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Resource Allocation and Performance Study for LTE Networks Integrated with Femtocells

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Abstract [1] — *Long-Term Evolution (LTE) networks comprising conventional cellular macrocells plus user-installed femtocells offer an economically viable solution to achieving high user capacity and upgrading to future fourth-generation systems. With the growing impetus for frequency reuse, the capacity of each user depends on not only the power spectral density of its own, but also on those of others in neighboring cells. Mitigating interference among macrocells and femtocells requires allocating physical resource dynamically in response to channel conditions. In this paper, we formulate the resource allocation problem as a utility optimization and develop a distributed algorithm for joint power control and user scheduling. The algorithm makes novel use of a class of fairness measures for determining user scheduling and is shown to be very efficient for realistic network parameters. Additionally, using a practical model for the LTE air interface that captures geographic distribution of users and buildings, we provide for a framework that allows comparison of different resource allocation algorithms. A variety of problem formulations, including femtocell density, resource tradeoff, and complexity-optimality tradeoff are derived and analyzed using a geometry-based stochastic LTE air interface model. Our analysis also offers useful guidelines for the planning and design of macrocells and femtocells.*

Keywords- Femtocells, Macrocells, LTE, Resource Allocation

I. INTRODUCTION

LTE defined by 3GPP is a highly flexible radio interface, which aims for a smooth evolution from earlier Universal Mobile Telecommunications System (UMTS) to future fourth generation systems [1]-[6]. It is in the process of being deployed to provide improvements in cellular capacity. However, due to the large attenuation loss for indoor users (especially at higher frequency) in LTE networks, femtocells are proposed to serve as small range indoor access points, which are installed by users and backhaul data through a broadband gateway over the Internet. It is shown that more than 50% of all voice calls and more than 70% of data traffic originates indoors [7]. Femtocells deployment amidst existing macrocells can efficiently take indoor traffic off expensive cellular networks, so that macrocell base-stations (BSs) can direct their resource to truly mobile users. The capacity and coverage of both macro- and femto-cells will benefit from the off-loading of traffic. Since the femtocell radio range (5 - 50 meters) is normally much smaller than the macrocell radius (300 – 2000 meters) [8], users served by femtocells experience superior signal reception and can greatly lower their transmit power to prolong battery life.

Macro- and femto-cell Base Stations (BSs) have to dynamically allocate physical resource to users in a time-frequency division manner, for varying channel conditions and quality of service criteria. The degrees of freedom available to the BSs to allocate, i.e. subcarriers, time slots, and transmit power, allow for optimized resource allocation algorithms that can decide which user to schedule in a time-frequency grid (i.e. user scheduling) and what transmit power to use for the scheduled user (i.e. power control). However, optimized resource allocation algorithms come with a double-edged sword – i.e., while they can provide for significantly improved network performance, when not applied carefully, they can cause high overheads and complexity thus rendering them both difficult and impractical for use in realistic systems.

In this paper, we consider the important and challenging problem of resource allocation in LTE network with femtocells and study their performance trades. Our contributions, in particular, are as follows: (i) We propose a novel fully distributed low overhead resource allocation (RA) algorithm that optimizes resource allocation on both the macro- and femto-cells. Using the well researched decomposition principles of power control and user scheduling [23]-[28], our RA algorithm allocates resources dynamically and improves LTE network capacity while also maintaining fairness among different users. (ii) We propose a new method of solving user scheduling by relaxing the discrete scheduling problem into a convex optimization problem and make use of a family of fairness measures as in [9] to map continuous scheduling decisions into binary decisions, interactively. Our method has low computational complexity, since it only requires a projected-gradient method and calculation of fairness measures. (iii) We use a practical LTE air interface model based on the WINNER 2 [29] project that employs stochastic geometry framework for modeling the random spatial distribution of users, femtocells, and buildings. Our simulation studies with realistic channel models shows about 75% of capacity improvement over round robin. (iv) We derive several problem formulations, including the impact of increasing femtocell density, physical resource tradeoff for network planning, and benefits of off-loading traffic using femtocells.

