Analysis of Blocking Probability in Noise- and Cross-Talk-Impaired All-Optical Networks

Yvan Pointurier, Maïté Brandt-Pearce, and Suresh Subramaniam

Abstract—In all-optical networks with no wavelength converters, signals are switched optically inside the nodes and therefore propagate over hundreds or thousands of kilometers with no electrical regeneration. Over such distances, physical impairments, such as intersymbol interference (ISI), amplifier noise, and leaks within nodes (cross-talk), accumulate and can lead to serious signal degradation, resulting in poor quality of transmission (QoT) as measured by signal bit-error rates. The role of routing and wavelength assignment (RWA) algorithms is to accommodate incoming calls in optical networks over a route and a wavelength. RWA algorithms block calls if a continuous wavelength from the source to the destination cannot be found (wavelength blocking) or when the QoT of the call is not acceptable (QoT blocking). Evaluating RWA algorithms via simulations is possible but time consuming, and hence analytical methods are needed. Wavelength blocking has been studied analytically in the past, but QoT blocking has never been analytically modeled to our knowledge. In this paper, we present an analytical method to evaluate blocking probability in all-optical networks, accounting for physical layer impairments. Our physical layer model includes ISI and noise, two static effects that only depend on the network topology, and also cross-talk, which depends on the network state. Simulations on three different topologies with various numbers of channels, representing small- to large-scale networks, show that our technique is suitable for quick and accurate dimensioning of all-optical networks: the accuracy of the blocking rates computed with the analytical method, taking only seconds or minutes to run, is the same as that of simulations, which take hours to run.

Index Terms—Optical networks; Routing and wavelength assignment; Physical impairments; Analytical modeling.

I. INTRODUCTION

All-optical networks have emerged as a solution to keep up with the always-increasing throughput demand. In today’s transport networks, data is transmitted over optical fibers, and optical-electrical-optical conversion is needed at the nodes to perform routing. These networks can achieve a throughput of up to several hundreds of Gbits/s using wavelength division multiplexed (WDM) channels. Yet optical fibers have a potential capacity of several tens of terabits/s. Although, as noted in [1], all-optical connections are not currently common service offerings, they offer significant infrastructure benefits in their ability to reduce cost, space, and power dissipation. Even in the case of current/near-future all-optical networks, where some regeneration may be desirable, an architecture using “islands of transparency” has been proposed, whereby a large network is split into smaller fully transparent networks separated by electrical regenerators. Each smaller transparent optical network is called an “island of transparency” [2]. Deploying such all-optical, dynamic networks where calls arrive and must be provisioned on-demand in near real-time is promising but also challenging, and novel issues have to be anticipated at the physical layer. Indeed, while perfect transmission and negligible bit-error rates (BER) are valid assumptions for optical networks with electrical regeneration, large all-optical networks, with paths that can reach several hundreds or thousands of kilometers with no regeneration other than amplification, are impaired by nonnegligible physical layer degradations [3,4]. Some of these impairments depend on the instantaneous traffic, and hence cross-layer techniques are needed to study them.

Manuscript received June 26, 2009; revised September 9, 2009; accepted September 11, 2009; published October 15, 2009 (Doc. ID 113425).

Y. Pointurier (e-mail: yvan@ieee.org) was with the University of Virginia. He is now with Athens Information Technology, 0.8 km Markopolou Avenue, Peania 19002, Athens, Greece.

M. Brandt-Pearce (e-mail: mb-p@virginia.edu) is with the Charles L. Brown Department of Electrical and Computer Engineering, University of Virginia, Charlottesville, Virginia 22904, USA.

S. Subramaniam (e-mail: suresh@gwu.edu) is with the Department of Electrical and Computer Engineering, The George Washington University, Washington, DC 20052.

