Joint Uplink Power Control, Beamforming and Bandwidth Allocation for Optimal QoS Assignment

Tian Lan[1], Prashanth Hande[1,2], Mung Chiang[1]
[1]Department of Electrical Engineering, Princeton University, NJ 08544, USA
[2]Flarion Technologies, NJ 07921, USA

Abstract— **This paper addresses the problem of joint resource allocation and QoS assignment in multi-cellular uplink of wireless data networks. We show how to jointly optimize three degrees of design freedom: transmit power, bandwidth allocation, and spatial diversity through antenna beamforming, in order to maximize the network-wide utility as a function of attained QoS. Many existing formulations of power control and antenna-beamforming can be viewed as special cases of this joint optimization. Our holistic framework leads to a quantitative understanding of the tradeoff among increasing power, partitioning the spectrum, and installing multiple antennas.**

The joint optimization is a globally coupled, non-convex problem. We solve it in several steps. First, we fix the bandwidth allocation and optimize transmit power across cells for a given beamforming vector. This extends naturally to the problem of jointly optimizing over receiver beamformer and transmit power, for which we develop a distributed algorithm that is proved to be convergent in general and globally optimal in the two user case, despite the non-convexity of this optimization. Next, the joint power-bandwidth optimization with fixed beamforming is shown to be a convex optimization, and we propose a distributed algorithm that computes the global optimum. Finally, an alternate maximization method is used to combine these two algorithms and shown to be convergent. Simulations show that the algorithm always converges to the jointly and globally optimal allocation, and demonstrate the tradeoff among power, bandwidth, and beamforming resources in maximizing network utility.

I. Introduction

A. Overview and Related Work

In contrast to cellular voice networks where all users demand the same QoS, wireless data networks need to optimize QoS assignment by allocating physical resources among interfering users. Such resources include transmit powers, bandwidth, and the spatial resource realized by antenna beamforming. This paper considers a joint optimization across these three degrees of design freedom, where the constraints come from the total bandwidth available, the interference limitations and the number of antennas that can be deployed. The objective is to enhance user experience as a function of attained QoS, and to maintain fairness among competing users. The utility function can capture both: it measures the efficiency of resource allocation and can enforce the notion of α-fairness [4]. Since the multi-cell uplink case is more mathematically and practically challenging than the single-cell or downlink cases, we focus on the following problem: maximizing network utility jointly over power control, bandwidth allocation, and beamforming for multi-cell uplink.

The problem of optimal power control has been studied extensively (eg, [1]-[3] [5] [10] [12] [11] [16] [17]). In particular, Foschini and Miljanic [1] showed that optimal power control for a fixed QoS assignment can be solved efficiently by an iterative power update algorithm. The algorithm allows a distributed implementation and has been embraced by operators of voice networks. Their solution since has been generalized by different authors [2] [3]. In later works, a different approach was provided [5] [10] [12] [11] [16], where a utility optimization framework was proposed to solve not only the power control for a fixed QoS, but also a joint power control and SIR optimization for varying QoS requirements. In a recent work, the authors of [17] formulated the problem based on the notion of 'load' and 'spillage', and showed that the joint power control and SIR optimization can be directly turned into an optimization over the load-factors. They further proposed a simple and fast load-spillage update algorithm that attains the optimal solution.

Alternatively, a higher QoS assignment can also be achieved by employing multiple antennas and receiver beamforming method at BSs. In this case, joint power control and beamforming has been considered in [8] [14] [13]. For the design objective of finding beamforming vectors to ensure a certain minimum SIR for each mobile user, the beamforming problem has been solved in [8] [14]. A separate result was provided in [13] dealing with optimal beamforming for max-min fairness. All these optimal schemes rely on channel measurements and feedback of each antenna gain. To overcome this difficulty, in another different approach termed opportunistic beamforming, the work in [7] proposes a scheme that only requires SIR feedback and is shown to be asymptotically optimal in the number of users.

The above set of approaches focus only on one set of the multiple resources. While analytically attractive, such approaches often give partial results to the more general problem of joint resource allocation. In this paper, our objective is to consider the problem of jointly optimizing power, bandwidth, and beamforming for QoS-based utility maximization. We tackle the overall problem in several steps.

First, we fix the bandwidth allocation and assume that the beamforming vectors are given by a separate algorithm that is either adaptively varying the beamforming vectors in some optimal sense, or simply varying the beamforming vectors in a pre-determined manner as in the opportunistic beamforming case. In either case, the ensuing QoS assignment should take into account the interference spilling into other users. We show that using fixed bandwidth allocation and given fixed beamforming vectors leads to a QoS assignment problem similar to the one that was dealt with in [17] with a different channel model to include beamformers. We propose a distributed mechanism to attain the globally optimal QoS allocation based on the notion of load and spillage developed in [17].

The problem then extends naturally to the formulation where the beamforming vectors are optimized along with the QoS assignment rather than being produced by a separate algorithm. We propose a distributed algorithm that is provably convergent. Empirical evidence suggests that the algorithm always converges to the global optimum, which we rigorously prove for the two use case.

Next, we assume fixed beamforming vectors and introduce the joint power-bandwidth optimization problem which turns out to be a convex optimization problem under the assumption that interference between users is averaged across the total bandwidth. This assumption is reasonable since interference averaging mechanisms are typically employed to combat inter-cell interference. We propose a distributed mechanism to attain the optimal allocation.

Finally, we tackle the non-convex optimization problem of jointly optimizing over power-bandwidth-beamforming, propose an algorithm that is provably convergent and empirically verify that it attains the global optimum in in all simulations.

B. Summary of Results, Organization, and Notation

The key results of this paper are summarized as follows:

- For existing beamformer realization schemes under fixed bandwidth allocation, generalizing our recent results in [17], we propose an interference management scheme to optimize the power allocation across cells (Problem A and Algorithm 1 in Section III).
- The problem extends to the more difficult optimization jointly over receiver beamforming and transmit powers, for which we develop a distributed algorithm that is proved to be convergent in general and globally optimal in the two user case, despite the nonconvexity of this optimization (Problem B and Algorithm 2 in Section IV).
- The joint power-bandwidth optimization with fixed beamforming is shown to be a convex optimization, and we propose a distributed algorithm that computes the global optimum (Problem C and Algorithm 3 in Section V).
- Finally, the joint optimization of power, bandwidth and beamformers is a globally coupled and non-convex problem, for which we propose an algorithm that is shown to be convergent (Problem OVERALL in Section II and Algorithm 4 in Section VI). The holistic framework not only recovers many existing results as special cases, it also leads to a quantitative understanding of the tradeoff among increasing power, partitioning the spectrum, and installing multiple antenna.
- Simulations in Section VII show that the algorithm always converges to the jointly and globally optimal allocation, and demonstrate the tradeoff among power, bandwidth, and beamforming resources in maximizing network utility.