The remainder of the paper is organized as follows. We begin with an overview of related work in Section II. In Section III, we introduce describe the system model. Section IV contains the details of our resource allocation algorithm based on fairness measures. Section V provides a description of the LTE air interface model. Section VI uses the resource allocation algorithm and the LTE air interface model to characterize several important tradeoffs and discuss their implications to planning and designing LTE networks. Section VII summarizes and concludes the paper.

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II. RELATED WORK

Prior research on femtocells has mainly focused on GSM and CDMA networks with single carrier, where frequency reuse structures among femtocells and macrocells have to be predetermined and are fixed for all time. The authors in [10] [11] have shown that it is more practical to split the radio frequency spectrum between femtocells and macrocells, thus negating any frequency reuse. The benefits of having a tilted antenna radiation pattern and macrocell-femtocell power ratio control is discussed [12]. Use of antenna sectoring and time-hopped CDMA is shown in [13] to achieve a higher user capacity for a shared spectrum network with femtocells.

In another line of work, for analytical tractability, femtocells are assumed to be regularly placed in macrocells, and physical channel attenuations are drawn from given distributions (e.g. Gaussian and Log-normal). These assumptions, while allowing closed-form solution, have limited applicability owing to the inherent variability in femtocell locations, building structure, and correlated fading in realistic scenarios. By approximating cross-cell interference by its average and ignoring interference among femtocells, the capacity of a CDMA network with femto- and macro-cells is derived in [14]-[16] for different cell-selection schemes. The analysis is accurate for up to eight femtocells per macrocell. This result is then extended in [13] to a large number of femtocells without approximating the interference statistics.

From an optimization perspective for networks with OFDMA, resource allocation formulated as an integer programming problem is shown to be NP hard and have exponential complexity [17]. For Digital Subscriber Line (DSL) applications, the resource allocation problem can be formulated either as non cooperative Nash games [18] [19] or as a utility maximization problem [20] [21] [22]. Several algorithms have been proposed to compute a Nash equilibrium solution using iterative waterfilling method (IWFA) [18] [19] or a global optimal solution using dual decomposition method [20] [21] [22]) for the utility maximization. Due to the nonconvex nature of the problem, these IWFA algorithms may be inefficient and converge to a stationary point with poor capacity. Significant effort has been made to establish conditions which can ensure the existence and uniqueness of a Nash equilibrium solution as well as the convergence of IWFA [18] [19]. In an attempt to analyze the performance of the dual decomposition algorithms, the duality gap of the resource allocation problem is studied in [17].

For fixed user scheduling, per-user water filling [23] or fixed point iteration with standard interference functions [24] can be applied to optimize the transmit power levels. To solve the user scheduling problem for fixed transmit power, several heuristic algorithms have been developed in [23]-[28]. In a separate work [25], a user scheduling algorithm that assigns subcarriers and time slots based on the ranking of their potential contribution to network utility is proposed. Similar work include [26] and [27], where scheduling decisions are made upon the group capacity and the ratio of signal to energy leaked in to neighboring, respectively. In a recent work, it is shown in [28] that the user scheduling problem can be solved optimally by formulating the problem as a maximal bipartite matching and solving it using a modified version of the Hungarian algorithm.

In summary, we note that the existing resource allocation algorithms may not be directly applied to LTE macrocells and femtocells due to several reasons as follows: (i) Instead of scheduling each individual subcarrier and time slot to users, user scheduling in LTE networks is decided in the unit of Physical Resource Blocks (PRBs), which contain a group of subcarriers and time slots, while power control is still performed for each subcarrier and time slot. The difference in granularity may cause large capacity loss when applying existing resource allocation algorithms to LTE networks. (ii)Unlike DSL applications where all subcarrier interfere with each other, LTE network use orthogonal transmission within each cell and universal frequency reuse at different cells. (iii)Existing resource allocation algorithms normally try to maximize the total network throughput or a linear utility function of capacity. Since the notion of fairness is not captured in the utility function, starvation may occur in network for users with bad channels. Therefore, we need to solve resource allocation for general utility functions capturing both notion of fairness and efficiency as is done in this paper.

