CS1311 March 10, 2016
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Midterm
TIME: 75 minutes

Problem 1: (25 points)
Let **R** be the set of real numbers.
a) Let f be a function from **R** to **R** where $f(x) = 10x - 30$ for all x. Prove that f is one-to-one and onto, and find the inverse function f^{-1}.
b) Let g be a function from **R** to **R** where $g(x) = x^2 + 1$ for all x. Prove that g is neither one-to-one nor onto.

Problem 2: (25 points)
a) Is $3^n + 2$ a prime for every non-negative integer *n*? Prove your answer.
b) Consider the following function $f(n)$ of whole numbers, defined recursively: $f(0) = 2$ and $f(n) = 3f(n-1) + 2$ for all $n \geq 1$. Prove by induction on *n* that $f(n) = 3^{n+1} - 1$ for all non-negative integers *n*.

Problem 3: (25 points)
a) Let $x_0 = 1$ and $x_n = 6x_{n-1} + 2n - 3$ for all $n \geq 1$. Solve this recurrence relation to find the value of x_n in terms of *n*.
b) Let $x_0 = 0, x_1 = 1,$ and $x_n = 7x_{n-1} - 10x_{n-2} + 4$ for all $n \geq 2$. Solve this recurrence relation to find the value of x_n in terms of *n*.

Problem 4: (25 points)
a) In how many ways can you select a committee of 5 people from a group of 12 people?
b) In how many ways can 8 runners finish a race if 2 runners will tie for 1st place, 3 will tie for 2nd place, and the remaining three runners will not tie with anybody.
c) How many positive integers ≤400 are divisible by 4? By 10? By 4 and 10? By 4 or 10? By neither 4 nor 10?