
…	…	…	…	…
M[i][0]	…	M[i][j]	…	M[i][n-1]
…	…	…	…	…
M[n-1][0]	…	M[n-1][j]	…	M[n-1][n-1]

- Mapping:
  - Row Major:MT[i][j] is mapped to:

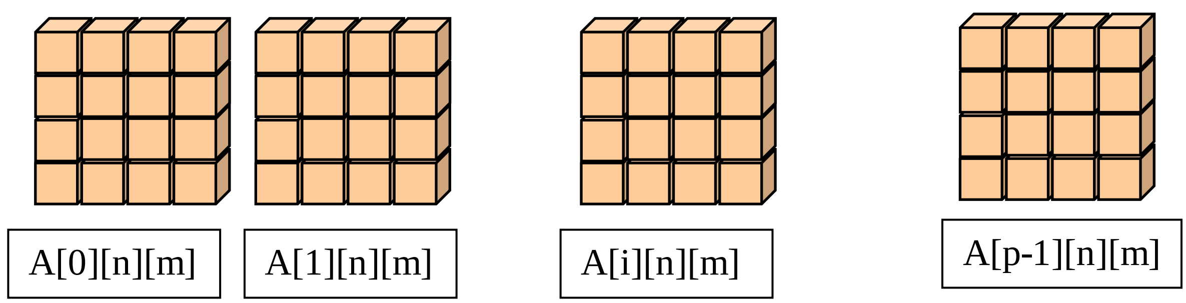
    @ of MT[i][j] = Base + (i*n+ j) * size_of_element

  - Column Major:MT[i][j] is mapped to:

    @ of MT[i][j] = Base + (j*n+ i) * size_of_element



This array can be interpreted as **p** 2D arrays of dimensional **n*m**:



The @ of The address of a[i][j][k] is:

Base+ (i*nm+ j*n+k) * size_of_element

- **Sparse Arrays**
    - Definition:
        - Arrays with many zero elements.
    - Types:
        - Arrays with one or more blocks of non-zero elements.
        - Randomly Distributed non-zero elements
- Triangular Matrices:
    - Upper and Lower
    - Lower:
        - Let MT be a one dimensional array: int **MT [n][n];**

$$\begin{cases} \mathbf{MT}[\mathbf{i}][\mathbf{j}] = \mathbf{0} & \mathbf{if\ i < j} \\ \mathbf{MT}[\mathbf{i}][\mathbf{j}] \neq \mathbf{0} & \mathbf{if\ i \geq j} \end{cases}$$

MT [0][0]	…	0		0
	0			
		0		
MT [i][0]		MT [i][j]	…	0
				0
				0
MT [n-1][0]		MT [n-1][j]		MT [n-1][n]

- **Implementation:**
  - **Store only the lower half**
  - Map MT into M' using **row-major scheme**
  - MT[i][j] is located in M'[$1+\frac{i}{2}+j$]

- **Applications:**
  - Symmetric Matrices:
    - M is a symmetric matrix iff M[i][j] = M[j][i] for i,j
  - Mapping:

    If i≥j then

    use the mapping function of the lower triangular matrix

    else

    interchange i and j in the mapping function of the lower triangular matrix

- **Randomly Distributed non-zero elements**
  - Characteristics of an element A[i][j] of a sparse matrix A:
    - row and column positions: i and j
    - The value of the entry: value
    - Implementation:
      - Store the non-zero elements in an array DA s.t. the first entry contains the following information:
        - First dimension
        - Second dimension
        - Max. number of elements: maxnonzero
        - int DA[maxnonzero][3];

- **Examples:**

```
public class Matrix {
      private int rows=0, columns=0;
      private double[][] data;

    // create r-by-c matrix of 0's
    public Matrix(int r, int c) {
        this.rows = r;
        this.columns = c;
        data = new double[r][c];
    }

       // create matrix based on 2d array
       public Matrix(double[][] d) {
        rows = d.length;
        columns = d[0].length;
        if ( rows > 0 && columns > 0 ){
                  data = new double[rows][columns];
                  for (int i = 0; i < rows; i++)
                        for (int j = 0; j < columns; j++)
                              data[i][j] = d[i][j];
             }
             else
                 System.exit(0);
//                  fatalError("Constructor " + r + ", " + c);
       }
       // Accessor method, get the number of rows in the Matrix.
        public int getRows() {
          return rows;
        }

        // Accessor method, get the number of columns in the Matrix.
        public int getColumns() {
          return columns;
        }
        // Accessor method, get an element of the Matrix.
        public double getEntry(int r, int c) {
          r = checkRowIndex(r);
          c = checkColumnIndex(c);
          return  data[r][c];
        }
        // Mutator: set an element of the Matrix.
        public Matrix setEntry(int r, int c, int val) {
          r = checkRowIndex(r);
          c = checkColumnIndex(c);
          data[r][c] = val;
          return this;
        }
        // Matrix addition
        public Matrix add(Matrix b) {
            AddcheckSize(b);
            Matrix result = new Matrix(b.getRows(), b.getColumns());
          for (int r = 0; r < rows; r++)
            for (int c = 0; c < columns; c++)
                  result.data[r][c] = data[r][c] + b.data[r][c];
          return result;
```

```
        }

        // Matrix Subtraction
        public Matrix sub(Matrix b) {
            //Implement this method
        }

        // Matrix Subtraction
        public Matrix transpose(Matrix b) {
            //Implement this method
        }

        // Make sure the row parameter r is valid(i.e., 1 <= r <= rows), then performs
        private int checkRowIndex(int r) {
          if ( r < 1 || r > rows )
           System.out.println("Row index " + r + " out of range");
            System.exit(0);
          return r - 1;
        }
        // Make sure the row parameter r is valid(i.e., 1 <= r <= rows), then performs
        private int checkColumnIndex(int c) {
          if ( c < 1 || c > columns )
           System.out.println("Row index " + c + " out of range");
            System.exit(0);
          return c - 1;
        }
        // Make sure the matrices have the same dimensions.
        private void AddcheckSize(Matrix b) {
          if ( rows != b.rows || columns != b.columns ){
            System.out.println("You cannot add two matrices of different sizes");
                  System.exit(0);
          }
        }
         // print matrix to standard output
          public void printMatrix() {
              for (int i = 0; i < rows; i++) {
                  for (int j = 0; j < columns; j++)
                      System.out.printf("%9.4f ", data[i][j]);
                  System.out.println();
              }
          }
}
```

- **Programming Assignment:**
  - Design and implement the missing operations in the Matrix ADT:
    - Subtraction
    - Multiplication
    - Transpose

  - Test your implementation.