III. SYSTEM MODEL AND PROBLEM FORMULATION

Consider an LTE network with n (femtocell and macrocell) base stations serving totally m mobile users. We use b_i to denote the BS serving mobile i for $i = 1,\dots,m$. Since users within the same cell are assigned orthogonal resources, the set of users who are possible to interfering with user i is given by

$$Ci = \{j : bj \neq bi, \forall j\} \quad (1)$$

Similarly, we define the set of users served by $BS\ k$ as

$$Bk = \{j : bj = \text{k}, \forall j\} \quad (2)$$

Radio resources in an LTE network are allocated in units of PRBs on a time-frequency grid. Each PRB has the size of $180kHz$ in the frequency domain and $0.5ms$ in the time domain. Therefore, we assume that the physical channel is divided into $T\ x\ F$ blocks, such that $(F \cdot 180)\ kHz$ is the total system bandwidth and $(T \cdot 0.5)\ ms$ is the time scale of dynamic resource allocations. Let $f_H(\cdot)$ be the random physical channel generation function which accepts set of network deployment parameters D (e.g. femtocell density, macrocell radius, and bandwidth) as input.

$$\{H_{bi,i,t,f}, \forall(t,f)\} = f_H(D) \quad (3)$$

Where $H_{bi,i,t,f}$ is the channel attenuation from user i to its base station b_i, for PRB (t,f). Let $P_{i,t,f}$ be the transmit power of user i on PRB (t,f) and $S_{i,t,f} = 1_{\{P_{i,t,f} > 0\}}$, $\forall i, t, f$ is the binary indicator of whether PRB (t,f) is assigned to user i. All channel attenuations are stored in matrix H, all transmit power in matrix P, and assignment decisions in S. The data rate of user i is given by a function of channel and power, as follows:

$$R_i = f_{R_i}(H, P, S) = \sum_{t=1}^{T}\sum_{f=1}^{F} \log\left(1 + \frac{1}{\Gamma} \cdot \frac{H^2_{b_{i,i,t,f}}\ P_{i,t,f}S_{i,t,f}}{\sum_{j\epsilon C_i} H^2_{b_{i,i,,t,f}} P_{j,t,f}S_{j,t,f} + N_{b_i}}\right), \forall i \quad (4)$$

Where N_{b_i} is the thermal noise power at BS b_i and Γ is an SINR gap, reflecting the loss over modulation and error coding. The problem now can be stated mathematically as follows:

Maximize $\sum_{i=1}^{m} U\,(R_i)$, (5)

Subject to $R_i = \sum_{t=1}^{T}\sum_{f=1}^{F} \log(\,1 + \frac{1}{\Gamma}\cdot\frac{H_{b_{i,i,t,f}}^2\ P_{i,t,f}S_{i,t,f}}{\sum_{j\epsilon C_i} H_{b_{i,i,,t,f}}^2 P_{j,t,f}S_{j,t,f} + N_{b_i}}\,), \forall i$

$\sum_{t=1}^{T}\sum_{f=1}^{F} \mathrm{P}_{i,t,f} \ \leq\ P_{max,i}, \forall i$

$S_{i,t,f}\ \epsilon\ \{0\ ,\ 1\}, \forall\ i,\ t, f$

$\sum_{i\epsilon B_k} S_{i,t,f} = 1, \forall\ t, f, k$

Variables $P_{i,t,f}$, $S_{i,t,f}$

Where $P_{max,\ i}$ is a maximum transmit power constraint for user i

$\sum_{i\epsilon B_k} S_{i,t,f} = 1, \forall\ t,\ f,\ k$, ensures that each PRB is assigned a single user within each cell, and

U (·) is the utility function.

