CS1311 **14th Apr 2016**

Homework 6 Solutions

Problem 1:

a)

- $x \,\bar{\vee}\, x = (x + x)' = x'$ because $x + x = x$ by the idempotence laws
- $x \,\bar{\vee}\, 1 = (x + 1)' = 1' = 0$ because $x + 1 = 1$ by the absorption laws.
- $x \,\bar{\vee}\, 0 = (x + 0)' = x'$ because $x + 0 = 0$ by the identity laws.

b)

- $x + y = ((x + y)')' = (x \,\bar{\vee}\, y)' = (x \,\bar{\vee}\, y) \,\bar{\vee}\, 0$ (the 1st equality comes from (z')'=z, the 2nd equality comes from the definition of $\bar{\vee}$, and the last comes from the 3rd equality of part (a) above)
- $x \cdot y = (x' + y')' = x' \,\bar{\vee}\, y' = (x \,\bar{\vee}\, 0) \,\bar{\vee}\, (y \,\bar{\vee}\, 0)$ (the 1st equality comes from De Morgan's laws, the 2nd comes from the definition of $\bar{\vee}$, and the last comes from the third equality of part (a) above)
- $x' = (x \,\bar{\vee}\, 0)$

Problem 2:

a)

x	y	z	w	$f(x, y, z, w)$	DNF Factors
0	0	0	0	0	
0	0	0	1	0	
0	0	1	0	0	
0	0	1	1	1	$\Rightarrow x'y'zw$
0	1	0	0	0	
0	1	0	1	1	$\Rightarrow x'yz'w$
0	1	1	0	1	$\Rightarrow x'yzw'$
0	1	1	1	1	$\Rightarrow x'yzw$
1	0	0	0	0	
1	0	0	1	1	$\Rightarrow xy'z'w$
1	0	1	0	1	$\Rightarrow xy'zw'$
1	0	1	1	1	$\Rightarrow xy'zw$
1	1	0	0	1	$\Rightarrow xyz'w'$
1	1	0	1	1	$\Rightarrow xyz'w$
1	1	1	0	1	$\Rightarrow xyzw'$
1	1	1	1	1	$\Rightarrow xyzw$

$$f = x'y'zw + x'yz'w + x'yzw' + x'yzw + xy'z'w + xy'zw' + xy'zw + xyz'w' + xyz'w + xyzw' + xyzw$$

b)

x	y	z	w	$g(x,y,z,w)$	DNF Factors
0	0	0	0	1	$\Longrightarrow x'y'z'w'$
0	0	0	1	1	$\Longrightarrow x'y'z'w$
0	0	1	0	1	$\Longrightarrow x'y'zw'$
0	0	1	1	0	
0	1	0	0	1	$\Longrightarrow x'yz'w'$
0	1	0	1	0	
0	1	1	0	0	
0	1	1	1	1	$\Longrightarrow x'yzw$
1	0	0	0	1	$\Longrightarrow xy'z'w'$
1	0	0	1	0	
1	0	1	0	0	
1	0	1	1	1	$\Longrightarrow xy'zw$
1	1	0	0	0	
1	1	0	1	1	$\Longrightarrow xyz'w$
1	1	1	0	1	$\Longrightarrow xyzw'$
1	1	1	1	1	$\Longrightarrow xyzw$

$$g = x'y'z'w' + x'y'z'w + x'y'zw' + x'yz'w' + x'yzw + xy'z'w' + xy'zw + xyz'w + xyzw' + xyzw$$

c)

x	y	z	w	$h(x,y,z,w)$	DNF Factors
0	0	0	0	1	$\Longrightarrow x'y'z'w'$
0	0	0	1	0	
0	0	1	0	0	
0	0	1	1	0	
0	1	0	0	0	
0	1	0	1	0	
0	1	1	0	0	
0	1	1	1	0	
1	0	0	0	0	
1	0	0	1	0	
1	0	1	0	0	
1	0	1	1	0	
1	1	0	0	0	
1	1	0	1	0	
1	1	1	0	0	
1	1	1	1	1	$\Longrightarrow xyzw$

$$h = x'y'z'w' + xyzw$$

Problem 3:

a)

x	y	z	f	f'
0	0	0	0	1
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	0

b)

$DNF\ of\ f = x'yz + xy'z + xyz' + xyz$

$CNF\ of\ f' = (x + y' + z') \cdot (x' + y + z') \cdot (x' + y' + z) \cdot (x' + y' + z')$

c)