Digital Object Identifier 10.1364/JOCN.1.000543
The role of routing and wavelength assignment (RWA) algorithms is to assign a route and a wavelength—the combination of which is called a lightpath [5]—to incoming calls in a network in order to satisfy an optimization goal, such as the minimization of the average call blocking probability in the network [6]. Since wavelength conversion is not yet mature for commercial deployment, a call in an all-optical network must use the same wavelength from the source to the destination, a constraint known as the wavelength continuity constraint. Failure to find a lightpath that meets the wavelength continuity constraint for an arriving call results in wavelength blocking for the call. Moreover, all-optical networks are subject to physical impairments that are static and depend on the network topology only and to other physical impairments that are dynamic and vary with the network state. Static impairments include intersymbol interference (ISI) caused by the interplay between linear (chromatic dispersion) and nonlinear [self-phase modulation (SPM)] propagation effects on signals, as well as filtering at the receiver, and amplified spontaneous emission (ASE) noise due to amplifiers [7], and causing optical signal-to-noise ratio (OSNR) reduction. Dynamic impairments include signal leaks within the nodes and that copropagate from the node where the leak occurs until the end of the lightpath; these leaks are called cross-talk and are described in [8]. Because of these impairments, the quality of transmission (QoT) of lightpaths assigned to incoming calls, as measured by their BER, may be beyond a threshold set by the network operator (typically the BER threshold is set between $10^{-9}$ and $10^{-15}$), depending on the network state. If, at admission time, the BER of a tentative lightpath assigned to a call is beyond the threshold, or if establishing the call would inject too much cross-talk and cause the BER of a lightpath already established in the system to cross the threshold, then the call cannot be accepted: it has to be rejected due to the QoT constraint, resulting in QoT blocking.

Evaluation by simulation of RWA for nontrivial network topologies, such as general mesh topologies, which was done in [4], where the impact of each of the aforementioned impairments (ASE noise, chromatic dispersion/SPM interaction, node cross-talk) was shown to strongly affect the call admission blocking rate, is a time-consuming process. For this reason, alternate analytical methods are needed. Although the problem of analytically computing blocking probability in all-optical networks has been studied in the past, using various models and assumptions, the physical layer has never been accounted for to this point in any analytical work. In this paper we present the first analytical method to evaluate QoT blocking in all-optical networks. The technique can be attached to any technique for calculating the wavelength blocking, which are numerous. In [9], a reduced load approximation scheme is developed and is applicable only to small networks due to its exponential computational complexity with the number of nodes in the network. Also, in [9], wavelength utilizations on different links are assumed to be independent. This independence assumption is oversimplifying, especially for sparse networks where nodal degree is low. The independence assumption is relaxed in [10–14]. However, other oversimplifying assumptions are made in [10]. The technique presented in [11], based on path decomposition, tackles fixed and alternate routing schemes, possibly with the presence of wavelength conversion in the network. In [12], the authors extend results from [14] and are able to determine the blocking due to outdated information in the network.

In [13], the problem of partial wavelength conversion is touched upon, and the wavelength blocking computation procedure is shown to be accurate for a variety of topologies. The iterative algorithm presented in [14] also yields very accurate results, while only making a two-link correlation assumption: the wavelength utilization on a link of a given route depends only on that of the one previous or the next link of the route. The wavelength blocking computation techniques presented in [13,14] are relatively simple and yield accurate results for large arbitrary topologies. To compute QoT blocking, our technique assumes that we can compute wavelength blocking for each route; however, our technique does not make assumptions on how this is done. For these reasons, and in order to demonstrate the independence of our algorithm with respect to wavelength blocking computation techniques, we use in turn those two different wavelength blocking computation algorithms [13,14] to compute QoT blocking in arbitrary networks. We show that our technique can be easily and independently combined with each of these techniques, yielding similar numerical results.

In this paper, we tackle the problem of analytically computing blocking probabilities in networks subject to both wavelength blocking and QoT blocking caused by static (ISI, ASE noise) and dynamic (cross-talk) effects. The model is applicable to metropolitan and regional all-optical networks, and, for very-large-scale networks divided into “islands of transparency,” to each of the islands. As in [13,14], we consider a single instance of routing and wavelength assignment, namely, fixed routing where routing tables are precomputed and contain a single path between any two nodes, and random wavelength assignment. Indeed, as is seen in [6], for instance, the impact of wavelength assignment on network performance as measured by blocking probability is less than an order of magnitude. The goal of the paper is to present a fast
algorithm that can be used to dimension the performance of a network, rather than computing exact blocking probabilities for specific RWA algorithms.

This paper is organized as follows. In Section II, we present our model and state the assumptions used throughout the paper. We present our technique to compute QoT blocking (including ISI, noise, and cross-talk) in Section III. In Section IV, we show how our QoT blocking algorithm integrates with a specific instance of the wavelength blocking computation algorithm. Our technique is validated by simulations on various network topologies for realistic physical layer parameters and we present a time complexity analysis in Section V.

