Performance Evaluation of Lossy DPCM Coding of Images Using Different Predictors and Quantizers

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Abstract

The phenomeal increases in the generation, processing, and transmission of digital images have created increasing demands on the storage capacities, the processing speeds, and on the bandwidth of communication. Typical applications include teleradiology, digital libraries, satellite imagery for earth resources, multimedia databases, and several others. Lossy image compression techniques enable the storage of digital images using reasonable space and their transmission with acceptable speed.

Lossy DPCM (differential pulse code modulation) technique for compression and decompression of images offers the advantages computational simplicity and ease of parallel implementation in hardware/software. In this paper we present results of evaluation of lossy DPCM scheme for coding images using a variety of combinations of quantizers, quantization levels, and prediction coefficients.

1 Introduction

The acquisition, generation, storage and processing of digital images in computers and their transmission over networks have grown tremendously in recent times and are expected to have explosive growth in the coming years. This is due to the great need for images in different applications on the one hand, and the rapid growth in computational power in personal computers, the development of input devices such as digital cameras and scanners, the availability of high-density storage devices, and the development of high-speed data communication networks, on the other hand. Despite the aforementioned technological advances, the demands placed on the processing speeds, storage capacities and on the bandwidth of communication seem to have been exceeding the availability.

Typical applications of digital images include teleradiology, satellite imagery of earth resources, information access from multimedia databases, photojournalism, graphic arts, and several others. Image compression is an attractive scheme to store images with reasonable amount of storage and transmit images with acceptable speed.

Digital images generally contain a significant amount of spatial and spectral redundancy. Image compression techniques take advantage of this redundancy to reduce the number of bits required to represent the image. The image compression schemes are broadly classified as either *lossless* or *lossy*, depending respectively on whether the compressed image can be *exactly* recovered or not. Generally, lossy schemes provide a much higher compression ratios than lossless schemes. Major characteristics to be considered in any compression scheme are the compression ratio (CR), the signal-to-noise ratio (SNR) of the reconstructed image with respect to the original, and the speed of encoding and decoding.

There are several lossy compression techniques such as (1) prediction coding, (2) transform coding, (3) block truncation coding, (4) vector quantization, and (5) subband coding [2]. The *differential pulse code modulation* (DPCM) is the most common approach to predictive coding. The basic idea behind this scheme is to predict the value of a pixel based on the correlation between certain neighboring pixel values, using certain *prediction coefficients*. The number of pixels used in the prediction is called the *order* of the predictor. Although a higher order predictor performs better than a lower order one, studies done on television images [5] and radiographs [6] have shown that there is only a marginal gain beyond a third-order predictor. The difference between the predicted value and the actual value of the pixels gives the differential (residual) image, which is much less correlated than the original image. The differential image is then quantized and encoded. The basic function of a quantizer is to map a large range of (possibly continuous) values onto a relatively smaller set of (discrete) values. Quantizer design may be based on either statistical or visual criteria. It is the quantization which sets apart the lossy DPCM scheme from the lossless DPCM scheme. The quantization lowers the bit rate and introduces error. To ensure that the predictions at both the encoder and decoder are identical, the encoder also uses the 'reconstructed pixel values' in its pixel value predictions.

In this paper, we evaluate lossy DPCM coding using five different third order predictors and three different quantizers – (1) uniform, (2) semi-uniform, and (3) Lloyd-Max, with the levels of quantization set to 2, 4, 8 and 16. The performance metric is the compression ratio (CR) and the corresponding signal-to-noise ratio (SNR).

The next section gives a description of the lossy DPCM algorithm. Experimental results are given in Section 3, followed by conclusions.

2 The Lossy DPCM Algorithm

A brief description of the lossy DPCM algorithm is first given followed by the pseudocode. The basic idea behind this scheme is to predict the value of a pixel based on certain neighboring pixel values, using certain *prediction coefficients*. The difference between the predicted value and the actual value of the pixels is the differential (residual) image, which is much less correlated than the original image. The differential image is then quantized and encoded.

The schematic for lossy DPCM coder is shown in Figure 1, along with a third-order predictor. Note that the decoder has access only to the reconstructed values of (previous) pixels while forming predictions of pixels. Since the quantization of the differential image introduces error, the reconstructed values generally differ from the original values. To ensure identical predictions at both the encoder and decoder, the encoder also uses the 'reconstructed pixel values' in its prediction. This is done by including the quantizer within the prediction loop as shown in Figure 1. (In essence, the decoder is built into the encoder).

