

Homework 1 Solution**Problem 1**

- a) $12 \in A$, $8 \notin B$, $6 \in B$, $10 \notin A$, $b \in C$, $b \notin B$, $\{3,5,11\} \subseteq A$, $\{1,3,6\} \not\subseteq B$,
 $\{5,6,10,11\} \subseteq C$
- b) $A \cup B = \{1,3,5,6,7,8,9,11,12,13\}$
 $A \cap B = \{3,6,12\}$
 $A - B = \{1,5,7,8,11,13\}$
 $A + B = \{1,5,7,8,9,11,13\}$
 $2^{A \cap B} = \{\emptyset, \{3\}, \{6\}, \{12\}, \{3,6\}, \{3,12\}, \{6,12\}, \{3,6,12\}\}$
 $(A \cap B) \times \{a, b\} = \{(3, a), (3, b), (6, a), (6, b), (12, a), (12, b)\}$
- c) $A \cup (B \cap C) = \{1,3,5,6,7,8,9,11,12,13\}$
 $A - (B \cap C) = \{1,3,5,7,8,11,12,13\}$
 $(A - B) - C = \{1,7,8,13\}$
 $(A + B) + C = \{a, b, 1,6,7,8,10,13\}$
 $A + (B + C) = \{a, b, 1,6,7,8,10,13\}$
- d) $|A \cup B| = 10$, $|A \cap B| = 3$, $|A - B| = 6$, $|A + B| = 7$, $|2^{A \cap B}| = 8$, $|(A \cap B) \times \{a, b\}| = 6$,
 $|A \cup (B \cap C)| = 10$, $|A - (B \cap C)| = 8$, $|(A - B) - C| = 4$,
 $|(A + B) + C| = 8$, $|A + (B + C)| = 8$

Problem 2

- a) $A \cup B$: The set of currently registered GW undergraduates who, in Spring 2016, are majoring in CS, or minoring in Economics, or both.
 $A \cap B$: The set of currently registered GW undergraduates who, in Spring 2016, are majoring in CS, and minoring in Economics.
 $A - B$: The set of currently registered GW undergraduates who, in Spring 2016, are majoring in CS, but not minoring in Economics.
 $A + B$: The set of currently registered GW undergraduates who, in Spring 2016, are either majoring in CS, or minoring in Economics.
 $A \cap C$: The set of currently registered GW undergraduates who, in Spring 2016, are majoring in CS, and taking CS 1311.
 $A + C$: The set of currently registered GW undergraduates who, in Spring 2016, are either majoring in CS, or taking CS 1311.
 $C - A$: The set of currently registered GW undergraduates who, in Spring 2016, are taking CS 1311, but not majoring in CS.
 $(A + B) \cap C$: The set of currently registered GW undergraduates who, in Spring 2016, are either majoring CS and taking CS 1311, or minoring Economics and taking CS 1311.

$A - (B \cup C)$: The set of currently registered GW undergraduates who, in Spring 2016, are majoring in CS, but neither minoring in Economics nor taking CS 1311.

b) $A \cap B \cap C, (A + B) - C, G - (A \cup B \cup C), C - (A \cup B)$

Problem 3

- | | |
|------|------|
| a) T | h) F |
| b) T | i) T |
| c) F | j) T |
| d) F | k) F |
| e) T | l) T |
| f) F | m) T |
| g) T | |

Problem 4

- a) There exists an American citizen, who is a movie director.
- b) Every American citizen has directed at least one movie.
- c) There exists an American citizen, who has directed all movies ever made.
- d) There exists a movie, which was directed by all American citizens.
- e) Every movie ever made was directed by at least one American citizen.
- f) There exists an American movie director, at least one movie was not directed by whom.
- g) There exists an American movie director, who hasn't directed any movie.

Problem 5

- a) $P(\text{Steven Spielberg})$
- b) $(\exists x \in A)(\neg P(x))$
- c) $(\exists x \in A)(\neg P(x))$
- d) $(\exists x \in A)(\exists y \in M)(\exists z \in M - \{y\})(P(x) \wedge Q(x, y) \wedge Q(x, z))$
- e) $(\exists x \in A)(\forall y \in M)(P(x) \wedge Q(x, y))$
- f) $(\exists x \in A)(\forall y \in M)(\neg Q(x, y))$
- g) $(\exists x \in M)(\forall y \in A)(\neg Q(y, x))$

Bonus Problem

- a) $(\forall x \in M)(\exists y \in A)(\exists z \in M - \{x\})(Q(y, x) \wedge Q(y, z))$
- b) $(\forall x \in A)(\exists y \in M)(Q(x, y) \Rightarrow P(x))$