We use the following notation. Vectors are denoted in bold small letter, e.g., $\mathbf{z}$, with their ith component denoted by z_i. Matrices are denoted by bold capitalized letters, e.g., $\mathbf{Z}$, with Z_{ij} denoting the $\{i, j\}$th component. Vector division $\mathbf{x}/\mathbf{y}$ and multiplication $\mathbf{xy}$ are considered component-wise, and vector inequalities denoted by $\succeq$ and $\preceq$ are component-wise inequalities. We use $\mathbf{D}(\mathbf{x})$ to denote a diagonal matrix whose diagonal elements are the corresponding components from vector $\mathbf{x}$. Subscripts $(\cdot)^T$ denotes the matrix transpose.

II. SYSTEM MODEL AND PROBLEM FORMULATION

A. System Model

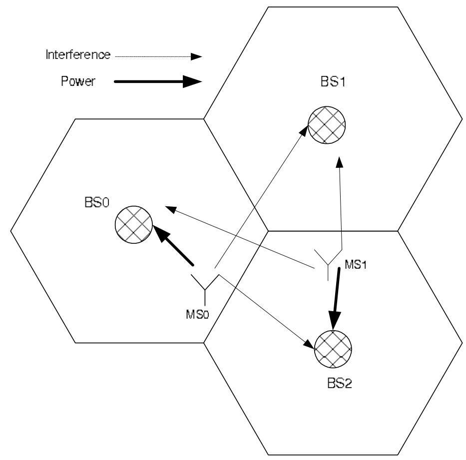

Fig. 1. An example of a multi-cellular network.

Consider a general multi-cell, multi-antenna setup where M mobiles, stations(MS) each equipped with a single antenna, establish links to N base-stations (BS), each equipped with an array of K antennas, as illustrated in Fig 1. We assume that each MS is served by one of the N BSs, thereby establishing M links. MS i is allocated a bandwidth b_i out of a total available bandwidth of B^m at each BS. We let σ_i be the receiving BS for link i. Then a bandwidth constraint is applied at each BS:

$$B^m = \sum_{j:\sigma_j = k} b_j, \text{ for } k = 1, \cdots, N. \tag{1}$$

In general, only MSs transmitting in the same bandwidth interfere with each other. However, a wireless system typically implements a mechanism that results in interference averaging across bandwidth between users in different cells. For example, the flash OFDM system employs frequency hopping to spread a signal of any bandwidth b_i across the entire bandwidth B^m. This allows us to define a QoS metric following the Shannon formula as a function of SIR γ_i and bandwidth b_i:

$$\beta(\gamma_i, b_i) = b_i \log_2(1 + B^m \gamma_i / b_i). \tag{2}$$

In this paper, we focus on the assignment of this QoS metric, although the methodologies can be applied to more general QoS metrics.

The multi-antenna channel gain from MS i to BS j is a $K\times 1$ vector, denoted by $\mathbf{h}_{ji}$. Let x_i be the information signal sent by mobile i. The BS receiver implements a linear beamformer denoted by $\mathbf{w}_i \in \mathbb{R}^K$. Our analysis is not restricted to spatially orthogonal beamforming. To capture the general corss-talk case, we let C_i denote the set of links whose transmitted power appear as interference to link i. The received signal at BS σ_i from MS i is given by

$$y_{\sigma_i i} = \left(\mathbf{w}_i^T \mathbf{h}_{\sigma_i i}\right) x_i + \sum_{j\in C_i} \left(\mathbf{w}_i^T \mathbf{h}_{\sigma_i j}\right) x_j + \mathbf{w}_i^T \mathbf{z}_{\sigma_i}. \quad (3)$$

where $\mathbf{z}_{\sigma_i}$ is an additive $K\times 1$ Gaussian noise with variance $\sigma^2 I$.

For notational convenience, the beamforming vectors are collected in a matrix $\mathbf{W} = \{\mathbf{w}_i\}$. Now the absolute path gain from MS i to BS σ_i is $(\mathbf{w}_i \mathbf{h}_{\sigma_i i})^2$. Define a $M\times M$ coupling matrix $\mathbf{G}(\mathbf{W})$ as a function of the beamforming vectors by

$$\left[\mathbf{G}(\mathbf{W})\right]_{ij} = \begin{cases} \frac{(\mathbf{w}_i^T \mathbf{h}_{\sigma_i j})^2}{(\mathbf{w}_i^T \mathbf{h}_{\sigma_i i})^2}, & \text{if } \ j \in C_i, \\ 0, & \text{if } j \notin C_i. \end{cases} \quad (4)$$

Using the fact that $G_{ij} = 0$ for $j \in C_i$, the total interference(including noise) at the BS serving MS i is given by,

$$q_i = \sum_{j=1}^{M} G_{ij} p_j + \eta_i, \quad (5)$$

where $p_i = E|x_i|^2$ is the transmit power from MS i and η_i is an equivalent noise given by

$$\eta_i = \frac{||\mathbf{w}_i||_2^2}{(\mathbf{w}_i^T \mathbf{h}_{\sigma_i i})^2}\sigma^2. \quad (6)$$

In vector form, equation (5) can be written as

$$\mathbf{q} = \mathbf{G}(\mathbf{W})\mathbf{p} + \boldsymbol{\eta}. \quad (7)$$

The SIR achieved by user i is defined by $\gamma_i = p_i/q_i$. Let $\mathbf{D}(\boldsymbol{\gamma})$ =diag$(\gamma_1, \ldots, \gamma_M)$. Applying (5), we get the basic system equations,

$$\mathbf{q} = \mathbf{G}(\mathbf{W})\mathbf{D}(\boldsymbol{\gamma})\mathbf{q} + \boldsymbol{\eta}, \quad (8)$$

and

$$\mathbf{p} = \mathbf{D}(\boldsymbol{\gamma})\mathbf{G}(\mathbf{W})\mathbf{p} + \mathbf{D}(\boldsymbol{\gamma})\boldsymbol{\eta}. \quad (9)$$

B. SIR Feasibility Region

Due to the interference between links, not all SIR vectors $\boldsymbol{\gamma}$ are achievable. An SIR vector $\boldsymbol{\gamma} \succ 0$ will be called *feasible* if there exists an interference vector, $\mathbf{q} \succeq 0$, and power vector $\mathbf{p} \succeq 0$, satisfying (8) and (9), respectively. Let $\rho(\cdot)$ denote the spectral radius function [1]. The following standard result gives a spectral radius characterization of SIR feasibility.

[1]Spectral radius is the maximum of the absolute value of the eigen-values of a matrix.