II. NETWORK AND CROSS-TALK MODEL

We present here our assumptions concerning the network, traffic, and cross-talk models used throughout this paper. In the all-optical networks modeled here, links represent unidirectional optical fibers. The number of wavelengths per link is fixed to a constant number \(C\) across the network and wavelength conversion is not available. Call durations are exponentially distributed with mean rate \(M(\lambda_{n_1,n_2}) = 1\), and call arrivals follow a Poisson process with mean rate \(\Lambda R_{(n_1,n_2)}\) for route \(R(n_1,n_2)\) from node \(n_1\) to \(n_2\). Since \(M=1\), the offered load in Erlangs on a route \(R\) is \(\Lambda R\). Denoting by \(V\) the set of nodes in a network, the total offered load in Erlangs in that network is \(\sum_{n_1,n_2 \in V,n_1 \neq n_2} \Lambda R_{(n_1,n_2)}\).

The physical layer and cross-talk in particular are modeled as follows. When a call is accepted in the network, it is assigned one lightpath, as shown in Fig. 1. Because of the physical impairments sustained by signals during their transmission on a lightpath, a signal may be too degraded at reception to ensure a minimal QoT as defined by the network operator. Denoting by \(\mu_1\) and \(\mu_0\) the means of the received “1” and “0” samples after electrical filtering and by \(\sigma_1\) and \(\sigma_0\) their respective standard deviations, the Q factor of a signal is defined as \(Q = (\mu_1 - \mu_0)/(\sigma_0 + \sigma_1)\). Using a Gaussian assumption [7], the BER and the Q factor of a signal are related by BER = 0.5 erfc(\(Q/\sqrt{2}\)) for uncoded on-off-keyed (OOK) signals. For instance, a BER of \(10^{-9}\) corresponds to a Q factor of \(Q = 6\).

In this paper, we account for three main physical impairments that are known to affect lightpaths in all-optical networks [15]: intersymbol interference, amplifier noise, and cross-talk. Each of these effects is accounted for as a noise variance in the Q factor of the lightpath; that is, for a given route \(R\),

\[
Q_R = \frac{\mu_{1,R} - \mu_{0,R}}{\sigma_{0,R} + \sigma_{1,R}} = \frac{\mu_{1,R} - \mu_{0,R}}{\sigma_{0,R} + \sqrt{\sigma_{1,R}^2 + \sigma_{0,R}^2 + \sigma_{X,R}^2}},
\]

where \(\sigma_{1,R} = \sqrt{\sigma_{1,R}^2 + \sigma_{n,R}^2 + \sigma_{X,R}^2}\) and \(\sigma_{0,R}\), \(\sigma_{n,R}\), and \(\sigma_{X,R}\) are the variance contributions due to intersymbol interference, amplifier noise, and cross-talk, respectively. Here we make the (usual) assumption of a high transmitter extinction ratio, such that ASE noise and cross-talk impairments can be ignored for the “0” bits. In addition, we assume that all signals are in the same polarization state, a worst-case scenario typically used to design networks (see, for instance, [8] in the context of cross-talk modeling).

We introduced ISI and ASE noise in Section I as static effects. Node cross-talk originates from signal leaks in the nodes, either at the demultiplexing stage or inside the switching fabric. The model for the origin of cross-talk we use here, “self-cross-talk,” was first detailed in [16], and we restate it here for clarity. In the “self-cross-talk” model, cross-talk is created by two lightpaths on different wavelengths entering an optical cross connect (OXC) by the same (input) port and exiting the OXC by the same (output) port. Because of imperfections at the input demultiplexer, a small part of the signal of each lightpath leaks onto the other.

Contrary to ISI and ASE noise, cross-talk is a dynamic effect, depending on what lightpaths are established in the network. We compute \(\sigma_{X,R}\) accounting for all cross-talk components on a lightpath by summing the variance contributions for each cross-talk term:

\[
\sigma_{X,R}^2 = n \sigma_{X,R}^2,
\]

where \(n\) is the number of cross-talk terms injected along route \(R\) and \(\sigma_{X,R}^2\) is the variance contribution of each of these cross-talk signals, assuming these are all equal.

