CS1311 January 12, 2015
Youssef

Homework 1
Due Date: January 28, 2016

Problem 1: (20 points)
Let A = {1, 3, 5, 6, 7, 8, 11, 12, 13}, B = {x | x is an integer that is a multiple of 3 and 1<x<14}, and C = {a, b, 5, 6, 9, 10, 11}.

a) Fill in the blanks with the most appropriate symbol ($\in, \notin, \subseteq, \not\subseteq$):
   12 … A, 8 … B, 6 ... B, 10 ... A, b ... C, b ... B, {3,5,11} ... A, {1,3,6} ... B, {5,6,10,11} ... C.
b) Evaluate: $A \cup B$, $A \cap B$, $A - B$, $A + B$, $2^{A \cap B}$, $(A \cap B) \times \{a, b\}$.
c) Evaluate: $A \cup (B \cap C)$, $A - (B \cap C)$, $(A - B) - C$, $(A + B) + C$, $A + (B + C)$.
d) Find the cardinality of each set in (b) and (c).

Problem 2: (20 points)
Let G be the set of all GW undergraduates who are registered in Spring 2016, and A, B and C be the following three subsets of G: The students in A are those majoring in CS, the students in B are those minoring in Economics, and C is the set of GW students registered in CS 1311 in Spring 2016.
a) Describe in understandable English prose the students in each of the following sets:
   $A \cup B$, $A \cap B$, $A-B$, $A+B$, $A \cap C$, $A+C$, $C-A$, $(A+B) \cap C$, $A-(B \cup C)$.
b) Express each of the following sets in terms of A, B, C, G and the different set operations:
   - The set of currently registered GW undergraduates who, in Spring 2016, are majoring in CS, minoring in Economics, and taking CS 1311.
   - The set of currently registered GW undergraduates who, in Spring 2016, are either majoring in CS or minoring in Economics, but not taking CS 1311 in either case.
   - The set of currently registered GW undergraduates who, in Spring 2016, are neither majoring in CS, nor minoring in Economics, nor taking CS 1311.
   - The set of currently registered GW undergraduates who, in Spring 2016, are non-CS majors and not minoring in Economics but taking CS 1311.

Problem 3: (20 points)
Let the proposition P(x) stand for "x is integer", and the proposition Q(x) stand for "x is a bird". Give the truth values (True or False) of the following propositions:
a) P(5)
b) P(-10)
c) P($\frac{1}{2}$)
d) P(6.7)
e) ($\exists$ a real number x) P(x)
f) ($\forall$ real number x) P(x)
g) ($\forall$ real number y)($\exists$ an integer x>y) P(x)
h) Q(snake)
i) Q(sparrow)
j) Q(owl)
k) ($\forall$ animal x)Q(x)

l) ($\exists$ animal x) Q(x)
m) ($\exists$ animal x) ($\neg$P(x))

Problem 4: (20 points)
Let P(x) stand for "x is a movie director" and Q(x,y) stand for "x is the director of movie y". Let A be the set of all American citizens, and M be the set of all movies ever made. Express each of the following statements in easily understandable English prose:
a) ($\exists$ x $\in$ A) P(x)
b) ($\forall$ x $\in$ A) ($\exists$ y $\in$ M) (Q(x, y))
c) ($\exists$ x $\in$ A) ($\forall$ y $\in$ M) (Q(x, y))
d) ($\exists$ y $\in$ M) ($\forall$ x $\in$ A) (Q(x, y))
e) ($\forall$ y $\in$ M) ($\exists$ x $\in$ A) (Q(x, y))
f) ($\exists$ x $\in$ A) ($\exists$ y $\in$ M) (P(x) $\wedge$ $\neg$Q(x, y))
g) ($\exists$ x $\in$ A) ($\forall$ y $\in$ M) (P(x) $\wedge\neg$Q(x, y))

Problem 5: (20 points)
Let P(x) and Q(x,y) be the same as in Problem 4. For each of the following sentences, express the sentence as a proposition or a predicate using P, Q, logical quantifiers ($\exists$ and $\forall$) and logical connectives ($\wedge$, $\vee$, and $\neg$):
a) Steven Spielberg is a movie director
b) There is at least one American citizen who is not a movie director
c) Not every American citizen is a movie director
d) There exists an American movie-director who has directed at least two movies
e) There exists an American movie-director who has directed every movie ever made
f) It is not the case that every American citizen has directed at least one movie
g) It is not the case that every movie ever made was directed by an American citizen

Bonus Problem: (5 points)

Let P(x), Q(x,y), A and M be the same as in Problem 4. Express each of the following sentences as a predicate using P, Q, logical quantifiers ($\exists$ and $\forall$), logical connectives ($\wedge$, $\vee$, and $\neg$), and implication (=>):
a) For every movie ever made, there exists an American citizen that has directed both that movie and at least another movie.
b) For every American citizen, if that citizen has directed a movie, then that citizen is a director.