CS 1311 February 16, 2016

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Homework 3

Due Date: March 1, 2016

Problem 1: (15 points)

Solve the following recurrence relations (that is, compute x_n in terms of n alone):

a) $x_0 = 2$, $x_n = 5x_{n-1} + 12$ for $n \geq 1$

b) $x_0 = 1$, $x_1 = 2$, $x_n = 8x_{n-1} - 15x_{n-2}$ for $n \geq 2$

c) $x_0 = 1$, $x_1 = 1$, $x_n = 2x_{n-1} + 8x_{n-2} + 12$ for $n \geq 2$

Problem 2: (20 points)

Solve the following recurrence relations (that is, compute x_n in terms of n alone):

d) $x_0 = 7$, $x_n = 4x_{n-1} + 9n+3$ for $n \geq 1$

e) $x_0 = 1$, $x_1 = 3$, $x_n = 8x_{n-1} - 15x_{n-2} + 16n+4$ for $n \geq 2$

Problem 3: (20 points)

a) How many ways are there to choose 4 people out of 10 people to form a 4-member committee?

b) In how many different orders can 9 runners finish a race if no ties are allowed?

c) In how many different orders can 9 runners finish a race if two people tie for the first place, three people tie for second place, and no ties otherwise?

d) A coin is tossed 8 times. Each outcome will be a sequence of 8 heads and/or tails. What is the number of outcomes where the number of heads is exactly 3?

e) Suppose that in a state, all car license plates consist of 3 capital letters followed by 4 digits.

   i. How many different license plates are possible?

   ii. How many possible plates start with "ABC" and end with an even digit?

   iii. How many plates are possible if all the letters and digits are distinct?

f) A combination lock requires three selections of numbers, each from 1 through 20.

   i. How many different "combinations" are possible?

   ii. How many "combinations" are possible if no number can be used twice?

g) One earn contains 22 balls where: 5 balls are red (labeled $R_1, R_2, \ldots, R_5$), 7 are blue (labeled $B_1, B_2, \ldots, B_7$), and 10 are white (labeled $W_1, W_2, \ldots, W_{10}$). You draw 9 balls from the urn. What is the number of possible outcomes where 2 of the drawn balls are red, 3 are blue, and 4 are white?

Problem 4: (20 points)

a) A particular brand of shirts comes in 6 colors (R,G, B, W, Y, and T), two styles (F for fitted and R for regular), and 4 sizes for each color and style (S, M, L, and XL). Express the different types of shirts as a product set, and give the cardinality of this set.

b) How many positive integers less than 200 are divisible by 6? By 9? By 6 and 9? By 6 or 9? By neither 6 nor 9?

c) Prove that there are $2^{n+1} - 2$ non-empty binary strings of up to n bits in length.

d) Let E and F be two non-empty sets where |E|=n and |F|=m for some positive integers n and m. Give the total number of possible functions from E to F, and prove your answer.

Problem 5: (20 points)

a) Consider the following algorithm segment:

```
for i=1 to n {
        for j=1 to i {
                X=X+1;
        }
}
```

What is the number of additions (+) performed by this code segment? Prove your answer. Note that your answer is an expression in n.

b) Consider the following algorithm segment:

```
for i=1 to n {
        for j=1 to i {
                for k=1 to j {
                        X=X+1;
                }
        }
}
```

What is the number of additions (+) performed by this code segment? Prove your answer. Note again that your answer should be an expression in n.

Bonus Problem: (5 points)

Consider the code of this function, called Compute, which takes one single input argument n, and returns an integer, where n must be a non-negative integer:

```
int Compute( int n){
        if (n==0 || n==1){
                return (n+1);
        }
        else{
                int a=Compute(n-1);
                int b=Compute(n-2);
                return (a+b);
        }
}
```

Let T(n) be the number of additions (+) performed by Compute(n). If you prefer, you can rename T(n) as x_n.

a) Derive a recurrence relation for x_n.

b) Solve the recurrence relation to determine the value of T(n) as an expression in n.