Solution to Homework 6

Problem 1: (20 points)

a)

x	y	$x \oplus y$
0	0	1
0	1	0
1	0	0
1	1	1

Table 1Truth table of $\oplus$

b)

$$x \oplus x = xx + x'x' = x + x' = 1$$
$$x \oplus x' = xx' + x'x = 0 + 0 = 1$$
$$x \oplus 0 = x0 + x'1 = 0 + x' = x'$$
$$x \oplus 1 = x1 + x'0 = x + 0 = x$$

c)

$$x \oplus y = xy + x'y' = yx + y'x' = y \oplus x$$

∴ Commutative.

x	y	z	$(x \oplus y) \oplus \mathbf{z}$	$x \oplus (y \oplus \mathbf{z})$
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	0
1	0	0	1	1
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

Table 2 Proof of associativity with truth table

∴ Associative.

x	y	z	$x \oplus (y + \mathbf{z})$	$(x \oplus y) + (x \oplus \mathbf{z})$
0	0	0	1	1
0	0	1	0	1
0	1	0	0	1
0	1	1	0	0
1	0	0	0	0
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1

Table 3 Disproof of distributivity over + with truth table

∴Not distributive over +.

x	y	z	$x \oplus (y \cdot z)$	$(x \oplus y) \cdot (x \oplus z)$
0	0	0	1	1
0	0	1	1	0
0	1	0	1	0
0	1	1	0	0
1	0	0	0	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

Table 4 Disproof of distributivity over · with truth table

∴ Not distributive over ·.

Problem 2: (20 points)

a) $f(x_1, x_2, x_3, x_4) = x_1 \cdot x_4 \cdot (x_2 + x_3)$

b) $f(x_1, x_2, x_3, x_4) = x_1 + x_2 + x_3 + x_4$

c) $f(x_1, x_2, x_3, x_4) = (x_1 x_4' + x_1' x_4) \cdot (x_2 x_3' + x_2' x_3)$

Problem 3: (20 points)

a)

x	y	z	f	f'
0	0	0	0	1
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	0

Table 5 Truth table of f and f'

b)

$DNF\ of\ f = x'y'z + x'yz' + xy'z' + xyz$

$CNF\ of\ f' = (DNF\ of\ f)' = (x + y + z') \cdot (x + y' + z) \cdot (x' + y + z) \cdot (x' + y' + z')$

c)

$DNF\ of\ f' = x'y'z' + x'yz + xy'z + xyz'$

$CNF\ of\ f = (DNF\ of\ f')' = (x + y + z) \cdot (x + y' + z') \cdot (x' + y + z') \cdot (x' + y' + z)$

Problem 4: (20 points)

a)

x	y	z	$f=xyz + x'z + x'yz'$	f'
0	0	0	0	1
0	0	1	1	0
0	1	0	1	0
0	1	1	1	0
1	0	0	0	1
1	0	1	0	1
1	1	0	0	1
1	1	1	1	0

Table 6 Truth table of $xyz + x'z + x'yz'$

$DNF\ of\ f = x'y'z + x'yz' + x'yz + xyz;$ $DNF\ of\ f' = x'y'z' + xy'z' + xy'z + xyz'$

$CNF\ of\ f = (x + y + z) \cdot (x' + y + z) \cdot (x' + y + z').(x' + y' + z)$

b)

x	y	z	$f = yz + xz' + x'z'$	f'
0	0	0	1	0
0	0	1	0	1
0	1	0	1	0
0	1	1	1	0
1	0	0	1	0
1	0	1	0	1
1	1	0	1	0
1	1	1	1	0

Table 7 Truth table of $yz + xz' + x'z'$

$DNF\ of\ f = x'y'z' + x'yz' + x'yz + xyz' + xyz;$ $DNF\ of\ f' = x'y'z + xy'z$

$CNF\ of\ f = (x + y + z') \cdot (x' + y + z')$

c)

x	y	z	w	$f = xyz + xy'zw + x'y'z'w' + x'yw' + y'zw'$	f'
0	0	0	0	1	0
0	0	0	1	0	1
0	0	1	0	1	0
0	0	1	1	0	1
0	1	0	0	1	0
0	1	0	1	0	1
0	1	1	0	1	0
0	1	1	1	0	1
1	0	0	0	0	1
1	0	0	1	0	1
1	0	1	0	1	0
1	0	1	1	1	0
1	1	0	0	0	1
1	1	0	1	0	1
1	1	1	0	1	0