Other types of cross-talk, which were presented in [16], are ignored here. For instance, cross-talk due to leaks in the switching fabric is generally weaker than cross-talk originating from the demultiplexers. Also, we do not distinguish between cross-talk coming from adjacent or nonadjacent channels. This assumption is accurate when the demultiplexer frequency response is flat in the cutoff region. Our method can be easily generalized to more than just the cross-talk model described above. For instance, a small modification of how cross-talk components are counted permits the accounting for switching fabric cross-talk. Such a modification is useful in cases where the primary

![Fig. 1. (Color online) Model of a transmission lightpath (plain line) used to compute the Q factor. Each node can inject one or more cross-talk component (dashed lines), and ASE noise can originate from each amplifier (dotted lines).](image)
source of cross-talk is the switching fabric, in the case of non-MEMS switches, for instance, [17]. Furthermore, our analytical method can compute blocking probabilities if both demultiplexer and switching cross-talk are present in the case where both cross-talk attenuations are the same. If nonadjacent cross-talk is actually weaker than adjacent cross-talk, then our technique actually would result in overdimensioning because the weaker nonadjacent channel cross-talk components would then be on an equal footing with the stronger adjacent channel cross-talk. Because our technique takes dynamic effects into account, all in all it reduces the amount of overdimensioning required at network design time.

In addition, although the work considers single-channel nonlinear effects, namely, the interaction between SPM and chromatic dispersion, we do not account here for nonlinear interchannel effects [cross-phase modulation (XPM), four-wave mixing (FWM)], which could be limiting impairments in certain next-generation all-optical networks. This assumption, however, is valid for metro or regional networks where routes are not more than a few hundreds of kilometers and where cross-talk is a major impairment [18].

III. Performance Analysis Under QoT Constraints

A. Overview

In this section, we assume that wavelength blocking probabilities are known, and we compute QoT blocking rates. More specifically, we compute the QoT blocking probability \( B^{(q)}_R \) due to ISI, noise, and cross-talk blocking for every route \( R \). Recall that a new call is blocked due to insufficient QoT if, at admission time, the \( Q \) factor of a tentative lightpath assigned to a call is beyond a predefined threshold or if establishing the call would inject too much cross-talk and cause the \( Q \) factor of a lightpath already established in the system to cross the threshold.

To compute the blocking probability due to QoT, we first assume that the wavelength blocking probabilities \( B^{(w)}_R \) are known and we determine the distribution of the number of cross-talk components \( XT_R \) that impair each route \( R \) in Subsection III.B. Then, we relate the blocking probability due to QoT to the cross-talk distributions in Subsection III.C.

The QoT computation algorithm is largely independent of the algorithm used to compute wavelength blocking \( B^{(w)}_R \) for every route \( R \). However, some algorithms, e.g., that presented in [14], rely on the computations of conditional blocking probabilities: let \( B^{(w)}_{R|X=m} \) be the blocking probability due to a lack of wavelength for route \( R \) given \( m \) wavelengths are free on link \( j \). We provide the necessary steps to interface with such algorithms as well as Section IV.

An overview of the algorithm used to compute blocking probabilities in all-optical networks with physical impairments is given in Algorithm 1. In each case, the QoT blocking computation algorithm is iterative and stops when a convergence criterion (e.g., blocking rate difference for each route between two consecutive iterations lower than a preset threshold) is met.

Algorithm 1 Blocking probability computation: main algorithm when no conditional blockings are used by the wavelength blocking computation algorithm.

1: Initialize \( B^{(w)}_R \) for all \( R \); using for instance [13].
2: Initialize \( B^{(w)}_R=0 \) and \( B_R=B^{(w)}_R \) for every route \( R \).
3: repeat
4: \( \mathbf{B}_R=B_R \).
5: Compute \( B^{(q)}_R \) for all \( R \); using Eqs. (3)–(8) as described in Subsections III.B and III.C.
6: Compute \( B_R \) for all \( R \); using Eq. (9).
7: until \( (B_R-\mathbf{B}_R)/B_R<\epsilon \) for every route \( R \)

Before we exhibit the details of the QoT blocking computation algorithm, we also make the following assumptions concerning the traffic model. These assumptions make analysis tractable and fast, yet approximate well the behavior of all-optical networks:

(A1) Routing is fixed with no alternate paths and wavelength assignment is a random pick, such that all wavelengths are statistically equivalent.

(A2) Wavelength occupancies on disjoint routes are independent of each other.

(A3) The establishments of lightpaths on two different routes are independent events.

