CS 6212 June 15, 2011

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MIDTERM EXAM

TIME: 2.5 Hours

Problem 1: (25 points)

Let G = (V,E) be the following weighted undirected graph: V = $\{1, 2, 3, 4, 5, 6, 7\}$ and E = $\{(1, 4), (3, 4), (3, 6), (2, 6), (4, 2), (2, 5), (1, 6), (4, 6), (1, 5), (4, 7), (7, 5)\}$, with respective weights 8, 11, 3, 6, 9, 20, 12, 3, 12, 16, 10.

a) Find a minimum spanning tree in G.

b) Designate node 1 as a sourse node. Find the distance between 1 and all the other nodes using the greedy algorithm. Show the value of the DIST array at every step.

Problem 2: (25 points)

Let $x[1 : n]$ be a **sorted** array. Define $y[1 : n]$ as follows: for all $i = 1, 2, \ldots, n$, $y[i]$ is the number of times the value of $x[i]$ repeats in the input array $x[1 : n]$. For example, if $n = 4$ and $x[1 : 4]$ is $7, 7, 10$, and 12, then $y[1] = y[2] = 2$ (because 7 occurs twice), $y[3] = 1$ (because 10 occurs once), and $y[4] = 1$.

a) Suppose the array $x[1 : 8]$ is: 1, 1, 5, 5, 5, 7, 13, 13. Give the array $y[1 : 8]$.

b) Write a divide-and-conquer algorithm that takes as input an arbitrary sorted array $x[1 : n]$, and returns as output the array $y[1 : n]$. Analyze the time of your algorithm.

Problem 3: (25 points)

The most frequent value in an array $A[1 : n]$ is the value that occurs the most in A. For example, if $A[1 : 7]$ is $1, 4, 2, 4, 2, 5, 4$, then the most frequence value in A is 4.

a) Write an algorithm that takes as input an arbitrary unsorted array $A[1 : n]$ and returns the most frequent value in A. (Hint: use sorting and your algorithm of Problem 2 above.)

b) Give the time complexity of your algorithm.

Problem 4: (25 points)

You have n employees and n jobs. It costs c_{ij} dollars for job i to be done by employee j (the c_{ij}'s are given input). The job assignment problem is to assign exactly one job to each employee in such a way that the sum of the costs of the n jobs is minimized.

a) Write a greedy algorithm for the job assignment problem.

b) What is the time complexity of this algorithm?

c) Show by a counter example that this greedy method does not always yield an optimal solution (Hint: You can find a counter example where n=2).