The design of a DPCM coder involves the optimization of the predictor and the quantizer. The inclusion of the quantizer in the prediction loop results in a complex dependency between the prediction error and the quantization error. However, the predictor and quantizer are usually optimized separately, since a joint optimization is usually complex. Under mean-squared error (MSE) optimization criterion, independent optimizations of the predictor and quantizer are good approximations to the jointly optimized solution [4].

The following notations are used in the subsequent algorithms.

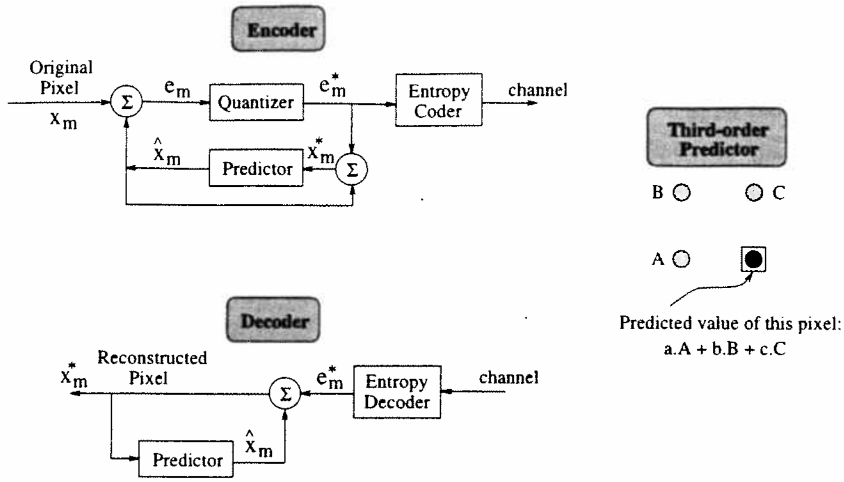

Figure 1: Lossy DPCM coding/decoding scheme.

I:	Original image.
$\hat{I}$:	Reconstructed image.
$N \times M$:	Image size (N rows, M columns of pixels).
a, b, c:	Prediction coefficients.
E^*:	Quantized residual image.
$CR[1:M]$:	Current row of reconstructed values.
$UR[1:M]$:	Upper row of reconstructed values.
$LC[1:N]$:	Leftmost column of reconstructed values.
Q:	Quantizer function – $Q(x)$ gives the quantized value of x.
Q^{-1}:	Dequantizer.
B:	Entropy coded bit stream.
k:	Number of quantization levels.
DV:	Vector of decision levels.
RV:	Vector of reconstruction levels.

Algorithm 2.1 LossyDPCM_Encode (**in:** I, a, b, c, Q, **out:** B)

begin
$E^*[1,1] \leftarrow Q(I[1,1])$.
$LC[1] \leftarrow CR[1] \leftarrow Q^{-1}(E^*[1,1])$.
{Work on top row. }
for $j = 2$ **to** M **do**
$E^*[1,j] \leftarrow Q(I[1,j] - CR[j-1])$.
$CR[j] \leftarrow Q^{-1}(E^*[1,j]) + CR[j-1])$.
endfor
{Work on leftmost column. }
for $i = 2$ **to** N **do**
$E^*[i,1] \leftarrow Q(I[i,1] - LC[i-1])$.
$LC[i-1] \leftarrow Q^{-1}(E^*[i,1]) + LC[i-1])$.
endfor
for $i = 2$ **to** N **do**
$UR[1:m] \leftarrow CR[1:M]$.
$CR[1] \leftarrow LC[i]$.
for $j = 2$ **to** M **do**
$p \leftarrow a \cdot CR[j-1] + b \cdot UR[j-1] + c \cdot UR[j]$. {*Prediction*}
$E^*[i,j] \leftarrow Q(I[i,j] - p)$.
$CR[j] \leftarrow Q^{-1}(E^*[i,j]) + p$.
endfor
endfor
$B \leftarrow$ ENTROPY_CODE(E^*).
end

Algorithm 2.2 LossyDPCM_Decode (**in:** B, a, b, c, Q, **out:** $\hat{I}$)

begin
$E^* \leftarrow$ ENTROPY_DECODE(B).
$\hat{I}[1,1] \leftarrow Q^{-1}(E^*[1,1])$.
{Work on top row. }
for $j = 2$ **to** M **do**
$\hat{I}[1,j] \leftarrow Q^{-1}(E^*[1,j] + \hat{I}[1,j-1]$.
endfor
{Work on leftmost column. }
for $i = 2$ **to** N **do**
$\hat{I}[i,1] \leftarrow Q^{-1}(E^*[i,1]) + \hat{I}[i-1,1]$.
endfor
for $i = 2$ **to** N **do**
for $j = 2$ **to** M **do**
$\hat{I}[i,j] \leftarrow Q^{-1}(E^*[i,j]) + a \cdot \hat{I}[i,j-1] + b \cdot \hat{I}[i-1,j-1] + c \cdot \hat{I}[i-1,j]$.
endfor
endfor
end

The overall schematic to do the coding/decoding and to determine the SNR and CR is shown in Figure 2.