Lemma 1: An SIR vector $\boldsymbol{\gamma} \succ 0$ is feasible if and only if $\rho(\mathbf{G}(\mathbf{W})\mathbf{D}(\boldsymbol{\gamma})) < 1$, when $\boldsymbol{\eta} \neq 0$. Given $\boldsymbol{\gamma}$, the transmitted powers to the MSs $\mathbf{p}$ is given by

$$\mathbf{p}(\boldsymbol{\gamma}, \mathbf{W}) = \left[\mathbf{I} - \mathbf{D}(\boldsymbol{\gamma})\mathbf{G}(\mathbf{W})\right]^{-1} \mathbf{D}(\boldsymbol{\gamma})\boldsymbol{\eta} \quad (10)$$

and the interference is given by

$$\mathbf{q}(\boldsymbol{\gamma}, \mathbf{W}) = \left[\mathbf{I} - \mathbf{G}(\mathbf{W})\mathbf{D}(\boldsymbol{\gamma})\right]^{-1} \boldsymbol{\eta} \quad (11)$$

For a given transmit power vector $\mathbf{p} \succ 0$, we define the following set of feasible SIR vectors

$$\boldsymbol{\Gamma}(\mathbf{W}, \mathbf{p}) = \{\boldsymbol{\gamma} : \left[\mathbf{I} - \mathbf{D}(\boldsymbol{\gamma})\mathbf{G}(\mathbf{W})\right]^{-1} \mathbf{D}(\boldsymbol{\gamma})\boldsymbol{\eta} \leq \mathbf{p}\}. \quad (12)$$

The set $\boldsymbol{\Gamma}(\mathbf{W}, \mathbf{p})$ includes all feasible SIR vectors that can be achieved with given $\mathbf{W}$ and $\mathbf{p}$. Practical networks also have finite limits on the maximum power. With channel measurements available, we can translate the transmit power constraints into receive power constraints denoted by $\mathbf{p}^m$.

C. Optimization Problem Formulation

A natural way to formulate the joint resource allocation problem is through a utility framework. Let $U_i(\beta_i)$ be a *utility*, representing the value to the overall network of allocating link i a QoS metric β_i given by (2). For a given bandwidth allocation b_i, we use $U_i(\gamma_i)$ instead of the more appropriate $U_i(\beta(\gamma_i, b_i))$. We use the following notation: $U_i'(\gamma_i) = \partial U_i/\partial \gamma_i$ and $U_i''(\gamma_i) = \partial^2 U_i/\partial \gamma_i^2$. We will make the following assumptions on the utility functions throughout this paper.

1) The utility functions $U_i(\beta_i)$ are strictly increasing, twice differentiable and concave in β_i. $U_i(\beta_i)$ is fair in the sense that, as $\beta_i \to 0$, $U_i(\beta_i) \to -\infty$, so that zero QoS assignment is not allowed in any utility maximization solution.
2) For fixed $\mathbf{b}$:
   (a) $U_i(\gamma_i)$ is concave in $\log \gamma_i$.
   (b) $U_i'(\gamma_i)$ and $U_i''(\gamma_i)$ are both bounded for all $\gamma_i \geq 0$.

In particular, the assumptions are satisfied for α-fair utilities [4] when $\alpha \geq 1$.

The utility maximization problem can be written as maximization of $\sum_i U_i(\beta_i)$ subject to SIR feasibility and bandwidth availability. We introduce a joint framework in **Problem OVERALL** for optimization across power, bandwidth and beamformers.

$$\begin{aligned} \text{maximize} \quad & \sum_i U_i(\beta_i) \quad (13) \\ \text{subject to} \quad & \beta_i = b_i \log(1 + B^m \gamma_i / b_i), \\ & \sum_{j:\sigma_j = k} b_j = B^m, \ \text{ for } k = 1, \cdots, N, \\ & \boldsymbol{\gamma} \in \boldsymbol{\Gamma}(\mathbf{W}, \mathbf{p}), \\ & \mathbf{p} \leq \mathbf{p}^m, \\ \text{variables} \quad & \boldsymbol{\gamma}, \boldsymbol{\beta}, \mathbf{W}, \mathbf{b}, \mathbf{p}. \end{aligned}$$

Problem OVERALL is a joint optimization over power $\mathbf{p}$, bandwidth $\mathbf{b}$, and beamforming vectors $\mathbf{W}$, in order to find the optimal assignment of SIR $\boldsymbol{\gamma}$ and QoS $\boldsymbol{\beta}$. It formulates an interesting tradeoff between the three system resources and can provide benchmarks for system designers to determine how much of each resource should be allocated to achieve a certain QoS.

We note that problem OVERALL is globally coupled and non-convex in general. The development of distributed algorithms for various special cases of this problem and their performance analysis will be the main thread for the rest of the paper.

III. Optimization over Power with Multiple Antennas

Consider now the case where the bandwidth assignment is fixed. We assume that the beamforming vectors are given to us by a separate algorithm that either *fixes* the beamforming vectors for each MS or alters the beamforming vectors according to a *fixed* pattern or even adaptively changes the beamforming vectors in response to some suitable feedback. For example, the opportunistic beamforming mechanism in [7] changes the beamforming vector in a fixed pattern so as to let every MS achieve the optimal beamforming configuration at which it's SIR is maximized. A scheduling mechanism such as the proportional fair scheduler then picks the MS that is closest to the maximal for transmission. The MS is then allocated a QoS according to the maximized SIR.

In a multi-cell network, however, the QoS assignment cannot be made solely based on the optimal beamforming configuration attained. We need a mechanism to manage the inter-cell interference in an optimal way so as to maximize the network wide efficiency. The algorithm in [17] is precisely such a mechanism for the single antenna case and we extend it below to the multiple antenna case.

The problem (**Problem A**) is defined as follows

$$\begin{aligned} \text{maximize} \quad & \sum_i U_i(\gamma_i) \\ \text{subject to} \quad & \boldsymbol{\gamma} \in \boldsymbol{\Gamma}(\mathbf{p}), \\ & \mathbf{p} \leq \mathbf{p}^m, \\ \text{variables} \quad & \boldsymbol{\gamma}, \mathbf{p}. \end{aligned} \tag{14}$$

Let $\boldsymbol{\nu}$ be the Lagrangian multiplier for the transmit power constraints. It has been shown in [17] that the set of feasible SIRs can be parameterized by a positive vector $\mathbf{s} \succ 0$,

$$\boldsymbol{\gamma}(\mathbf{s}) = \mathbf{s}/\mathbf{r}(\mathbf{s}), \tag{15}$$

where the division of the vectors is component-wise, and $\mathbf{r}(\mathbf{s})$ is the following M-dimensional vector:

$$\mathbf{r}(\mathbf{s}) = \mathbf{G}^T\mathbf{s} + \boldsymbol{\nu}. \tag{16}$$

Vectors $\mathbf{s}$ and $\mathbf{r}$ are known as the load vector and the spillage vector respectively. They have an interesting interpretation: The spillage factor r_i, of the form $r_i = \sum_j G_{ji}s_i + \nu_i$, represents the effect of interference due to link i on other links in the network weighted by the loads of each link. Link i, with an SIR γ_i, loads the network with $s_i = r_i\gamma_i$ in the sense that link i is less tolerant by a factor of s_i to interference from other links in the network.

