CS 1311 April 7, 2016

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Homework 6

Due Date: April 21, 2016

Problem 1: (20 points)

Let (B,+, •,’ , 0, 1) be a Boolean algebra. Define the following operation $\bar{\nabla}$ (called NOR):

$$x \mathbin{\bar{\nabla}} y = (x+y)'.$$

a) Evaluate $x \mathbin{\bar{\nabla}} x$, $x \mathbin{\bar{\nabla}} 1$, $x \mathbin{\bar{\nabla}} 0$, all in terms of x, 0, 1, and ’.

b) Express x+y and x•y and x’ using the operation $\bar{\nabla}$ but without using +, • , or ’.

Problem 2: (20 points)

a) Suppose that there is a panel of 4 judges for some game, and that after each player completes his/her presentation, each judge enters his/her vote of yes or no (yes=1, no=0) into a machine. The machine tallies the votes, and returns 1 (that is, *pass*) if at least two judges vote yes. Otherwise, the machine returns 0 (for *fail*). Express the working of the machine as a Boolean function of four variables (the 4 judges’ votes).

b) Assume there is a contest where each contestant gives a speech, and there are 4 judges that vote “*good*” or “*bad*” for each contestant. The interest is in whether the contestant is non-controversial, that is, if the four votes do not split 50-50. Therefore, a machine tallies the votes, and returns 1 if the contestant is found non-controversial; otherwise, the machine returns 0. Express the behavior of the machine as a Boolean function of four variables, where a good-vote=1 and a bad-vote=0.

c) Same problem as in (b) except that here the machine is supposed to tell if the judges are unanimous in their votes, in which case the machine returns 1; otherwise, the machine returns 0. Express the behavior of the machine as a Boolean function of four variables, where a good-vote=1 and a bad-vote=0.

Problem 3: (20 points)

Let B = {0, 1} be a Boolean algebra and let f: $B^3 \rightarrow B$ be a Boolean function such that f(1, 1, 0) = f(1, 0, 1) = f(0, 1, 1) = f(1,1,1)=1 and f(x, y, z) = 0 for all other (x, y, z) in B^3.

a) Give the truth table of f and f’ (the complement of f).

b) Write f in disjunctive normal form and f’ in conjunctive normal form.

c) Write f in conjunctive normal form and f’ in disjunctive normal form.

Problem 4: (20 points)

Minimize each of the following Boolean expressions using Karnaugh maps. Show the Karnaugh maps.

a) x’y’z’ + xz’ + xy’z

b) xz’ + yz + y’z

c) x’y’z’ + x’yz’w’ + xyzw + xy’w + yz’w

d) xzw + yz’w + xy’z’ + y’z’w’ + xyz’

Problem 5: (20 points)

Let f, g, and h be three Boolean functions of four variables each, where f(x, y, z, w) = 1 if and only if the binary string xyzw has an even number of 0s; g(x, y, z, w) = 1 if and only if exactly three of the four variables are equal, and h(x, y, z, w) = 1 if and only if the leftmost bit and rightmost bit of xyzw are different or the two middle bits of xyzw are equal.

a) Give the truth tables of f, g, and h.
b) Express f, g and h in DNF and CNF.
c) Minimize f, g, and h using Karnaugh maps.

Bonus Problem: (5 points)
Note that every integer 0 through 3 can be represented with a 2-bit string (0 as 00, 1 as 01, 2 as 10, and 3 as 11). Let xy and zw be a two 2-bit binary strings. We say that xy $\geq$ zw if the integer represented by xy is $\geq$ the integer represented by zw. Let f(x,y,z,y) be a function where f(x,y,z,w)=1 if xy $\geq$ zw; otherwise, f(x,y,z,w)=0. Thus, f can be viewed as a Boolean function, and x, y, z, and w can be viewed as Boolean variables. Write the truth table of f, and then minimize f using Karnaugh maps.