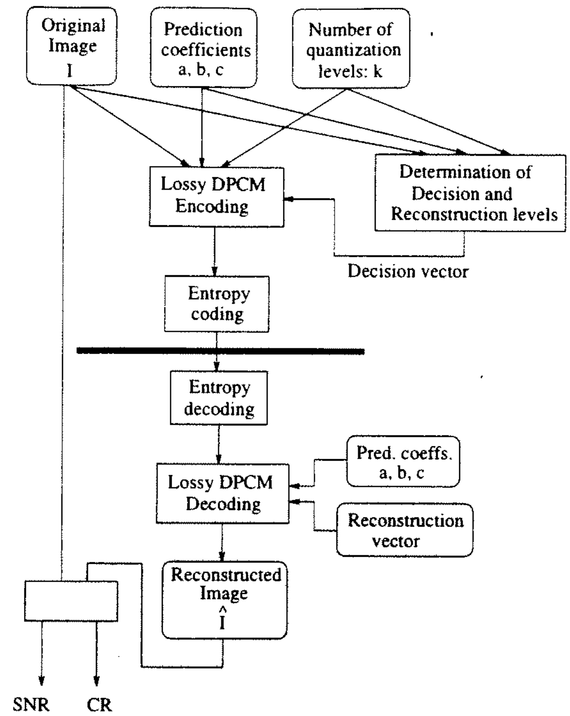

Figure 2: Schematic for determining SNR and CR.

2.1 Description of the quantizers

The basic function of a quantizer is to map a large range of (possibly continuous) values onto a relatively smaller set of (discrete) values. In the context of image coding and decoding, a quantizer has a *decision vector* and a *reconstruction vector*. The decision vector determines the *quantization level* l for any given pixel x: $l = Q(x)$, and the reconstruction vector determines the *reconstructed pixel* $\hat{x}$ value for a given quantization level: $\hat{x} = Q^{-1}(l)$. The essential difference between the different quantizers is in terms of how the decision and reconstruction vectors are determined.

The three types of quantizers used in the evaluation of the lossy DPCM coding of images – (1) uniform, (2) semi-uniform, (3) Lloyd-Max, are briefly described below, together with the algorithm for the derivation of the decision and reconstruction levels for each of the quantizers. Note that since the quantization is applied to the residual image, the design of the quantizers are based on the pixel statistics of the residual (differential) image.

1. **Uniform quantizer** In this case, the decision vector consists of k equally spaced intervals from the minimum pixel value to the maximum pixel value in the differential (residual) image, where k is the number of quantization levels. The reconstruction vector consists of the mid-points of the decision intervals. The pseudocode for the determination of decision and reconstruction vectors for the uniform quantizer is given below.

Algorithm 2.3 UNIFORMQUANTIZERLEVELS (**in:** I, a, b, c, k, **out:** DV, RV)

```
begin
  E ← ResidualImage(I, a, b, c).
  m1 ← MIN(E); m2 ← MAX(E);
  Δ ← ⌊(m2 − m1)/k⌋.
  DV[1] ← m1.
  DV[i] ← DV[i − 1] + Δ, 2 ≤ i ≤ k.
  DV[k + 1] ← m2.
  RV[1] ← DV[1] + Δ/2.
  RV[i] ← RV[i − 1] + Δ, 2 ≤ i ≤ k.
end
```

2. **Semi-uniform quantizer** In semi-uniform quantizer, the decision levels are the same as those in case of uniform quantizer. However, the reconstruction levels are the *centroids* of the decision intervals. The centriods lie at the center of the mass of the probability density enclosed by the the two adjacent decision levels. The centroids are determined based on the number of pixels in the decision intervals and the pixel values. The pseudocode for the determination of decision and reconstruction vectors for the uniform quantizer is given below.