Thus problem (14) can be transformed into an optimization over the load vector $\mathbf{s}$ and power price vector $\boldsymbol{\nu}$. The minimization over $\boldsymbol{\nu}$ is carried out using the gradient update for the Lagrangian multipliers with an appropriate step size δ_ν.

$$\nu_i[t+1] = [\nu_i[t] + \delta_\nu(p_i[t] - p^m{}_i)]^+. \tag{17}$$

The maximization over $\mathbf{s}$ is accomplished by an ascent update in the direction

$$\Delta s_i = \frac{U_i'(\gamma_i)\gamma_i}{q_i} - s_i. \tag{18}$$

That (18) is indeed an ascent update is shown in [17]. We summarize the algorithm below. It is guaranteed to converge and the solution can be shown to achieve the global optimal [17].

Algorithm 1 (**Power Control for Optimal SIR Assignment**):

- Parameters: step size $\delta_s > 0, \delta_\nu > 0$ and utility functions $\{U_i(\beta_i)\}$.
- Initialize: Arbitrary positive vectors $\mathbf{s}[0]$, $\boldsymbol{\nu}[0]$.

1) Compute the spillage-factor $r_i[t]$ according to (16).
2) Assign SIR values $\gamma_i[t] = s_i[t]/r_i[t]$.
3) Mobiles adjust maximum power $\mathbf{p}$ until actual SIR γ_{real} converges to the target SIR:
$$p_i^+[t] = \frac{\gamma_i[t]}{\gamma_{\text{real}}}p_i[t].$$
4) Measure the resulting interference $q_i[t]$.
5) Update the load factor $s_i[t]$ in the ascent direction given by by (18),
$$s_i[t+1] = s_i[t] + \delta_s\Delta s_i[t].$$
6) Update power price according to (17).

Continue: $t := t + 1$.

IV. Joint Optimization over Beamforming and Power

We keep the bandwidth allocation b_i for each link fixed but optimize over beamformers rather than accept the beamformers from a separate algorithm. Problem OVERALL reduces to a joint beamforming and power optimization (**Problem B**) as follows

$$\begin{aligned} \text{maximize} \quad & \sum_i U_i(\gamma_i) \\ \text{subject to} \quad & \boldsymbol{\gamma} \in \boldsymbol{\Gamma}(\mathbf{W}, \mathbf{p}), \\ & \mathbf{p} \leq \mathbf{p}^m, \\ \text{variables} \quad & \boldsymbol{\gamma}, \mathbf{W}, \mathbf{p}. \end{aligned} \tag{19}$$

Note that for fixed beamforming vectors $\mathbf{W}$, problem (19) reduces to Problem A for which a solution was proposed using an ascent update on the load-factor $\mathbf{s}$. We follow a similar approach here and construct an update for the beamforming vectors in an ascent direction. Consider the Lagrangian for problem (19) with respect to s and power constraint prices $\boldsymbol{\nu}$:

$$\mathcal{L}(\mathbf{s},\mathbf{W},\boldsymbol{\nu}) = \sum_i U_i(\gamma_i(\mathbf{s},\mathbf{W})) + \boldsymbol{\nu}^T(\mathbf{p}^m - \mathbf{p}(\boldsymbol{\gamma}(\mathbf{s},\mathbf{W}),\mathbf{W})), \quad \text{where } \boldsymbol{\gamma}(\mathbf{s},\mathbf{W}) = \mathbf{s}/(\mathbf{G}^T(\mathbf{W})\mathbf{s}+\boldsymbol{\nu}). \tag{20}$$

The solution to the primal problem satisfies the Lagrangian stationarity equation in the KKT optimality conditions [18], given by

$$(\partial L/\partial \boldsymbol{\gamma})^T \frac{\partial \boldsymbol{\gamma}}{\partial \mathbf{s}} = \mathbf{0}. \tag{21}$$

$$\sum_i U_i'(\gamma_i)\frac{\partial \boldsymbol{\gamma}_i}{\partial \mathbf{w}_j} = \frac{\partial(\boldsymbol{\nu}^T\mathbf{p})}{\partial \mathbf{w}_j}. \tag{22}$$

Using the results in Appendix A, condition (21) implies $U_i'(\gamma_i) = r_i q_i$ and an ascent direction on s can be found.

To construct the optimal beamforming vectors, we consider condition (22). First, we obtain the following equation

$$\begin{aligned}\boldsymbol{\nu}^T\mathbf{p}^m &= \boldsymbol{\nu}^T(\mathbf{I}-\mathbf{D}(\boldsymbol{\gamma})\mathbf{G}(\mathbf{W}))^{-1}D(\boldsymbol{\gamma})\boldsymbol{\eta},\\ &= \boldsymbol{\nu}^T\mathbf{D}(\boldsymbol{\gamma})\left(\mathbf{I}-\mathbf{D}(\boldsymbol{\gamma})\mathbf{G}(\mathbf{W})^T\right)^{-T}\boldsymbol{\eta},\\ &= \mathbf{s}^T\boldsymbol{\eta}.\end{aligned} \tag{23}$$

Using (23), the first-order condition on $\mathbf{w}_k$ results in

$$0 = \sum_i U_i'(\gamma_i)\frac{\partial \boldsymbol{\gamma}_i}{\partial \mathbf{w}_j} - \frac{\partial(\boldsymbol{\nu}^T p)}{\partial \mathbf{w}_j} \tag{24}$$

$$\begin{aligned}&= \sum_i U_i'(\gamma_i)\frac{\partial \boldsymbol{\gamma}_i}{\partial \mathbf{w}_j} - s_j\frac{\partial \boldsymbol{\eta}_i}{\partial \mathbf{w}_j}\\ &= \Bigg[\sum_i \frac{U_i'(\gamma_i)\gamma_i^2}{s_i}\left(\frac{(\mathbf{w}_j^T\mathbf{h}_{\sigma_j i})^2}{\mathbf{w}_j^T\mathbf{h}_{\sigma_j j}} - \mathbf{w}_j^T\mathbf{h}_{\sigma_j i}\mathbf{h}_{\sigma_j i}^T\right)\\ &\quad -\sigma^2 I + \frac{||\mathbf{w}_j||_2^2\sigma^2}{\mathbf{w}_j^T\mathbf{h}_{\sigma_j j}}\Bigg]\cdot\frac{2s_j}{(\mathbf{w}_j^T\mathbf{h}_{\sigma_j j})^2}.\end{aligned} \tag{25}$$

This implies the optimal beamforming vector must satisfy

$$\left(\sigma^2 I + \sum_i \frac{U_i'(\gamma_i)\gamma_i^2}{s_i}\mathbf{h}_{\sigma_j i}\mathbf{h}_{\sigma_j i}^T\right)\mathbf{w}_j = c\cdot \mathbf{h}_{\sigma_j i}, \tag{26}$$

where c is a scalar constant depending on $\mathbf{w}_k$. The amplitude and phase of the beamforming vectors have no effect on the SIR. Without loss of generality, we assume $c = 1$. Plugging in (18), the optimal beamforming vector is derived as follows

$$\mathbf{w}_j = \left[\sigma^2\mathbf{I} + \sum_i\left(1+\frac{\Delta s_i}{s_i}\right)p_i\mathbf{h}_{\sigma_j i}\mathbf{h}_{\sigma_j i}^T\right]^{-1}\mathbf{h}_{\sigma_j i}. \tag{27}$$

Such $\mathbf{w}_i$ maximizes the Lagrangian for a given load factor s.