The utility function, U (·), is used to capture various design objectives, such as throughput efficient and allocation fairness. In one embodiment the utility function, U (·), can be from the family of widely applied α - fair utilities,

$$\sum_i U_\alpha\ (x_i), \text{ where } U_\alpha\ (x) = \begin{cases} \frac{x^{1-\alpha}}{1-\alpha}, & \alpha \geq 0,\ \ \alpha \neq 1 \\ \log(x), & \alpha = 1 \end{cases} \quad (6)$$

The α - fair utilities model was chosen as a tradeoff between throughput efficient and allocation fairness. A maximize of the α - fair utility function satisfies the definition of α – fairness: a maximization of the log utility function (α = 1) is proportionally fair, and a maximization of the α-fair utility function with $\alpha \rightarrow\ \infty$ is max-min fair.

IV. Our Optimized Resource Allocation Algorithm

The resource allocation problem formulated in Section III is NP-hard due to the integer constraints placed on $\{\mathrm{S}_{i,t,f}\ , \forall\ i, t, f\}$. We therefore propose the following decomposition.

<u>Problem 1A</u> is the power optimization over P for fixed scheduling S and is represented mathematically as follows and which is solved for all users jointly:

Maximize $\sum_{i=1}^{m} U\,(R_i)$, (7)

Subject to $R_i = \sum_{f=1}^{F} log\,(1 + \frac{1}{\Gamma}\cdot\frac{H_{b_{i,i,t,f}}^2\ P_{i,t,f}S_{i,t,f}}{\sum_{j\epsilon C_i} H_{b_{i,i,,t,f}}^2 P_{j,t,f}S_{j,t,f} + N_{b_i}}\,), \forall i$

$\sum_{f=1}^{F} P_{i,f} \ \leq\ P_{max,i}, \forall i$

Variables $P_{i,f}$

In LTE networks, since interference (in the denominator of the rate function above) only comes from users in neighboring cells, the problem can be further decomposed across cells that reduce the amount of message-passing among different cells. Thus minimizes the space Ci, and the monotonicity property of the α - fair utility is seen to apply. The problem then becomes a standard (iterative) water-filling problem and maximizes rates R_i for all users i = 1,…, m independently and is as follows:

Maximize R_i

Subject to $R_i = \sum_{f=1}^{F} log\,(\,1 + \frac{1}{\Gamma}\cdot\frac{H_{b_{i,i,t,f}}^2\ P_{i,t,f}S_{i,t,f}}{\sum_{j\epsilon C_i} H_{b_{i,i,,t,f}}^2 P_{j,t,f}S_{j,t,f} + N_{b_i}}\,), \forall i$ (8)

$\sum_{f=1}^{F} P_{i,f} \ \leq\ P_{max,i}, \forall i$

Variables $P_{i,f}$

Where the utility function U(·) is omitted due to its monotonicity over R_{bi}. Let λ_i be the Lagrangian multiplier for the transmit power constraint of user i. Then the Lagrangian for the power control problem of user i is as follows:

$$L\,(P_{i,f},\, \lambda_i) = \sum_{f=1}^{F} log\,(\,1 + \frac{1}{\Gamma}\cdot\frac{H_{b_{i,i,t,f}}^2\ P_{i,t,f}S_{i,t,f}}{\sum_{j\epsilon C_i} H_{b_{i,i,,t,f}}^2 P_{j,t,f}S_{j,t,f} + N_{b_i}}\,) \quad (9)$$

This is a convex optimization whose solution can be given in close form using the Karush-Kuhn-Tucker (KKT) conditions as described below.

<u>Solution 1A</u>

Therefore, the solution to Problem 1A is as follows:

$$P_{i,f,}^{*}\,(\lambda_i) = \begin{cases} \left(\frac{1}{\lambda} - \frac{\Gamma(\sum_{j\epsilon C_i} H_{b_{i,i,f}}^2\ P_{i,f}\,S_{i,f} + N_{b_i}\ \ )}{H_{b_{i,i,f}}^2}\right)^{+} \\ 0, \end{cases} \quad (10)$$

Where $(x)^+$ = max(x,0) is a projection to the set of non-negative numbers, and $\frac{1}{\lambda_i}$ is the water level satisfying $\sum_{f=1}^{F} P_{i,f}^{*}\,(\lambda_i\,) \leq P_{max,i}$