$DNF\ of\ f' = x'y'z' + x'y'z + x'yz' + xy'z'$

$CNF\ of\ f = (x + y + z) \cdot (x + y + z') \cdot (x + y' + z) \cdot (x' + y + z)$

Problem 4:

a)

x	y	z	f
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

	y		y'	
x		1	1	1
x'			1	
	z	z'		z

$f = xz' + xy' + y'z'$

b)

x	y	z	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

	y		y'	
x	1	1	1	1
x'	1			1
	z	z'		z

$f = x + z$

c)

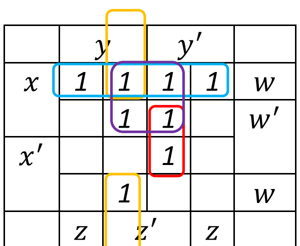
x	y	z	w	f
0	0	0	0	1
0	0	0	1	1
0	0	1	0	0
0	0	1	1	0
0	1	0	0	1
0	1	0	1	1
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	1
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	0
1	1	1	1	1

	y		y'		
x	1	1	1	1	w
					w'
x'		1	1		
		1	1		w
	z	z'		z	

$f = xw + x'z'$

d)

x	y	z	w	f
0	0	0	0	1
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	0
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	0
1	1	1	1	1



$f = xw + xz' + y'z'w' + yz'w$

Problem 5:

a)

x	y	z	w	f	g	h
0	0	0	0	1	0	1
0	0	0	1	0	1	1
0	0	1	0	0	1	0
0	0	1	1	1	0	1
0	1	0	0	0	1	0
0	1	0	1	1	0	1
0	1	1	0	1	0	1
0	1	1	1	0	1	1
1	0	0	0	0	1	1
1	0	0	1	1	0	1
1	0	1	0	1	0	1
1	0	1	1	0	1	0

1	1	0	0	1	0	1
1	1	0	1	0	1	0
1	1	1	0	0	1	1
1	1	1	1	1	0	1

b)

$$DNF\ of\ f = x'y'z'w' + x'y'zw + x'yz'w + x'yzw' + xy'z'w + xy'zw' + xyz'w' + xyzw$$

$$CNF\ of\ f = (x + y + z + w') \cdot (x' + y + z' + w) \cdot (x + y' + z + w) \cdot (x + y' + z' + w') \cdot (x' + y + z + w) \cdot (x' + y + z' + w') \cdot (x' + y' + z + w)(x' + y' + z' + w')$$

$$DNF\ of\ g = x'y'z'w + x'y'zw' + x'yz'w' + x'yzw + xy'z'w' + xy'zw + xyz'w + xyzw'$$

$$CNF\ of\ g = (x + y + z + w) \cdot (x + y + z' + w') \cdot (x + y' + z + w') \cdot (x + y' + z' + w) \cdot (x' + y + z + w') \cdot (x' + y + z' + w) \cdot (x' + y' + z + w) \cdot (x' + y' + z' + w')$$

$$DNF\ of\ h = x'y'z'w' + x'y'z'w + x'y'zw + x'yz'w + x'yzw' + x'yzw + xy'z'w' + xy'z'w + xy'zw' + xyz'w' + xyzw' + xyzw$$

$$CNF\ of\ h = (x + y + z' + w) \cdot (x + y' + z + w) \cdot (x' + y + z' + w') \cdot (x' + y' + z + w')$$

c)

f	y		y'		
x	1		1		w
		1		1	w'
x'	1		1		
		1		1	w
	z	z'		z	

$$f = x'y'z'w' + x'y'zw + x'yz'w + x'yzw' + xy'z'w + xy'zw' + xyz'w' + xyzw$$

g	y		y'		
x		1		1	w
	1		1		w'
x'		1		1	
	1		1		w
	z	z'		z	

$$g = x'y'z'w + x'y'zw' + x'yz'w' + x'yzw + xy'z'w' + xy'zw + xyz'w + xyzw'$$

h	y		y'		
x	1		1		w
	1	1	1	1	w'
x'	1		1		
	1	1	1	1	w
	z	z'		z	

$h = yz + y'z' + xw' + x'w$

Bonus Problem:

X	y	z	w	f
0	0	0	0	1
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	1
0	1	0	1	1
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	0
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

	y		y'		
x	1	1	1		w
	1	1	1	1	w'
x'		1	1		
		1			w
	z	z'		z	

$f = xy + xz' + xw' + yz' + z'w'$