To compute the optimal beamforming vectors in (27), $\Delta s_i/s_i$ is required at all BS. If there exists a message-exchanging mechanism across BSs in the network, these quantities can be made available at each BS. Thus beamforming vectors in (27) can be solved at each BS distributively.

If message-exchanging across BSs is not allowed, we update the load-factor s until it converges for every $\boldsymbol{\nu}$ and $\mathbf{w}_i$. The convergence implies $\Delta s_i = 0$, for $i = 1,\ldots,M$. The beamforming vectors obtained in (27) then become

$$\mathbf{w}_j = \left[\sigma^2 I + \sum_i p_i\mathbf{h}_{\sigma_j i}\mathbf{h}_{\sigma_j i}^T\right]^{-1}\mathbf{h}_{\sigma_j i}. \tag{28}$$

Such $\mathbf{w}_i$ is in fact the minimum-mean-square-error (MMSE) filter [14]. The matrix inversion is known as a whitening filter, which can be easily obtained at each BS using a power spectrum estimation and matrix inverse. This beamforming vector update removes the need for BS message-exchange although more iterations are required.

Both (27) and (28) give ascent updates for the beamforming vectors. This allows us to apply an alternate optimization method. The proposed algorithm for joint beamforming and power optimization is summarized as below.

*Algorithm 2 (***Joint Power Control and Beamforming):**

- Parameters: step size $\delta_s > 0, \delta_\nu > 0$ and utility functions $\{U_i(\gamma_i)\}$.
- Initialize: Arbitrary positive vectors $s[0], \nu[0]$.

1) BS updates beamforming vectors $\mathbf{w}_i$,
   (a) according to (27) if BS message-exchanging is enabled.
   (b) according to (28) otherwise.
2) Apply step 1) to 5) in Algorithm 1 with the following
   (a) one iteration only if BS message-exchange is enabled.
   (b) multiple iterations until load-spillage factors converge.
3) Power control for each mobile $p_i^+[t] = p_i[t]\gamma_i[t]/\gamma_{\text{act}}$.
4) Update power price according to (17).

Continue: $t := t+1$.

Theorem 1: (**Convergence**) For sufficiently small step size $\delta > 0$, Algorithm 2 converges to a stationary point of problem (19).

Proof: For a fixed price vector ν, Algorithm 2 employs an alternate maximization of the Lagrangian,

$$F(\mathbf{s},\mathbf{W}) = \mathcal{L}(\mathbf{s},\mathbf{W},\boldsymbol{\nu}). \tag{29}$$

For fixed $\mathbf{s}[t]$, $\mathbf{W}[t]$ is updated such that objective F is maximized by $\mathbf{W}[t+1]$. For fixed $\mathbf{W}[t+1]$, $\mathbf{s}[t]$ is updated in an ascent direction according to (18). Thus, the alternate maximization generates a sequence of non-decreasing objective values and it converges if $F(\mathbf{s},\mathbf{W})$ is upper bounded. To show this, we consider a strictly positive $\boldsymbol{\nu}$ and observe

$$\lim_{\mathbf{p}\to+\infty} F(\mathbf{s},\mathbf{W}) = -\infty. \tag{30}$$

This implies that F is indeed upper bounded for any strictly positive $\boldsymbol{\nu}$. In practice, we can make $\boldsymbol{\nu}$ arbitrarily close to 0 according to the tolerance set for our optimization.

The update for price vectors $\boldsymbol{\nu}$ and $\boldsymbol{\lambda}$ correspond to the gradient for the Lagrangian multipliers. In practice, both the load factor $\mathbf{s}$, beamforming vector $\mathbf{W}$ and price $\boldsymbol{\nu}$ are updated simultaneously. Such a joint update can be proven to converge to the optimum of problem (19) if the Lagrangian is concave and the maximized Lagrangian for a given price is convex in the price. This is true when problem (19) is convex. ■

Theorem 2: (**Optimality**) For a network with one BS and two mobile users, the optimization problem is convex and the stationary point is indeed a global optimal if $p_i^m \le \frac{\sigma^2}{a||\mathbf{h}_{\sigma_j i}||^2}$ is satisfied, where a is a channel correlation factor defined by

$$a = \sqrt{1 - \frac{((\mathbf{h}_{\sigma_1 1}^H \mathbf{h}_{\sigma_2 2})^2)}{(||\mathbf{h}_{\sigma_1 1}||^2 \cdot ||\mathbf{h}_{\sigma_2 2}||^2)}}. \quad (31)$$

Proof: We show that under the given conditions, the joint beamforming and power optimization problem can be transformed into a convex optimization problem using a logarithmic change of variables. A proof for this fact is provided in Appendix B. Thus, the optimization problem is convex over variables $\log \gamma_i$ and $\log p_i$, and the convergence is indeed towards a global optimum. ■

V. Joint Optimization over Bandwidth and Power

In this section, we assume beamforming vectors are fixed and the utility optimization is over power and bandwidth. Optimization over bandwidth can be accomplished by a suitable allocation of tones in an OFDM network or an allocation of p-n sequences in a CDMA network. A cellular network typically implements an interference averaging mechanism where the power allocated on any given tone or p-n sequence interferes equally with every tone or p-n sequence in neighboring cells. For example, the flash-OFDM system is an OFDM system that employs frequency hopping patterns that spreads the interference from one tone equally across all the tones in a neighboring cell. The interference averaging mechanism simplifies the interference structure in the joint power-bandwidth optimization problem and results in a convex optimization problem.

We begin our analysis by proposing a joint power-bandwidth optimization (**Problem C**) as follows

$$\begin{aligned}
\text{maximize} \quad & \sum_i U_i(\beta_i) \qquad (32)\\
\text{subject to} \quad & \beta_i = b_i \log(1 + B^m \gamma_i / b_i),\\
& \sum_{j:\sigma_j = k} b_j = B^m, \text{ for } k = 1, \cdots, N,\\
& \boldsymbol{\gamma} \in \boldsymbol{\Gamma}(\mathbf{p}),\\
& \mathbf{p} \le \mathbf{p}^m,\\
\text{variables} \quad & \boldsymbol{\gamma}, \boldsymbol{\beta}, \mathbf{b}, \mathbf{p}.
\end{aligned}$$

It can be verified that β is a concave function of b_i and the problem is a convex optimization. The Lagrangian for the primal problem (32) can be written down as

$$\begin{aligned}
\mathcal{L}(\boldsymbol{\gamma}, \mathbf{b}, \boldsymbol{\nu}, \boldsymbol{\lambda}) = \; & \sum_i U_i(\beta_i) + \sum_k \nu_k (p_k^m - p_k(\boldsymbol{\gamma}))\\
& + \sum_k \lambda_k (B^m - \sum_{i:\sigma_i = k} b_i) \qquad (33)
\end{aligned}$$

where we have introduced the bandwidth prices λ_k at BS k in addition to the power prices ν_i. The solution to the primal problem satisfies the Lagrangian stationarity equation in the KKT [18] optimality conditions, given by

$$\begin{aligned}
U_i'(\beta_i) \frac{\partial \beta_i}{\partial \gamma_i} &= \boldsymbol{\nu}^T \frac{\partial \mathbf{p}}{\partial \boldsymbol{\gamma}} \mathbf{e}_i.\\
U_i'(\beta_i) \frac{\partial \beta_i}{\partial b_i} &= \lambda_{\sigma_i}. \qquad (34)
\end{aligned}$$