Thus, the solution to Problem 1A is equivalent to determining the water-level for each user. Once water-levels are known, a set of transmit power and potential data rates of assigning PRB f to different users can be computed as follows:

$$r_{i,f}\,(\lambda) = \log\,(\,1 + \frac{1}{\Gamma}\cdot\frac{H_{b_{i,i,t,f}}^2\ P_{i,t,f}S_{i,t,f}}{\sum_{j\epsilon C_i} H_{b_{i,i,,t,f}}^2 P_{j,t,f}S_{j,t,f} + N_{b_i}}\,). \quad (11)$$

Rate $r_{i,f}\,(\lambda)$ is the potential data rate that user i can achieve on PRB (t,f) (independent of the scheduling), and only depends on parameter (λ).

<u>Problem 1B</u>

For a set of fixed water levels $\frac{1}{\lambda_i}$, Problem 1 can be reduced to a scheduling problem of assigning PRBs to users as follows:

Maximize $\sum_{i=1}^{m} U\,(R_i)$, (12)

Subject to $R_i = \sum_{f=1}^{F} S_{i,f}\cdot r_{i,f}\,(\lambda)$

$S_{i,f}\,\epsilon\ \{0, 1\}, \forall\ i, f$

$\sum_{i\ \epsilon\ B_k} S_{i,f} = 1, \forall\ i, f$

Variables $S_{i,f}$

If the utility is linear in R_i, Problem 1B becomes a maximal weighted bipartite matching problem in graph theory and can be solved by a modified version of the Hungarian algorithm with polynomial complexity F^3. Using a standard optimization technique, the integer constraint $S_{i,f}\,\epsilon\ \{0, 1\}$ is relaxed to a continuous constant $0 \leq\ S_{i,f}\ \leq\ 1$, so that each BS needs to solve the following problem:

Maximize $\sum_{i\ \epsilon\ B_k} U\,(R_i)$ (13)

Subject to $R_i = \sum_{f=1}^{F} S_{i,f}\cdot r_{i,f}\,(\lambda)$

$0 \leq\ S_{i,f} \leq 1, \forall\ i, f$

$\sum_{i\ \epsilon\ B_k} S_{i,f} = 1, \forall\ f$

Variables $S_{i,f}$

Next, the solution is projected to the space of binary scheduling decisions, satisfying $S_{i,f}\ \epsilon\ \{0, 1\}$ and $\sum_{i\ \epsilon\ B_k} S_{i,f} = 1$.

The following family of fairness measures developed through axiomatic theory $\{\mathrm{F}(\mathrm{x}_{1\ldots}\ \mathrm{x}_{\mathrm{m}}) = \left[\sum_{i=1}^{m}\left(\frac{x_i}{\sum x_i}\right)^{1-\beta}\right]^{\frac{1}{\beta}}\}$is applied to find the physical resource block with the most biased or unfair scheduling vector f* = arg $\min_f F(\{S_{i,f} | i \in B_k\}), \forall k$.

Physical resource block *(t, f)* is then assigned to the user with the largest element of $\{S_{i,f} \mid i \in B_k\}$. To assign all physical resource blocks, the above procedure is repeated. The solution is summarized as follows:

SOLUTION 1B

While $i < m$ {

- solve equation (x) to obtain $S_{i,f^*} = \mathbf{1}_{\{Si,f^* = \max i\, Si,f^*\}}$
- Find the most biased scheduling as
  - f*= arg $\min_f F(\{S_{i,f} | i \in B_k\}), \forall k$
- Assign $S_{i,f^*} = \mathbf{1}_{\{Si,f^* = \max i\, Si,f^*\}}$
- $i = i + 1$

} repeat

V. LTE AIR INTERFACE MODEL

The LTE air interface model that we use is a geometry-based stochastic radio channel model based on WINNER II [29] project radio channel models.

The physical layout of the modeled radio network is created as follows. The user can select one or more propagations scenarios and construct a network environment including houses or buildings, femtocell base stations (BS), macrocell base stations, and mobile stations (MS or UE) which are randomly placed in a hexagonal grid containing deployed macrocells. A number of houses, N_{house}, each of a size w x w, are uniformly distributed over the area. A femtocell base station can be placed in each house with a uniform distribution and serve a number of indoor mobile stations in the house, $N_{UE, in}$. A number of outdoor mobile stations, $N_{UE, out}$, are uniformly placed over the area as well.

