

Homework Assignment 1

Due Sep. 10

Please answer each of the below questions. Remember, you may work together, but everyone MUST type up and understand their solutions. Solutions that appear copied will be considered a violation of the academic honesty code. Also, you must list all people you work with on this homework as well as any outside resources (e.g., web search, books, etc.) that you use.

1. Imagine that I roll a 6-sided die and record the value, v . Then, I ask you to guess v . After you make your guess, g , I then reveal a hint, h , which is chosen randomly such that $h \neq v$ and $h \neq g$. I then give you the option to keep your original guess or to switch your guess. Should you a) switch your guess, b) stay with your original guess, or c) does it not matter? Explain your reasoning.

To answer this question, let V be a random variable denoting the value of the die. Let H be a random variable denoting the value of the hint. Let G_1 be a random variable denoting your original guess and let G_2 be a random variable denoting your switched guess.

Compute the following probabilities:

- For some value $h \in \{1, 2, 3, 4, 5, 6\}$, compute $\Pr[H = h]$. (Hint: consider what happens when $g = v$ and when it doesn't.)
 - Compute $\Pr[G_1 = V | H = h]$. (Hint: Recall that by Bayes' Theorem $\Pr[A|B] = \frac{\Pr[B|A]\Pr[A]}{\Pr[B]}$)
 - Compute $\Pr[G_1 \neq V | H = h]$
 - Compute $\Pr[G_2 = V | H = h]$
2. Suppose n people are in an ice-cream shop which sells f different flavors of ice-cream. Each person likes at least $f/4$ of the flavors. Show that there exists an ice-cream flavor that is liked by at least $n/4$ people.

Hint: Use proof by contradiction

3. Exercise 2.3 in the book.
4. Exercise 2.4 in the book.
5. Exercise 2.6 in the book.