Evaluating the gradient (see Appendix A), we have

$$\begin{aligned}
U_i'(\beta_i) \frac{\partial \beta_i}{\partial \gamma_i} &= r_i q_i.\\
U_i'(\beta_i) \frac{\partial \beta_i}{\partial b_i} &= \lambda_{\sigma_i}. \qquad (35)
\end{aligned}$$

If $\beta_i = b_i \log(1 + B^m \gamma_i / b_i)$, then we have

$$\begin{aligned}
U_i'(\beta_i) \frac{B^m}{1 + B^m \theta_i} &= r_i q_i.\\
U_i'(\beta_i)(\log(1 + B^m \theta_i) - \frac{B^m \theta_i}{1 + B^m \theta_i}) &= \lambda_{\sigma_i}. \qquad (36)
\end{aligned}$$

where $\theta_i = \gamma_i / b_i$ has the interpretation of SIR per tone. To proceed further, we fix the utility function to proportional utility $U_i(\beta_i) = \log(\beta_i)$. In that case, the KKT conditions reduce to

$$\begin{aligned}
\frac{B^m}{b_i(1 + B^m \theta_i) \log(1 + B^m \theta_i)} &= r_i q_i.\\
1/b_i &= r_i q_i \theta_i + \lambda_{\sigma_i}. \qquad (37)
\end{aligned}$$

To construct a distributed mechanism to achieve the utility optimization over $\mathbf{b}$ we introduce the following bandwidth update direction

$$\Delta b_i = \frac{1}{\lambda_{\sigma_i} + r_i q_i \theta_i} - b_i, \qquad (38)$$

and the following bandwidth price update

$$\lambda_k[t+1] = [\lambda_k[t] + \delta(\sum_{i:\sigma_i = k} b_i) - B^m]^+, \quad k = 1, \ldots, N \quad (39)$$

The algorithm is as follows.

*Algorithm 3 (***Joint Power-Bandwidth Allocation):**

- Parameters: step size $\delta_s > 0, \delta_w > 0$ and utility functions $\{U_i(\beta_i)\}$.
- Initialize: Arbitrary positive vectors $\mathbf{s}[0], \mathbf{b}[0], \boldsymbol{\lambda}[0]$.

1) Compute the spillage-factor $r_i[t]$ according to (16).
2) Assign SIR values $\gamma_i[t] = s_i[t] / r_i[t]$.
3) Mobiles adjust maximum power $\mathbf{p}$ until actual SIR γ_{real} converges to the target SIR:

$$p_i^+[t] = \frac{\gamma_i[t]}{\gamma_{\text{real}}} p_i[t].$$

4) Measure the resulting interference $q_i[t]$.
5) Update the load factor $s_i[t]$ in the ascent direction given by (18)
$$s_i[t+1] = s_i[t] + \delta_s \Delta s_i[t].$$
6) Update bandwidth allocation in the direction given by (38)
$$b_i[t+1] = b_i[t] + \delta_w \Delta w_i[t]$$
7) Update power price according to (17) and bandwidth price according to (39).

Continue: $t := t + 1$.

Theorem 3: (**Convergence and Optimality**) For sufficiently small step sizes δ_ν and $\delta_{\mathbf{w}}$, Algorithm 3 converges to the globally optimal solution of problem (32).

Proof: For given price vectors ν and $\boldsymbol{\lambda}$, we use the load-spillage characterization and consider Lagrangian (20) as a function of the load vector and the bandwidth allocation vector:

$$F(\mathbf{s}, \mathbf{b}) = \mathcal{L}(\boldsymbol{\gamma}(\mathbf{s}), \mathbf{b}, \boldsymbol{\nu}, \boldsymbol{\lambda}). \tag{40}$$

Based the results in [17], $\Delta \mathbf{s}$ in Algorithm 2 is an ascent update with respect to a fixed bandwidth allocation $\mathbf{b}$. The bandwidth update is also in an ascent direction since Δw_i defined in (38) is indeed a gradient of $F(\mathbf{s}, \mathbf{b})$. In addition, it can be shown that the Lipschitz continuity property is satisfied by $F(\mathbf{s}, \mathbf{b})$ which implies the existence of a non-zero step size δ such that $F(\mathbf{s}[t+1], \mathbf{b}[t+1]) \geq F(\mathbf{s}[t], \mathbf{b}[t])$. With the Lipschitz property, the descent lemma in [17] implies the convergence to the Lagrangian maximization.

The updates for price vectors $\boldsymbol{\nu}$ and $\boldsymbol{\lambda}$ correspond to the gradient for the Lagrangian multipliers if we allow convergence to the Lagrangian maximum over $\mathbf{s}$ and $\mathbf{b}$ at every prices $\boldsymbol{\nu}[t]$ and $\boldsymbol{\lambda}[t]$. Of course, in practice, both the load factor $\mathbf{s}$, bandwidth allocation $\mathbf{b}$, and prices $\boldsymbol{\nu}[t]$ and $\boldsymbol{\lambda}$ are updated simultaneously. Such a joint update can be proven to converge to the optimum of the original constrained optimization problem if the Lagrangian is concave and the maximized Lagrangian for a given price is convex in the price. This is indeed true and therefore the algorithm converges to the global optimum. ■

VI. Optimization over Three Physical Resources

We now propose an algorithm for the problem OVERALL introduced in Section II. This joint resource optimization over all variables is non-convex. We propose an algorithm that alternates between two sets of variables $\{\boldsymbol{\gamma}(\mathbf{s}), \mathbf{W}\}$ and $\{\boldsymbol{\gamma}(\mathbf{s}), \mathbf{b}\}$. For a fixed $\mathbf{b}$, we solve a joint optimization over $\{\boldsymbol{\gamma}(\mathbf{s}), \mathbf{W}\}$, and for a fixed $\mathbf{W}$ we solve the joint optimization over $\{\boldsymbol{\gamma}(\mathbf{s}), \mathbf{b}\}$ and iterate. By applying Algorithm 2 and Algorithm 3 iteratively, we propose a distributed greedy algorithm for problem OVERALL. The ascent updates for subproblems are summarized in table II.

The entire procedure of our proposed algorithm is summarized below.

TABLE I

Ascent updates for resource optimization.