After the deployment, there are $N_{UE} = N_{UE, in} \cdot N_{house} + N_{UE, out}$ mobile stations and $N_{BS} = N_{house} + N_{site}$ base stations in the network which result in $N_{channel} = N_{UE} \cdot N_{BS}$ communication channels. The channels are denoted as $i = 1 \ldots N_{channel}$, where each channel i represents a base station – mobile station pair (s_i, m_i) or (b_i, i).

Next, the user parameters are generated. The WINNER II models generate user parameters based on the physical layout and propagation scenario created earlier. These parameters are classified into two sets: large scale and small scale parameters. The large scale parameters include the following: delay spread and distribution; angle of departure spread and distribution; angle of arrival spread and distribution; shadow fading standard deviation; and Ricean K-factor. The large scale parameters are drawn randomly from tabulated distribution functions. The small scale parameters include the following: scaling parameters for delay distribution; cross-polarization power ratios, number of clusters, cluster angle spread, and distribution, cluster angle spread of arrival, etc. The small scale parameters are drawn randomly from tabulated distribution functions and random LS parameters.

In addition, location dependent parameters are generated. The location dependent parameters can include d_i, ϕ_i, $d_{out, i}$, and $d_{in,i}$. For each channel i, d_i denotes the distance from base station s_i to mobile station m_i and ϕ_i denotes the departure or arrival angle at base station s_i. The distance $d_{out, i}$ is the distance from base station s_i to the wall next to the mobile station location when the mobile station location is placed outdoors. The distance $d_{in,i}$ is the perpendicular distance from the wall to the mobile station, and ϕ_i is the angle between the line-of-sight (LOS) to the wall and a unit vector normal to the wall.

The channel impulse response (CIR) is computed for each channel i and the channel attenuation for each subcarrier f_k in channel i. Figure 1 shows the steps used to make these calculations.

In short, the CIR for each channel is computed as a function of four components: antenna gain, $H_{AG, i}$; path loss, $H_{PL, i}$; shadow fading, $H_{SF, i}$; and multi-fading, $H_{MF, i}(\tau)$. The CIR for each channel i is represented mathematically as follows:

$$H_i(\tau) = H_{AG, i} \cdot H_{PL, i} \cdot H_{SF, i} \cdot H_{MF, i}(\tau)$$

Where i represents (b_i, i).

Once the CIR is known for a particular channel i, then the channel attenuation is computed for each subcarrier.

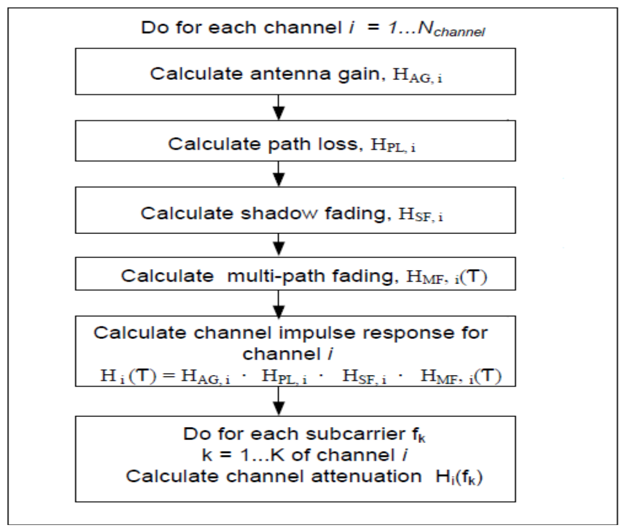

Figure 1: Channel Impulse Response and Attenuation Calculations

VI. PERFORMANCE ANALYSIS

A. Performance improvement of Resource Allocation

Consider a network where 50 macro-users and 19 femto users are uniformly random deployed over an area of 6 macrocells and 10 femtocells. Each macrocell has a radius of 1000 meters and each femtocells (house) has a size of 7 *x* 7 meters. For the uplink, the transmit power is chosen to be $P_{max, macro}$ = 21dBm and $P_{max,femto}$ = 21dBm with an noise figure of F_n = 7dB at all base stations. The LTE network is operating over band 2.3-2.5 GHz with a bandwidth of 5MHz. We formulate the LTE resource allocation problem (5) with the above parameters and solve the problem using the proposed distributed algorithm.

