CS 1311 February 14, 2019
Youssef Homework 3
Due Date: February 28, 2019

Problem 1: (15 points)
Solve the following recurrence relations (that is, compute x_n in terms of n alone):
a) $x_0 = 1$, $x_n = 6x_{n-1} + 10$ for $n \geq 1$
b) $x_0 = 1$, $x_1 = 3$, $x_n = 6x_{n-1} - 8x_{n-2}$ for $n \geq 2$
c) $x_0 = 4$, $x_1 = 10$, $x_n = 7x_{n-1} - 12x_{n-2} + 18$ for $n \geq 2$

Problem 2: (15 points)
Solve the following recurrence relations (that is, compute x_n in terms of n alone):
a) $x_0 = -3$, $x_n = 6x_{n-1} + 10n + 2$ for $n \geq 1$
b) $x_0 = 9$, $x_1 = 15$, $x_n = 7x_{n-1} - 12x_{n-2} + 18n + 3$ for $n \geq 2$

Problem 3: (20 points)
a) In how many different orders can 7 people line up in one single line for some service ?
b) In how many ways can you choose 7 people out of 10 to form a 7-member committee?
c) In how many different orders can 30 runners finish a race if 5 people tie for the first place, 4 people tie for the second place, 3 people tie for the third place, 2 people tie for the fourth place, and no ties otherwise?
d) Suppose that in a state, all car license plates consist of 6 characters where each character can be a capital letter or a decimal digit, such that each license plate number starts with two letters and has at least one digit. How many different license plates are possible?
e) In how many different orders can n cars line up in a gas station of 2 parallel gas pumps? (Hint: think of the 2 lines as one single combined line with one identical dummy car between the 1st and 2nd original lines.)

Problem 4: (20 points)
a) A die is thrown 10 times, where the die has 6 faces labeled 1, 2, …, 6. Each outcome will be a sequence of 10 faces (i.e., 10 digits), where each face is one of the 6 values.
   i. What is the number of possible outcomes where face 4 comes up exactly 5 times?
   ii. What is the number of possible outcomes where face 3 comes up an even number of times?
   iii. What is the number of possible outcomes where each face is even
b) A coin is tossed 10 times. Each outcome is a sequence of 10 heads and/or tails.
   i. What is the number of possible outcomes where the number of tails is exactly 4? At most 4? At least 4?
   ii. What is the number of possible outcomes where the number of heads is odd?
   iii. What is the number of possible outcomes where we get as many heads as tails?
c) One urn contains 25 balls: 8 balls are red (labeled $R_1, R_2, \ldots, R_8$), 5 are blue (labeled $B_1, B_2, \ldots, B_5$), and 12 are white (labeled $W_1, W_2, \ldots, W_{12}$). You draw 10 balls from the urn. What is the number of possible outcomes where 5 of the drawn balls are red, 2 are blue, and 3 are white?

Problem 5: (10 points)
a) A trousers vendor carries 10 different styles, where each style comes in 6 different color and 8 different sizes. How many different kinds of trousers does this vendor carry?
b) How many positive integers less than 300 are divisible by 8? By 12? By 8 and 12? By 8 or 12? By neither 8 nor 12?

Problem 6: (20 points)

a) Let n be a positive even integer. Consider the following algorithm segment:

```
ft= n/3;
st=2*ft;
X=n;
for i=1 to n {
    m=i % 3;
    if (m == 0)
        for j=1 to ft
                X=X+5;
    else if (m == 1)
        for j=ft+1 to st
                X=X-1;
    else
        for j= st+1 to n
                X=X+j*j;
}
```

Give the number of additions (+), number of subtractions (-), and number of "mod" operations (%) performed by this code segment. Prove your answers. Note that your answers are expressions in n. (Note: you can assume that n is a multiple of 3)

b) Consider the code of this function, Compute:

```
int Compute( int n){   // n is a non-negative integer
        if (n==0){
                return (1);
        }
        else{
                int a=Compute(n-1);
                int x=0;
                int m = n*n;
                for i=1 to m
                        x=x/i + a*i*i;
                return (x);
        }
}
```

Let T(n) be the number of operations (+, * and /) performed by Compute(n). You can rename T(n) as x_n. Derive a recurrence relation for x_n, and solve it to determine the value of T(n).

Bonus Problem: (5 points)

We have one die and one coin. We will run the following experiment: First we throw the die; we take the value of the resulting face (call that value k), and throw the coin k times in a row. The resulting sequence of k heads/tails is called the *outcome* of the experiment.

What is the number of possible outcomes where we have exactly one head? Exactly two heads? As many heads as tails?