Algorithm 2.4 SEMIUNIFORMQUANTIZERLEVELS (**in:** I, a, b, c, k, **out:** DV, RV)

```
begin
  E ← ResidualImage(I, a, b, c).
  m1 ← MIN(E); m2 ← MAX(E);
  Δ ← ⌊(m2 − m1)/k⌋.
  DV[1] ← m1.
  DV[i] ← DV[i − 1] + Δ, 2 ≤ i ≤ k.
  DV[k + 1] ← m2.
  s[1 : k] ← 0; n[1 : k] ← 0.
  for i = 1 to N do
    for j = 1 to M do
      l ← Locate(E[i, j], DV). {l is in the range: 1 ≤ l ≤ k}
      {Locate the interval in DV where the pixel E[i, j] falls.}
      n[l] ← n[l] + 1.
      s[l] ← s[l] + E[i, j].
    endfor
  endfor
  for l = 1 to k do
    RV[l] ← s[l]/n[l].
  endfor
end
```

3. **Lloyd-Max quantizer** In the Lloyd-Max quantizer, optimum decision and reconstruction levels are obtained, while quantization error is minimized with respect to the mean-square-error metric [7, 8]. This results in non-uniform decision regions. The decision levels are halfway between the neighboring reconstruction levels and the reconstruction levels are the *centroids* of the two adjacent decision levels. These are determined by the iterative algorithm given below.

Algorithm 2.5 Lloyd-MaxQuantizerLevels (**in:** I, a, b, c, k; **out:** DV, RV)

1. **begin**
2. $E \leftarrow$ ResidualImage(I, a, b, c).
3. *Start with an initial estimate of RV.*
   For example, RV[i]'s could be the uniform reconstruction levels.
   $RV[1] = \texttt{MIN}(E) + \frac{\Delta}{2}$.
   $RV[i] = RV[i-1] + \Delta,\ 2 \leq i \leq k$.
   where $\Delta = \frac{\texttt{MAX}(E) - \texttt{MIN}(E)}{k}$.
4. $DV^{old}[1] \leftarrow DV^{new}[1] \leftarrow \texttt{MIN}(E)$.
5. $DV^{old}[k+1] \leftarrow DV^{new}[K+1] \leftarrow \texttt{MAX}(E)$.
6. $DV^{old}[i] \leftarrow (RV[i-1] + RV[i])/2,\ 2 \leq i \leq k$.
7. **do**
8. UpdateRV(E, DV, RV).
9. $DV^{new}[i] \leftarrow (RV[i-1] + RV[i])/2,\ 1 \leq i \leq k$.
10. $error \leftarrow \sqrt{\sum_i (DV^{new}[i] - DV^{old}[i])^2}$.
11. $DV^{old}[i] \leftarrow DV^{new}[i],\ 2 \leq i \leq k+1$.
12. **until** $(error < \epsilon)$.
13. **end**

Algorithm 2.6 UpdateRV (**in:** E, DV; **out:** RV)

1. **begin**
2. $s[1:k] \leftarrow 0$; $n[1:k] \leftarrow 0$.
3. **for** $i = 1$ **to** N **do**
4. **for** $j = 1$ **to** M **do**
5. $l \leftarrow$ Locate$(E[i,j], DV)$. {*l is in the range:* $1 \leq l \leq k$}
   {Locate the interval in DV where the pixel $E[i,j]$ falls.}
6. $n[l] \leftarrow n[l] + 1$.
7. $s[l] \leftarrow s[l] + E[i,j]$.
8. **endfor**
9. **endfor**
10. **for** $l = 1$ **to** k **do**
11. $RV[l] \leftarrow s[l]/n[l]$.
12. **endfor**
13. **end**

3 Experimental Results

The Lossy DPCM scheme was implemented using three different types of quantizers – (1) Uniform, (2) Semi-uniform, and (3) Lloyd-Max quantizer, with different quantization levels – 2, 4, 8, and 16, and five different sets of prediction coefficients (three third-order and two first-order) – $\{0.5, 0, 0.5\}, \{0.75, -0.5, 0.75\}, \{1, -1, 1\}, \{1, 0, 0\}$, and $\{0, 0, 1\}$. The experiments were performed to determine the signal to noise ratio (SNR) and compression ratio (CR) for each combination of the above parameters. The plots of the SNRs and CRs obtained using the three different quantizers, with the various prediction coefficients, and quantization levels are shown in Figures 3 and 4 respectively. It is observed that for all predictors and quantization levels, the Lloyd-Max quantizer gives the highest SNR, followed by the semi-uniform and the uniform quantizers. The highest CR is given by the semi-uniform quantizer followed by the uniform and Lloyd-Max quantizers. When SNR and CR are considered together, the semi-uniform quantizer provides good performance.