$$\sum_{i=1}^{m} U(Ri) = \frac{1}{m} \sum_{i=1}^{m} \log(R_i) \quad (14)$$

We use a logarithm utility function such that the solution to (6) maximizes a geometric mean of rates and therefore, achieves a proportional fairness [31], [32] among all users.

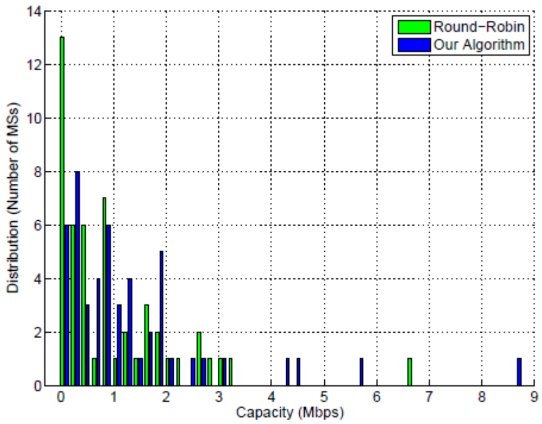

Figure 2 Distribution of data rates for LTE network with 50 macro users and 19 femto users

Figure 2 compares the resulting distribution of rates to that of a round-robin PRB allocation with iterative-waterfilling power allocation. It shows that our algorithm not only improves average rate by about 75%, but also balance rates of different users. The fraction of users with rates higher than 200Kbps increases from 81.1% to 91.3%, i.e. *Prob* $\{R_i > 200Kbpss\}$= 91.3%.

B. Cost and Resource Trades

For an allocation policy $\emptyset(\cdot)$ in the basic problem (5), we can write the utility achieved by $\emptyset(\cdot)$ as a function of deployment parameters D and transmit power budget P_{max}:

$$\sum_{i=1}^{m} U(Ri) = \sum_{i=1}^{m} U\left(f_{Ri}\,(H,P,S)\right) \quad (15)$$
$$= \sum_{i=1}^{m} U\left(f_{Ri}\left(f_H(D), \emptyset(f_H\,(D), P_{max})\right)\right)$$
$$\triangleq f_U\,(\emptyset, D, P_{max})$$

The function $f_U(.)$ that maps deployment parameters and transmit power budget to achievable utility is more than an intellectual curiosity, it leads to a quantitative understanding of the engineering tradeoff among increasing power, changing redeployment (e.g. cell size and antenna array), and utilizing more bandwidth. Figure 3 illustrates an achievable (by allocation algorithm $\emptyset$), equal-utility contour surface, defined by

$$(P,K,F): U = f_U(\emptyset, D, \mathrm{P}max), \text{for } \mathrm{P}max = 1 \cdot P, (K,F) \in D \quad (16)$$

Where $(K, F) \in D$ denotes a deployment D satisfying resource constraints K and F, U is a desired utility level and all BSs have the same amount of resources: transmit power P, total bandwidth F, and number of antennas K. Each axis in Figure 3 represents the value on one of the three resources, and each point on the surface states that for this set of resources, the resource allocation given by algorithm $\emptyset$ attains the same utility value U as other points on the surface. A more optimal resource allocation algorithm, which achieves utility U with lower resource requirements, is able to push down the equal utility surface in Figure 3.

Imposing a cost model on top of this Pareto-optimal surface will further provide a cost-effectiveness analysis to the wireless cellular operator. For example, since bandwidth is a scarce resource in a wireless cellular network, a large cost function should be associated with the total bandwidth usage, whereas smaller costs are assigned to power and number of antennas. This choice of costs would result in a resource bundle of (F = 1*MHz; P* = 5*dB; K* = 4), in which the total bandwidth usage has been minimized. This is in contrast to a different resource bundle of (F = 4.37*MHz; P* = 1*dB; K* = 1) lying on the same tradeoff surface, where number of antennas and transmit power has been minimized.

