CS 1311 April 11, 2019

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Homework 6

Due Date: April 25, 2019

Problem 1: (20 points)

Let (B,+, •,′, 0, 1) be a Boolean algebra. Define the following operation$\oplus$: $x \oplus y = xy + x'y'$.

a) Give the truth table of $\oplus$.

b) Evaluate $x \oplus x$, $x \oplus x'$, $x \oplus 0$, and $x \oplus 1$. Your answers should be x, 0, 1, or x'.

c) Is $\oplus$ commutative? Associative? Distributive over +? Distributive over •? Prove your answers

Problem 2: (20 points)

a) In a competition judged by a panel of 4, after each player completes his/her presentation, each judge enters his/her vote of yes or no (yes=1, no=0) into a machine. The machine tallies the votes, and returns 1 (that is, *pass*) if the 1st judge, 4th judge, and at least one more judge vote yes. Otherwise, the machine returns 0 (for *fail*). Express the working of the machine as a Boolean function of four variables (the 4 judges' votes).

b) Same as (a) except that this time the machine returns 1 if at least one judge votes yes.

c) Same as (a) except this time we are interested to see if the 1st and 4th judges disagree and the 2nd and 3rd judges disagree. If so, the machine returns 1; otherwise, it returns 0. Express the behavior of the machine as a Boolean function of four variables.

Problem 3: (20 points)

Let B = {0, 1} be a Boolean algebra and let$f$: $B^3 \rightarrow$ B be a Boolean function such that $f(x, y, z) = 1$ if an odd number of variables have the value 1; otherwise, $f(x, y, z) = 0$.

a) Give the truth table of f and f'(the complement of f).

b) Write f in disjunctive normal form andf' in conjunctive normal form.

c) Write$f$ in conjunctive normal form and f' in disjunctive normal form.

Problem 4: (20 points)

For each of the following Boolean expressions, give the truth table, and put the expression in DNF and CNF.

a) xyz + x'z + x'yz'

b) yz + xz' + x'z'

c) xyz + xy'zw + x'y'z'w' + x'yw' + y'zw'

d) xzw' + xyz' + y'z'w' + x'y'z' + yz'w'

Problem 5: (20 points)

Minimize each of the expressions of problem 4 using Karnaugh maps. Show the Karnaugh maps.

Bonus Problem: (5 points)

a) Let x and y be two Boolean variables (i.e., each can be 0 or 1 only). Also, let $f(x, y) = 1$ if $x = y$, and $f(x, y) = 0$ otherwise. Express f as a Boolean expression in terms of x and y.

b) Note that every integer 0 through 7 can be represented with a 3-bit string (0 as 000, 1 as 001, 2 as 010, 3 as 011, 4 as 100, 5 as 101, 6 as 110, and 7 as 111). Let x and y be two integers between 0 and 7 inclusive, expressed in binary as $x_2x_1x_0$ and $y_2y_1y_0$, respectively. Finally, let f be a function where $f(x, y) = 1$ if $x = y$; otherwise, $f(x, y) = 0$. Express f as a Boolean expression in terms of the 6 bits x_2, x_1, x_0, y_2, y_1, and y_0, where each bit is treated as a Boolean value. Show your reasoning.