1	1	1	1	1	0

Table 8 Truth table of $xyz + xy'zw + x'y'z'w' + x'yw' + y'zw'$

$$DNF\ of\ f = x'y'z'w' + x'y'zw' + x'yz'w' + x'yzw' + xy'zw' + xy'zw + xyzw' + xyzw$$

$$DNF\ of\ f' = x'y'z'w + x'y'zw + x'yz'w + x'yzw + xy'z'w' + xy'z'w + xyz'w' + xyz'w$$

$$CNF\ of\ f = (x + y + z + w')(x + y + z' + w')(x + y' + z + w')(x + y' + z' + w')$$
$$(x' + y + z + w')(x' + y' + z + w)(x' + y' + z + w')$$

d)

x	y	z	w	$f = xzw' + xyz' + y'z'w' + x'y'z' + yz'w'$	f'
0	0	0	0	1	0
0	0	0	1	1	0
0	0	1	0	0	1
0	0	1	1	0	1
0	1	0	0	1	0
0	1	0	1	0	1
0	1	1	0	0	1
0	1	1	1	0	1
1	0	0	0	1	0
1	0	0	1	0	1
1	0	1	0	1	0
1	0	1	1	0	1
1	1	0	0	1	0
1	1	0	1	1	0
1	1	1	0	1	0
1	1	1	1	0	1

Table 9 Truth table of $xzw' + xyz' + y'z'w' + x'y'z' + yz'w'$

$$DNF\ of\ f = x'y'z'w' + x'y'z'w + x'yz'w' + xy'z'w' + xy'zw' + xyz'w' + xyz'w + xyzw'$$

$$DNF\ of\ f' = x'y'zw' + x'y'zw + x'yz'w + x'yzw' + x'yzw + xy'z'w + xy'zw + xyzw$$

$$CNF\ of\ f = (x + y + z' + w)(x + y + z' + w')(x + y' + z + w')(x + y' + z' + w)$$
$$(x + y' + z' + w')(x' + y + z + w')(x' + y + z' + w')(x' + y' + z' + w')$$

Problem 5: (20 points)

a)

	y	y'		
x	1			
x'	1	1		1
	z	z'	z	

Table 10 Karnaugh map of $xyz + x'z + x'yz'$

$$yz + x'y + x'z$$

b)

	y	y'		
x	1	1	1	
x'	1	1	1	
	z	z'	z	

Table 11 Karnaugh map of $yz + xz' + x'z'$

$$y + z'$$

c)

	y	y'			
x	1			1	w
	1			1	
	1	1	1	1	w'
x'					w
	z	z'	z		

Table 12 Karnaugh map of $xyz + xy'zw + x'y'z'w' + x'yw' + y'zw'$

$$x'w' + xz$$

d)

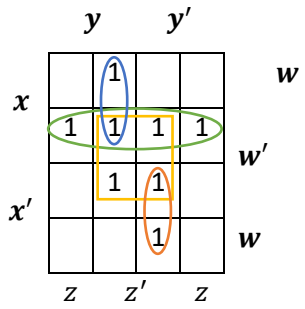

Table 13 Karnaugh map of $xzw' + xyz' + y'z'w' + x'y'z' + yz'w'$

$$xw' + z'w' + xyz' + x'y'z'$$

Bonus Problem: (5 points)

a) $f(x, y) = xy + x'y'$

b) $f(x_2, x_1, x_0, y_2, y_1, y_0) = (x_2y_2 + x_2'y_2') \cdot (x_1y_1 + x_1'y_1') \cdot (x_0y_0 + x_0'y_0')$ because each pair of the bits needs to agree.