Resource	Update method
Power Control	$s_i[t+1] = s_i[t] + \delta_s \Delta s_i$
	$\Delta s_i = \frac{U_i'(\gamma_i)\gamma_i}{q_i} - s_i$
Bandwidth Allocation	$b_i[t+1] = b_i[t] + \delta_w \Delta w_i[t]$
	$\Delta w_i = \frac{1}{\lambda_i + \gamma_i q_i \theta_i} - b_i$
Beamforming	$\mathbf{w}_j = \left[\sigma^2 I + \sum_i \left(1 + \frac{\Delta s_i}{s_i}\right) p_i \mathbf{h}_{\sigma_j i} \mathbf{h}_{\sigma_j i}^T\right]^{-1} \mathbf{h}_{\sigma_j i}$

*Algorithm 4 (***Joint optimization of three resources***):*

- Parameters: step size $\delta_s > 0, \delta_\nu > 0, \delta_w > 0$ and utility functions $\{U_i(\gamma_i)\}$.
- Initialize: Arbitrary positive vectors $\{s[0], \nu[0], \mathbf{b}[0]\}$.

1) Perform step 1) to 4) in Algorithm 3 until it converges.
2) Use the resulting $\mathbf{s}^+[t]$ as an initial input for the next step.
3) Perform step 1) to 7) in Algorithm 2 until it converges.
4) Assign the resulting load factor to $\mathbf{s}[t+1]$.

Continue: $t := t + 1$.

Theorem 4: (**Convergence**) Algorithm 4 converges to a stationary point of problem OVERALL.

Proof: Algorithm 4 alternatively solves two sub-problems of problem OVERALL such that utility performance in one sub-problem is carried over to the other sub-problem whenever an algorithm transition occurs. The output load-factor $\mathbf{s}[t]$ of one algorithm is used as an initial input for the next iteration. Each iteration is thus an increasing update for network utility. Convergence is guaranteed if the utility function is upper bounded. This is true since power, bandwidth and multi-antenna channel gain are all finitely bounded by constraints in problem OVERALL. The ascent argument and bounded utility ensure the convergence to a stationary point. ■

All of load factor $\mathbf{s}$, beamforming vector $\mathbf{W}$ and bandwidth B can be updated at the same time. This simultaneous update works well for problem OVERALL in practice.

VII. Simulation Results

We summarize the system parameters used for all numerical simulations. Channel matrices are drawn from an i.i.d. Gaussian distribution with mean 0 and variance 1. Maximum power and bandwidth constraints are assumed to be $p_i^m/\sigma^2 =$ 0dB and $B^m = $ 1MHz respectively unless otherwise stated. The utility function is chosen to be a logarithmic function $U_i(\beta_i) = \log(\beta_i)$. We adopt a uniform step size $\delta = 0.01$ for all updates.

In the first set of simulations, we present the convergence behavior of Algorithms 1, 2 and 3, for a network with $N = 1$ BS, $M = 1$ users and $K = 3$ BS antennas. The same downlink channel is used, but with fixed beamforming vectors $\mathbf{w}_i = \mathbf{1}$ for Algorithm 1 and 2, and fixed bandwidth allocation $b_i =$

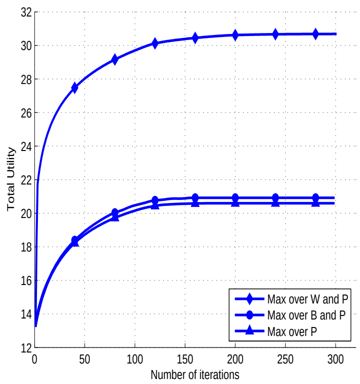

Fig. 2. Plot of the convergence behavior of Algorithms 1, 2, and 3.

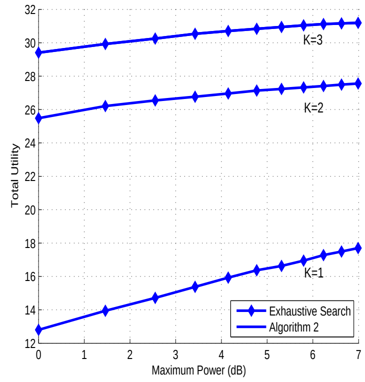

Fig. 3. Plot of optimal utility v.s. transmit power constraint.

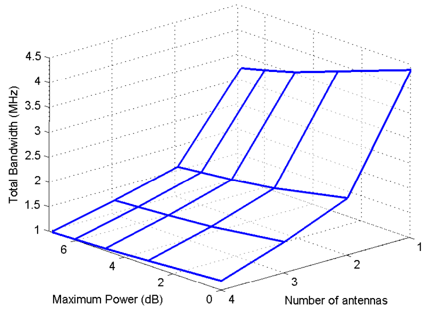

Fig. 4. A Pareto-optimal tradeoff surface among different resource allocations that achieve the same utility.

B^m/M for Algorithm 3. In Fig. 2, the total utility is plotted against the number of iterations. All algorithms are observed to be very efficient. We also compare the relative benefits of beamforming v.s. bandwidth allocation. As illustrated in Fig. 2, the joint optimization over beamforming and power provides substantial QoS improvement over power control alone, whereas the joint optimization over bandwidth and power does not offer as much benefits.

Next we numerically verify the global optimality of Algorithm 3. Consider two cells each with $M = 4$ mobiles. Suppose that each mobile is capable of transmitting a maximum power of p^m. We can solve the optimal total utility of the joint optimization over beamforming and power using either Algorithm 3 or an exhaustive search. Fig. 3 illustrates the optimal total utility v.s. maximum transmit power averaged on 10 different channel realizations, in which the utility-power curves plotted by Algorithm 3 and the exhaustive search always coincide. Thus the global optimality of Algorithm 3 is numerically confirmed. The three curves shown in Fig. 3 correspond to different numbers of antennas with $K = 1, 2$ and 3. Since a logarithmic utility $U_i(\beta_i) = \log(\beta_i)$ is considered, the optimal total utility in Fig.3 reflects the geometrical mean of data rates β_i of all links. We note that adding more antennas to a network results in a significant QoS improvement; as much as a 75% utility increase can be gained by upgrading the network from one to two BS antennas. However, the benefits of adding more antennas shows diminishing marginal returns; upgrading from two to three BS antennas doesn't offer as much improvement.

Finally, we illustrate the optimal utility as a function of three resource parameters: transmit power p^m, total bandwidth B^m and number of BS antennas K. In a network with $M = 1$ users, we use Algorithm 4 presented in Section VI for solving the joint utility optimization problem under these three constraints. Fig. 4 illustrates a Pareto-optimal tradeoff surface for the optimized utility contour over 3-D bases, by plotting a set of resource bundles $\{B^m, p^m, K\}$ pertaining to the same optimal utility. This plot provides system design benchmarks based on a cost-effectiveness analysis. More precisely, each resource can be assigned a cost based on its usage, and network operators then decide how to distribute their budget on the Pareto-optimal surface according to their individual preferences to minimize their individual cost. For example, since bandwidth is a scarce resource in a wireless cellular network, a large cost function should be associated with the total bandwidth usage, whereas smaller costs are assigned to power and number of antennas. This choice of costs would result in a resource bundle of $\{B^m = 1M, p^m = 5, K = 4\}$, in which the total bandwidth usage has been minimized. This is in contrast to a different resource bundle of $\{B^m = 4.37M, p^m = 1, K = 1\}$ lying on the same tradeoff surface, where number of antennas and transmit power has been minimized.