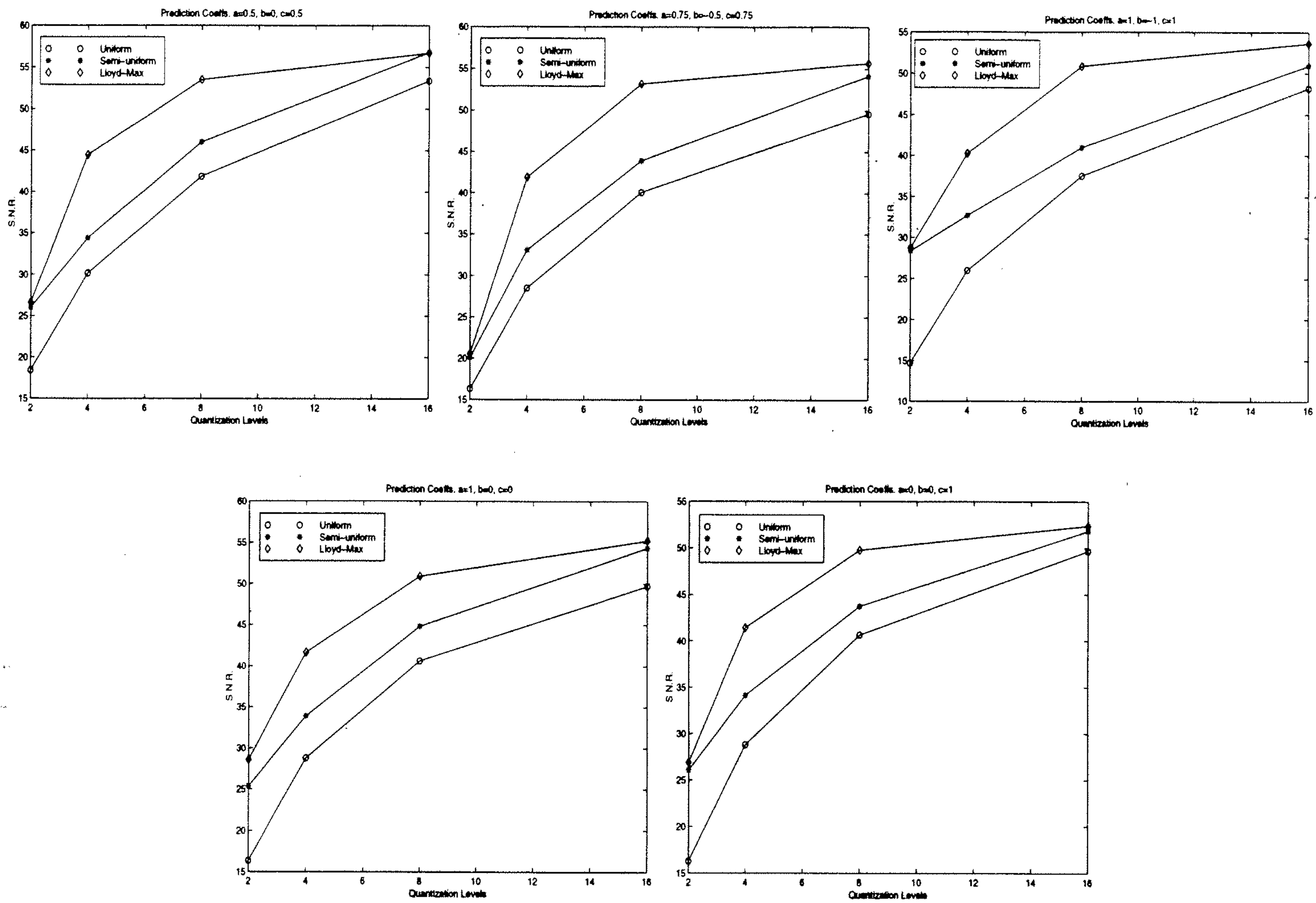

Figure 3: SNRs of lossy DPCM with different quantizers and prediction coefficients.

Note: The objective of the evaluation was to find the best combination of quantizers and quantization levels. The entropy coding used was basic Huffman coding. Higher compression could be achieved by using run-length encoding followed by Huffman coding.

The original image and the reconstructed images for a few cases are given here.

Reconstructed images for other cases will be given in the full paper.

