CS1311 March 7, 2019
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Midterm
TIME: 75 minutes

Problem 1: (25 points)
Let $\mathbb{R}$ be the set of real numbers, and N the set of integers.

a) Let f be a function from $\mathbb{R}$ to $\mathbb{R}$ where $f(x) = 10x + 35$ for all x. Prove that f is one-to-one and onto, and find the inverse function f^{-1}.
b) Let g be a function from $\mathbb{N}$ to $\mathbb{R}$ where $g(x) = 4\sqrt{x} - 7$ for all x. Is g one-to-one? Onto? Prove your answer.

Problem 2: (25 points)
a) Let a and b be two positive real numbers. Prove by induction on n that
$$(\forall\, n \in \mathbb{N})((a+b)^n \geq a^n + b^n).$$
b) Prove by induction on positive integer n that
$$1 \times 3 + 2 \times 4 + 3 \times 5 + \cdots + n(n+2) = \frac{n(n+1)(2n+7)}{6}.$$
c) Prove that for every positive integer n, $n(n+1)(2n+7)$ is divisible by 6.

Problem 3: (25 points)
a) Let $x_0 = 0$ and $x_n = 2x_{n-1} + n$ for all $n \geq 1$. Solve this recurrence relation to find the value of x_n in terms of n.
b) Let $x_0 = 0, x_1 = 4,$ and $x_n = 2x_{n-1} + x_{n-2}$ for all $n \geq 2$. Solve this recurrence relation to find the value of x_n in terms of n.

Problem 4: (25 points)
a) In how many ways can you arrange 8 people in a waiting line?
b) Same as (a) except this time the 8 people are 5 women and 3 men and all the women are ahead of the men?
c) Same as (a) except this time the 8 people are 5 women and 3 men and no two men are next to each other.
d) How many 7-letter strings are there where 3 of the letters are a's, 2 are b's, and 2 are c's?
e) A coin is tossed 8 times. Each outcome is a sequence of 8 heads and/or tails. What is the number of possible outcomes where the number of heads is higher than the number of tails?