Homework 1 Solution

Problem 1

a) $7 \in A$, $6 \notin B$, $10 \notin B$, $4 \notin A$, $b \in C$, $b \notin B$, $\{2,5,12\} \subseteq A$, $\{3,5,6\} \not\subseteq B$, $\{a,b,5\} \subseteq C$

b) $A \cup B = \{1,2,3,5,6,7,9,10,11,12\}$
$A \cap B = \{5,7\}$
$A - B = \{1,2,6,10,11,12\}$
$A + B = \{1,2,3,6,9,10,11,12\}$
$2^{A \cap B} = \{\emptyset, \{5\}, \{7\}, \{5,7\}\}$
$(A \cap B) \times \{a,b\} = \{(5,a),(5,b),(7,a),(7,b)\}$

c) $A \cup (B \cap C) = \{1,2,5,6,7,10,11,12\}$
$A \cap (B \cup C) = \{5,6,7\}$
$A - (B \cup C) = \{1,2,10,11,12\}$
$(A - B) - C = \{1,2,10,11,12\}$
$(A + B) + C = \{1,2,3,4,5,9,10,11,12,a,b,c\}$
$A + (B + C) = \{1,2,3,4,5,9,10,11,12,a,b,c\}$

d) $|A \cup B| = 10$, $|A \cap B| = 2$, $|A - B| = 6$, $|A + B| = 8$, $|2^{A \cap B}| = 4$, $|(A \cap B) \times \{a,b\}| = 4$, $|A \cup (B \cap C)| = 8$, $|A \cap (B \cup C)| = 3$, $|A - (B \cup C)| = 5$, $|(A - B) - C| = 5$, $|(A + B) + C| = 12$, $|A + (B + C)| = 12$

Problem 2

a) $A_{50} \cap I_{100,000}$: The set of all Americans who are at most 50 years old with individual annual income less than \$100,000.
$A_{30} \cap \overline{I}_{1,000,000}$: The set of all Americans who are at most 30 years old with individual annual income at least \$1,000,000
$\overline{A}_{17} \cup A_{65}$: The set of all Americans.
$|B \cap \overline{I}_{100,000}| > |H \cap \overline{I}_{100,000}|$: Among all Americans whose individual annual income is at least \$100,000, there are more people whose highest education is a Bachelor's degree than those whose highest education is a High school degree.
$|B \cap \overline{I}_{100,000}| > |B \cap I_{100,000}|$: Among Americans whose highest education is a Bachelor's degree, more people make more than \$100,00 annually.
$(H \cap (A_{50} - A_{40})) \cup (B \cap (A_{30} - \overline{A}_{20}))$: The set of all Americans between

age 40 and 50 with highest education of High school degree and Americans between age 20 and 30 with highest education of Bachelor's degree.

b) $\overline{A}_{17} \cap B \cap \overline{I}_{10,000,000}$
$0.80|H| < |H \cap I_{50,000}|$
$|\overline{A}_{50} \cap \overline{I}_{1,000,000,000}| > |A_{50} \cap \overline{I}_{1,000,000,000}|$
$(A_{49} - A_{17}) \cap (I_{1,000,001} - I_{1,000,000})$

Problem 3

a) True	b) False	c) False	d) True	e) False
f) True	g) True	h) True	i) False	j) True

Problem 4

a) At any time t, any person alive is/was/will be a musician.

b) There will be at time in the future, every person alive then will be a musician.

c) There was a time t, any person alive then was never a musician.

d) There will be a time in the future, when no person alive ahs any children alive.

e) There will be a time in the future, when every person alive has a living child or still has a parent alive.

f) Human will be extinct in the future.

g) Alice is alive and has a grandchild alive.

h) Alice is alive but her parents are dead.

i) Musician Elvis is no longer alive. There was a time when he was not a musician.

j) Everyone is born at some point in time.

Problem 5

a) $(\forall t)(\exists x \in A_t)(\exists y \in A_t)(M_t(x) \wedge \neg M_t(y))$

b) $(John \in A_n) \wedge (\exists x \in A_n, \exists y \in A_n, x \neq y)(P(John, x) \wedge P(John, y))$

c) $(Rebecca \in A_{1960}) \wedge (\forall t < 1960)(Rebecca \notin A_t)$

d) $(\exists t > n)(\forall y \in A_t)(\exists x \in A_t)(P(x,y) \wedge M_t(x))$

e) $(\forall t)(\forall x \in A_t)(\exists t^{'} < t)(x \notin A_{t'})(\neg M_t(x))$

f) $(\forall t)(\forall x \in A_t)(\exists y \in A_t, \exists z \in A_t, y \neq z)(P(y,x) \wedge P(z,x))$

g) $(Dan \in A_n) \wedge ((\exists t < n)(\exists x \in A_t)(P(Dan,x))) \wedge ((\forall y \in A_n)(\neg P(Dan,y)))$

h) $|A_n| = 7,000,000,000$

i) $(\exists t < n)(\exists t^{'} < t)(|A_t| < |A_{t'}|)$

j) $(\forall t \geq n)(\forall t^{'} > t)(|A_{t'}| > |A_t|)$

Bonus Problem

a) $(\forall t)(\forall x \in A_t)(\forall t^{'} > t)(M_t(x) \Rightarrow M_{t'}(x))$

b) There is a mistake in this part, therefore this part is removed from the problem.

c) $(\exists t > n)(\forall x \in A_t)(x \in A_{t+100} \Rightarrow x \in A_{t+150})$