4 Conclusions

Lossy image compression techniques enable the storage of digital images using reasonable space and their transmission with acceptable speed. Lossy DPCM technique for compression and decompression of images offers the advantages of computational simplicity and ease of parallel implementation in software/hardware. In this paper, the lossy DPCM coding of images was evaluated using a variety of combinations of (1) quantizers, (2) quantization levels, and (3) prediction coefficients. The Lloyd-

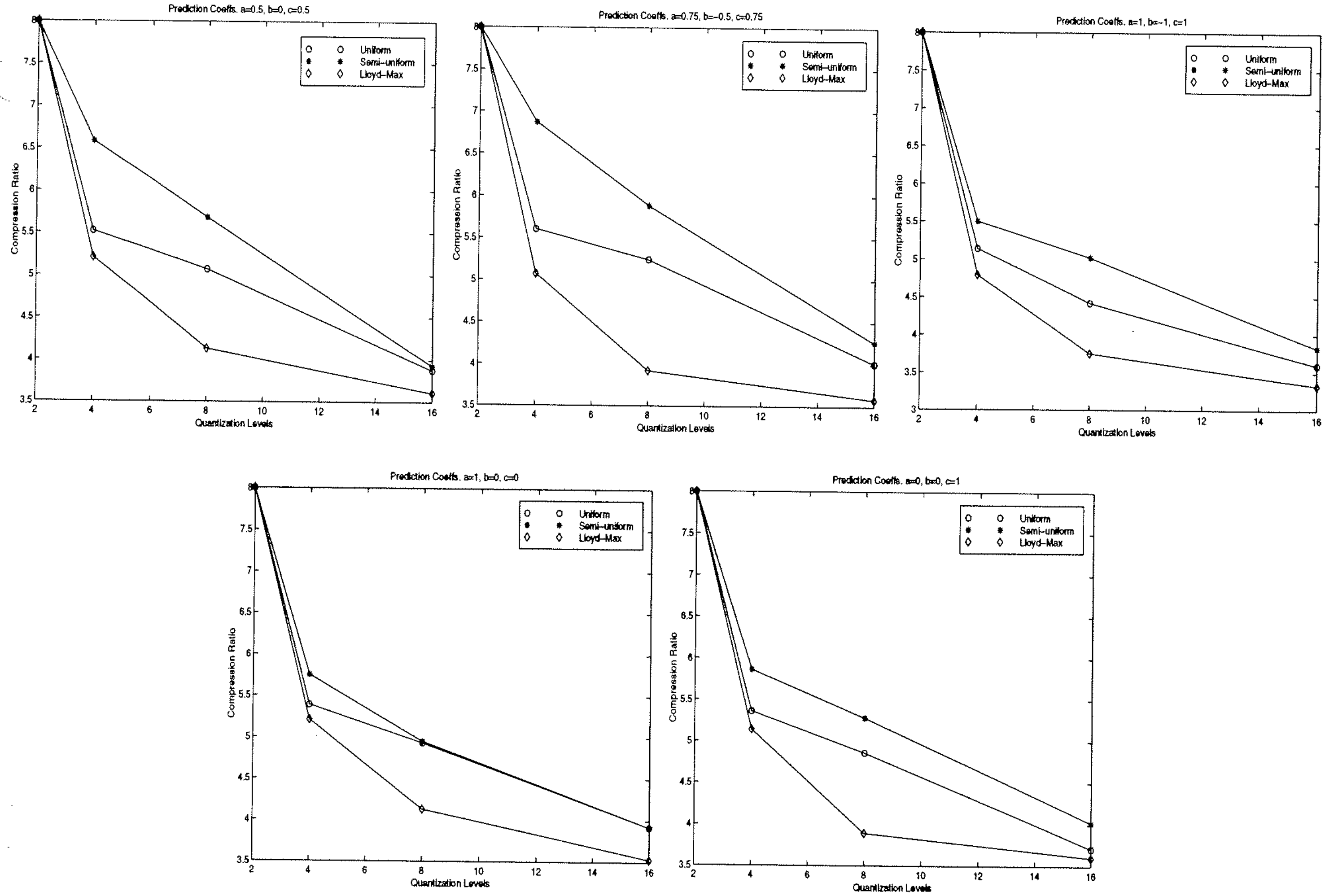

Figure 4: CRs of lossy DPCM with different quantizers and prediction coefficients.

Max quantizer performs better in terms of SNR, while the semi-uniform quantizer performs better in terms of CR.

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Figure 5: Original and reconstructed images.