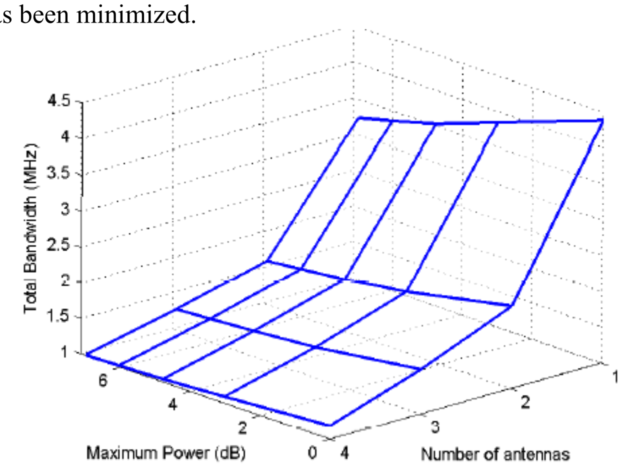

Figure 3: Pareto-Optimal tradeoff surface among different resource allocations that achieve the same utility. Each axis represents the value of one resource

C. Optimality and Complexity Trades

LTE resource allocation can be performed on the time scale of 1*ms*. A good resource allocation algorithm not only tends to optimize performance (measured by utilities), but also minimizes implementation complexity (measured by overhead or execution time). Since achieving better optimality normally requires more complicated resource allocation algorithms, suboptimal algorithms with lower complexity are of interest for practical implementation, especially in a dynamic scenario with fast fading. We can formulate an optimality complexity tradeoff for different resource allocation algorithms. Let $C(\emptyset,H,P_{max})$ be the number of CPU cycles for algorithm $\emptyset$ to converge for deployment D and power constraint P. For algorithm $\emptyset$ the optimality-complexity tradeoff is given by

$$\{(\tau,\delta): \tau = C(\emptyset, D, P_{max}), \delta = U_{opt} - f_U(\emptyset, D, P_{max})\} \quad (17)$$

Where U_{opt} is the optimal utility value over all feasible resource allocations obtained by exhaustive search. This tradeoff provides a unifying framework for comparing different resource allocation algorithms in Section IV.

D. Femtocell Density

Finally, using the utility formulation in Equation 15 we can study the behavior of network performance with respect to the change of network deployment parameters. For instance, in order to analyze how the benefits of using femtocells scale as density of femtocells increases, we can formulate the following problem

$$U_{\emptyset}(n_{femto}) = E_D[f_U(\emptyset, D, P_{max})|n_{femto} \in D \quad (18)$$

Where n_{femto} is the number of femtocells and $E_D[.]$ is an expectation over different deployment and channel realizations.

The expected utility is modeled as a function of the femtocell cell density. Intuitively, there should exist a threshold, above which adding more femtocells would cause network performance to saturate. Equation (18) gives a formulation for studying LTE network performance over a variety of deployment parameters and thus offers important guidelines that network designers and deployment personal can use to maximize their gains.

VII. CONCLUSION

LTE networks comprising macrocells plus femtocells are beginning to offer economically viable solutions to achieving high user capacity. The above coupled with the growing impetus for frequency reuse, underscores the need for efficient resource allocation mechanisms in such networks. In this paper, by formulating the resource allocation problem as an optimization problem we develop a distributed algorithm that makes use of a class of fairness measures for determining user scheduling. As indicated by our results, our algorithm is shown to be very efficient for realistic network parameters. We also propose a realistic air interface model for LTE, Macro- and Femto-cell networks. Additionally, we provide formulations that can be used by network designers and engineers to understand the impact of femtocell density, resource tradeoffs, and complexity-optimality tradeoffs and thus assist them with the planning and design of macrocells and femtocells for better return of investment (ROI). As next steps, we propose to analyze the impact of scale and varying mixes of traffic on the overall quality of service.

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