CS 1311 March 1, 2016

Youssef Homework 4

Due Date: March 22, 2016

Problem 1: (20 points)

Let Y be the set of all YouTube videos available on Januray 1, 2016, where each video is identified by its URL. For each of the following relations on the set Y, state without proof if the relation is reflexive, symmetric, antisymmetric, transitive, an equivalence relation, and/or a partial order. In the case of equivalence relations, explain what every equivalence class signifies. In the case of partial orders, indicate if the set Y has a minimum, a maximum, minimals, and/or maximals, and explain what minimum, maximum, minimal and maximal mean in that context.

a) x R y if x and y have the same number of "views".
b) x R y if x and y have the same number of "likes".
c) x R y if (x has fewer "likes" than y) or (x is the same video as y).
d) x R y if x has fewer "dislikes" than y .

Problem 2: (20 points)

An **alphabetic string** is any sequence of one or more English letters. For example, *abc*, *Apple*, *q*, and *hxTrR* are alphabetic strings. Let **S** be the set of all alphabetic string of one or more letters, and let **L** be the set of all the <u>lower-case</u> alphabetic strings of one or more letters. Let *f*: **S**→**L** be a function that maps every string in S into its lower-case counterpart (for example, *f*(*Apple*)=*f*(*APPLE*)=*f*(*apple*)=*f*(*APpLe*)=*apple*)**.** Let α, β, and γ stand for arbitrary alphabetic strings.

a) Let R be following relation in **S**: α R β if the first letter of α and the first letter of β are equal and have the same case.
   i. Prove that R is an equivalence relation.
   ii. How many equivalence classes does R have?
   iii. Characterize in words the equivalence class of the string *apple*.
b) Let R be following relation in **S**: α R β if *f*(α)=*f*(β), meaning that α and β correspond to the same lower-case string.
   i. Prove that R is an equivalence relation.
   ii. Find the equivalence class of the string *app*.
   iii. How many strings are in the equivalence class of the string *apple*?

Problem 3: (20 points)

Take the same set **L** as in Problem 2.

a) Let R be following relation in **L**: α R β if the string α is lexicographically before (or at) β, that is, α would occur before β in the dictionary or the two strings are equal.
   i. Prove that R is a partial order.
   ii. Does **L** have a minimum with respect to R? If so, what is it?
   iii. Does **L** have a maximum? If so, what is it?
   iv. If E is the subset of all lower-case strings of at most 5 letters each, does E have a maximum? If so, what is it? Does E have maximals?
b) Let R be following relation in **L**: α R β if the string α is a prefix of the string β.
   i. Prove that R is a partial order.

ii. Does **L** have a minimum with respect to R? If so, what is it?
iii. Does **L** have minimals? If so, what are they?

c) Use the same relation R as in (b), and let E={*apple, applebee, app, be, beef, bee, bean, been, can, cancun, canister*}, which is a subset of **L**. Draw the graph of R for the set E only, and also draw the Hasse diagram of R for E only.
d) Does E have a maximum or a minimum with respect to R? If so, identify them, but if not, identify the maximals and minimals of E.

Problem 4: (20 points)
Let R and S be two relations in a set A. Let R∩S be the new relation in A defined as follows:

x R∩S y if x R y and x S y.

a) Prove that if R and S are reflexive, then R∩S is reflexive.
b) Prove that if R and S are symmetric, then R∩S is symmetric.
c) Prove that if R or S is antisymmetric, then R∩S is antisymmetric.
d) Prove that if R and S are transitive, then R∩S is transitive.

Problem 5: (20 points)
Let **Z** be the set of all integers, and define R to be the following relation in **Z**:

x R y if (x-y) is a multiple of 4.

Note: an integer n is a multiple of 4 if and only if there is an integer q where n=4q.

a) Prove that R is an equivalence relation.
b) For every integer *x*, let [*x*] = *x* modulo 4. That is, [*x*] is an integer *r* where $0 \le r \le 3$, such that when x is divided by 4, the remainder is *r*. Prove that for all integers *x*, *x* R [*x*].
c) Find the equivalence classes of R

Bonus Problem: (5 points)
Let **N** bet the set of natural numbers, and R the following relation in **N×N:**

(m,n) R (m',n') if $m+n'=m'+n$.

a) Prove R is an equivalence relation.
b) Denote by [*m*,*n*] the equivalence class of (*m*,*n*), and let E be the set of all the equivalence classes of R. Let *f*: **E**→**Z** where $f([m,n])=m-n$, and **Z** is the set of integers. Prove that *f* is one-to-one and onto.