CS1311 May 3, 2016
Youssef

Final
TIME: 2 Hours

Problem 1: (25 Points)

a. Prove by induction on n that $1 \times 3 + 2 \times 5 + 3 \times 7 + \cdots + n \times (2n+1) = \frac{n(n+1)(4n+5)}{6}$ for all positive integers n. Conclude that $\forall$ integers $n \geq 1$, $n(n+1)(4n+5)$ is divisible by 6.
b. Let $x_0 = 0, x_n = 2x_{n-1} + n \; \forall \; n \geq 1$. Prove by induction that $x_n = 2^{n+1} - n - 2 \; \forall \; n \geq 0$.
c. Solve the following recurrence relation: $x_0 = 1$, and $x_n = 4x_{n-1} + 3n - 1 \; \forall \; n \geq 1$.
d. Solve the following recurrence relation: $x_0 = 0$, and $x_n = 3x_{n-1} + 3^n \; \forall \; n \geq 1$. (Hint: look for x_n of the form $(an + b)3^n$.)

Problem 2: (25 points)

a. Let *F* be the set of Facebook accounts at this moment, and define in *F* the following relation *R*: *x R y* if *x* has fewer "friends" than *y*. Is R reflexive? Symmetric? Antisymmetric? Transitive? An equivalence relation? A partial order? Prove your answers.
b. Let *S* be the set of all lower-case English alphabetic strings, and define the following relation *R* in *S*: α *R* β if the two strings α and β have the same length and each letter in α appears in β and each letter in β appears in α. Prove that *R* is an equivalence relation, and compute the equivalence class of each of the following strings *aa*, *ab*, *abc*, and *aab*. Also, find the cardinality of the equivalence class of a string α of length *n* where all the *n* letters are distinct.

Problem 3: (25 points)

Let G = (V,E) be the following undirected graph: V = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12} and E = {(1,2) , (2,5) , (1,4) , (4,5) , (1,3) , (3,9) , (1,9) , (3,8) , (3,7) , (8,7), (3, 6), (3, 10), (6, 10), (6, 11), (6, 12), (11, 12), (8,10}.

a. Perform depth-first search on G starting from node 1. Show the depth-first search tree, and provide its depth (which is the distance between the root and a bottommost leaf). Is G connected? Why?
b. Perform breadth-first search on G starting from node 1. Show the breadth-first search tree, and provide its depth.
c. Does G have articulation points? If so, list them.
d. Does G have a Hamiltonian cycle? If yes, show one, and if no, why not?
e. Does G have an Eulerian cycle? If yes, show one, and if no, why not?

Problem 4: (25 points)

a. Let *f(x,y,z) = x'y + y'z' + yz + xz* be a Boolean function. Give the truth table of *f*, and put *f* in DNF and CNF.
b. Minimize each of the following Boolean expressions using Karnaugh maps.
   i. *x'y' + yz + y'z' + xz*
   ii. *yz +x'z'+xy'z+x'z*
   iii. *x'yw' + xy'w + x'z'w' + x'y'zw'*
   iv. *x'yz+x'y'w'+yz'w'+xy'z'w'+y'zw+xyzw*