VIII. Conclusion and Future Work

In this paper, we study the joint resource allocation problem in a multi-cell multi-antenna wireless network. The main goal is to achieve an optimal QoS assignment through joint power control, bandwidth allocation and beamforming across the entire network. The proposed optimization problem is globally coupled and non-convex in general. To overcome this difficulty, we make use of an alternate optimization method with spillage-load updates. The algorithm is distributed and provably convergent. Even proving global optimality is still in progress, we show optimality for several special cases. Our unified resource allocation is novel in the sense that it allows a theoretical computation of the tradeoff between increasing power, partitioning the spectrum and installing multiple antenna, making the result applicable to practical network designs.

Our mechanism can be extended to the multi-cell downlink case through an uplink-downlink duality result [14] that we have extended to the multiple cell case. The duality can be exploited to provide an algorithm for joint downlink beamforming, power control and bandwidth allocation to achieve an optimal QoS assignment.

Finally, fading and mobility will be integrated into our general framework in future works. For example, our proposed algorithm requires precise channel information at BSs, and it would be interesting to generalize the result to the case of fading channels where very limited information can be fedback, and analyze the tradeoff between random beamforming vs. optimal beamforming. Similarly, the issue of network topology change would also be covered by introducing user mobility and soft handoff models.

Appendix A: Gradient Evaluation

We state the following results on gradients, skipping the proofs due to space limitation. If $\mathbf{p}(\gamma)$ and $\mathbf{q}(\gamma)$ are given according to (10) and (11), then

$$\frac{\partial \mathbf{p}}{\partial \gamma} = (\mathbf{I} - \mathbf{D}(\gamma)\mathbf{G})^{-1}\mathbf{D}(\mathbf{q}), \tag{41}$$

$$\frac{\partial \mathbf{q}}{\partial \gamma} = (\mathbf{I} - \mathbf{G}\mathbf{D}(\gamma))^{-1}\mathbf{G}\mathbf{D}(\mathbf{q}). \tag{42}$$

If $\boldsymbol{\gamma} = \boldsymbol{\gamma}(\mathbf{s}, \boldsymbol{\nu})$ for some fixed vector $\boldsymbol{\nu}$, then

$$\frac{\partial \gamma}{\partial \mathbf{s}} = \mathbf{D}(1/\mathbf{r})(\mathbf{I} - \mathbf{G}\mathbf{D}(\gamma))^T. \tag{43}$$

If $L(\mathbf{s}, \mathbf{W}, \boldsymbol{\nu})$ is the Lagrangian defined in Section IV and $F(\mathbf{s}) = L(\mathbf{s}, \mathbf{W}, \boldsymbol{\nu})$ for some fixed $\mathbf{W}$ and $\boldsymbol{\nu}$, then

$$\begin{aligned}\nabla F^T(\mathbf{s}) &= (\partial F/\partial \gamma)^T \frac{\partial \gamma}{\partial \mathbf{s}}, \\ &= (\mathbf{U}'(\gamma) - \mathbf{r}\mathbf{D}(\mathbf{q}))^T \mathbf{D}(1/\mathbf{r})(\mathbf{I} - \mathbf{G}\mathbf{D}(\gamma))^T.\end{aligned}$$

Further, if the ascent direction $\Delta \mathbf{s}$ is defined in (18), we can show that $\nabla F^T \Delta \mathbf{s} = 0$ if and only if $\nabla F(\mathbf{s}) = 0$.

Appendix B: Proof for Beamforming Convexity

For a network with one BS and two mobile users, to show problem (19) is indeed a convex optimization with a logarithmic change of variable, we only need to show that the SIR constraints

$$\gamma_i \geq \frac{(\mathbf{w}_i^T \mathbf{h}_{\sigma_i i})^2 p_i}{\sum_{j \in C_i} (\mathbf{w}_i^T \mathbf{h}_{\sigma_i j})^2 p_j + \sigma^2 ||\mathbf{w}_i||^2}, \tag{44}$$

can be transformed into a convex function. To this end, we plug the optimal beamforming vector (27) into (44) above. This gives

$$p_i \mathbf{h}_i^H \left(\sigma^2 I + \sum_{j=1}^{K} p_j \mathbf{h}_j \mathbf{h}_j^H \right)^{-1} \mathbf{h}_i \geq \frac{\gamma_i}{1 + \gamma_i}. \tag{45}$$

where $\mathbf{h}_i$ denotes $\mathbf{h}_{\sigma_i i}$ for simplicity. To cancel the $p_i \mathbf{h}_i \mathbf{h}_i^H$ term in the matrix inverse, we can apply the Matrix Inverse Lemma

$$(A + BCD)^{-1} = A^{-1} - A^{-1}B(C^{-1} + DA^{-1}B)^{-1}DA^{-1}. \tag{46}$$

Denote $A = \sigma^2 I + \sum_{j \in C_i i} p_j \mathbf{h}_j \mathbf{h}_j^H$. By simple manipulations, we obtain $p_i \mathbf{h}_i^H A^{-1} \mathbf{h}_i \geq \gamma_i$ as an equivalent SIR constraint.

Now let $K = 2$. We establish a sufficient condition such that the SIR constraint for user one is convex:

$$p_1 \mathbf{h}_1^H \left(\sigma^2 I + p_2 \mathbf{h}_2 \mathbf{h}_2^H\right)^{-1} \mathbf{h}_1 \geq \gamma_1. \tag{47}$$

The observation is to apply the Matrix Inverse Lemma agian, which gives

$$p_1 \mathbf{h}_1^H \mathbf{h}_1 - \frac{p_1 p_2 \left(\mathbf{h}_1^H \mathbf{h}_2\right)^2}{\sigma^2 + p_2 \mathbf{h}_2^H \mathbf{h}_2} \geq \gamma_1 \sigma^2. \tag{48}$$

Now we define a to be the channel correlation factor as in Theorem 2. Then (48) becomes

$$\frac{p_1}{\gamma_1 \sigma^2} \mathbf{h}_1^H \mathbf{h}_1 \geq \frac{1 + p_2 \mathbf{h}_2^H \mathbf{h}_2 / \sigma^2}{1 + p_2 a \mathbf{h}_2^H \mathbf{h}_2 / \sigma^2}. \tag{49}$$

Now, we take log on both sides of (49) and it can be easily shown that constraint (49) is convex in $\log p_i$ and $\log \gamma_i$ when $p_2 \leq \sigma^2/(a \cdot ||\mathbf{h}_2||_2^2)$. Thus a sufficient condition for convexity is $p_2^m \leq \sigma^2/(a \cdot ||\mathbf{h}_2||_2^2)$, since the transmit power p_2 is bounded by the maximal power ${p^m}_2$. When this sufficient condition for convexity holds, problem (19) is convex and the stationary point is indeed a globally optimal solution.

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