CS1311 January 15, 2019
Youssef

Homework 1
Due Date: January 31, 2019

Problem 1: (20 points)
Let A = {1, 2, 5, 6, 7, 10, 11, 12}, B = {x | x is an odd integer and $3 \le x \le 10$}, and C = {a, b, c, 4, 5, 6}.

a) Fill in the blanks with the most appropriate symbol ($\in$, $\notin$, $\subseteq$, $\nsubseteq$):
7 … A, 6 … B, 10 ... B, 4... A, b ... C, b ... B, {2, 5, 12} ... A, {3, 5, 6} ... B, {a, b, 5} ... C.
b) Evaluate: $A \cup B$, $A \cap B$, $A - B$, $A + B$, $2^{A \cap B}$, $(A \cap B) \times \{a, b\}$.
c) Evaluate: $A \cup (B \cap C)$, $A \cap (B \cup C)$, $A - (B \cup C)$, $(A - B) - C$, $(A + B) + C$, $A + (B + C)$.
d) Find the cardinality of each set in (b) and (c).

Problem 2: (20 points)
Suppose the universal set U is the set of all living Americans. Let $A_{n\ be}$ the set of all Americans who are at most n years old (for any non-negative integer n), I_m the set of all Americans whose individual annual income is < m dollars (for any positive integer m), B is the set of all Americans whose highest education is a Bachelor's degree, and H is the set of all Americans whose highest education is a high school degree. It is assumed in this problem that all people in question are currently living without explicitly stating so, that a minor is not 18 years old yet, and an adult is anyone who is at least 18 years old.

a) Describe in understandable English prose the people in each of the following sets/expressions:
$A_{50} \cap I_{100,000}$; $A_{30} \cap \bar{I}_{1,000,000}$; $\bar{A}_{17} \cup A_{65}$; $|B \cap \bar{I}_{100,000}| > |H \cap \bar{I}_{100,000}|$;
$|B \cap \bar{I}_{100,000}| > |B \cap I_{100,000}|$; $(H \cap (A_{50} - A_{40})) \cup (B \cap (A_{30} \cap \bar{A}_{20}))$;
b) Express each of the following sets/statements in terms of U, B, H, A_n, I_m, the set operations ($\cap$,$\cup$,$-$, and complementation), set cardinality, numbers, number operations, and the comparison relations ($=, \neq, <, >, \le, \ge$)
   - The set of American adults whose highest education is a Bachelor's degree and make at least \$10M per year
   - Among Americans whose highest education is a high school degree, more than 80% make less than \$50K per year.
   - There are more American billionaires over 50 than American billionaires under 50.
   - The set of American adults under 50 who make exactly \$1M per year.

Problem 3: (20 points)
Let the proposition G(x) stand for "x believes in God", S(x) stand for "x is a scientist", P(x,y) stand for "x is a parent of y", where the universe of discourse U is the set of all living people. For each of the following propositions, state if it is True or False:

a) S(Peter Higgs)
b) S(Bill Clinton)
c) $(\forall x \text{ in } U)\ G(x)$
d) $(\exists x \text{ in } U)\ G(x)$
e) $(\forall x \text{ in } U)\ S(x)$
f) $(\exists x \text{ in } U)\ (S(x) \land G(x))$
g) $(\exists x \text{ in } U)\ (\neg S(x) \land \neg G(x))$
h) $(\exists x \text{ in } U)\ (G(x) \lor S(x))$
i) $(\forall x \text{ in } U)\ (\exists y \text{ in } U)\ (P(x,y))$
j) $(\exists x \text{ in } U)\ (\exists y \text{ in } U)\ (P(x,y) \land \neg G(x) \land G(y))$

Problem 4: (20 points)
Let A_t be the set of all people alive at time t, and let n stand for the present time (i.e., **n**ow); thus, A_n is the set of all people currently alive. Let $M_t(x)$ stand for "at time t, person x is/was/will be a musician", and let P be as in Problem 3. Express each of the following statements in easily understandable English prose:

a) $(\forall t)(\forall x \in A_t)\ M_t(x)$
b) $(\exists t > n)(\forall x \in A_t)\ (M_t(x))$
c) $(\exists t < n)(\forall x \in A_t)\ (\neg M_t(x))$
d) $(\exists t > n)(\forall x \in A_t)(\forall y \in A_t)\ (\neg P(x,y))$
e) $(\exists t > n)(\forall x \in A_t)\ ((\exists y \in A_t)\ P(x,y) \vee (\exists y \in A_t)\ P(y,x))$
f) $(\exists t > n)(A_t = \emptyset)$
g) $(\text{Alice} \in A_n) \wedge (\exists t \leq n)(\exists x \in A_t)(\exists y \in A_n)(P(\text{Alice},x) \wedge P(x,y))$
h) $(\text{Alice} \in A_n) \wedge (\forall x \in A_n)\ (\neg P(x, \text{Alice}))$
i) $(\text{Elvis} \notin A_n) \wedge ((\exists t<n)(\text{Elvis} \in A_t \wedge M_t(\text{Elvis})) \wedge ((\exists t'<t)(\text{Elvis} \in A_{t'} \wedge \neg M_{t'}(\text{Elvis}))$
j) $(\forall t)(\forall x \in A_t)\ (\exists t'<t)(x \notin A_{t'})$

Problem 5: (20 points)
Express each of the following sentences as a predicate using the notations of Problem 4, as well as logical quantifiers ($\exists$ and $\forall$), logical connectives ($\wedge$, $\vee$, and $\neg$), $\in$, $\notin$, $\emptyset$, set-cardinality, integers, numerical relations ($<$, $\leq$, $=$, and $\neq$), variables (like x, y, z), parentheses, and nothing else:

a) In every generation, there are musicians and there are non-musicians
b) John is alive and has at least two living children
c) Rebecca was born in 1960
d) There will be a time where every person has a parent who is a musician
e) Nobody is born a musician
f) Everyone has at least two parents
g) Dan had children but they are all dead now
h) The world population now is 7 billion
i) There was a time when the world population decreased
j) From now on, the world population will always increase

Bonus Problem: (5 points)
Using the same symbols as in problem 5, as well as implication (=>) and addition (+), express each of the following sentences as a predicate or a proposition:

a) Once a musician, always a musician
b) Mary lived from 1950 to 2010, and, between 1900 and 2000, she was a musician every other year
c) There will come a time when if a person survives to 100 years of age, s/he will live at least another 50 years.