Assumptions (A1) and (A2) are made in other papers focusing on wavelength blocking, most notably, in [13,14]. In particular, the wavelength equivalence assumption (A1) prevents the inclusion of cross-channel effects (XPM, FWM, differentiation between adjacent and nonadjacent cross-talk); the removal of this assumption is left for future work and this paper is applicable to cases where cross-channel effects are not limiting factors in the quality of transmission of signals. We would like to emphasize, however, that this wavelength independence assumption can be mitigated; for instance, accounting for XPM and FWM could be done by assuming a worst-case, static case, whereby all channels are in use, although this would result in network overdimensioning. Hence, our technique could also be applicable to more general network scenarios than small- or medium-scale metro/regional networks, at the expense of the accuracy of the dimensioning. In addition, our analysis can be extended to the case of dynamic routing where the route is not dependent on the network state, e.g., when
probabilistically picking routes from a set of candidates, but analyzing state-dependent routing requires new methods. Assumption (A3) is used further in this work; numerical results show that the resulting approximations do not much affect the accuracy of the method.

In addition, we consider that the system is ergodic. The blocking probability for a route in a network is actually a time average (average of blockings per unit of time), but the analytical method computes ensemble averages (average of blockings over the set of possible network states). Therefore, the blocking probabilities we compute are equivalent to blocking rates. Moreover, event orders are not important to compute these rates. More specifically, in a real network, if call $C_1$ arrives and is blocked because establishing $C_1$ would cause the QoT of some other call $C_2$ already established to drop below the predefined threshold, then $C_1$ is rejected; in our analysis, since it is the QoT of $C_2$ that drops below threshold, $C_2$ is rejected. Averaging over all network states in the analysis yields the blocking rate in the real network. This ergodicity principle does not hold if more than two connections are involved simultaneously; for instance, it could be argued that, if $C_1$, $C_2$, and $C_3$ arrive, $C_1$ can be established, but $C_2$ and $C_3$ would drive the QoT of $C_1$ beyond threshold if they were established and are hence blocked. Our analysis counts only the blocking of $C_1$ rather than the two blockings of $C_2$ and $C_3$, leading to a QoT blocking probability underestimation. However, such very specific situations where three connections or more arrive almost concurrently and share the same resources and where the QoT of one of them is close to the threshold are likely to be much less probable than the concurrent arrival of only two connections as described above, where ergodicity holds; hence the impact of this underestimation can be expected to be limited, as will be seen with numerical results in Section V.

B. Distribution of the Number of Cross-Talk Terms

Denote by $U_R(k)$ the probability that $k=0,\ldots,C$ calls are established on route $R$; $U_R(k)$ accounts only for calls that use exactly route $R$, not for calls that use only part of route $R$ or that use a route that includes $R$. To determine the distribution $U_R$ for each route $R$, using assumptions (A1) and (A2), we approximate $U_R$ as a binomial random variable. Then, the utilization of a wavelength on some route $R$ can be viewed as a Bernoulli trial with probability of success $p_R$, the probability that the corresponding call is established. The probability that a call is established on route $R$ is the per-wavelength arrival rate for calls on route $R$ multiplied by the probability that a call is actually accepted on $R$; that is,

$$p_R = \frac{\Lambda_R}{M_R} C = \frac{\Lambda_R}{C},$$

(3)
since, with no loss of generality, the service rates $M_R$ are assumed to be unity.

The probability that exactly $k$ calls are established on route $R$ is therefore

$$U_R(k) = \binom{C}{k} p_R^k (1-p_R)^{C-k}, \quad k = 0,1,\ldots,C. \quad (4)$$

Let $I_R^R = \{R_1,\ldots,R_p\}$ be the set of the routes that are potential sources of cross-talk for lightpaths established on route $R$. The set $I_R^R$ is determined for each route $R$ as follows.

Consider route $R$ consisting of the sequence of $s$ nodes $(r_1,\ldots,r_s)$. Consider a route $R'$ consisting of the sequence of nodes $(r'_1,\ldots,r'_t)$. Route $R'$ can inject self-cross-talk at node $r_n$ ($n=1,\ldots,s$) of $R$ if the input ports for $R$ and $R'$ at node $r_n$ are the same or if the output ports for $R$ and $R'$ at node $r_n$ are the same. That is, $R' \in I_R^R$ when

- $n=1$ (if $r_n$ is the first node of $R$); $R$ and $R'$ start with the same link, i.e., $(r_1, r_2) = (r'_1, r'_2)$. In this case, cross-talk occurs at the multiplexer used to add wavelengths to the network.
- $n=s$ (if $i$ is the last node of $R$); $R$ and $R'$ end with the same link, i.e., $(r_{s-1}, r_s) = (r'_{s-1}, r'_s)$.
- $1 < n < s$ (the other cases): $R$ and $R'$ share two consecutive links separated by node $r_n$, i.e., there exists $v \in \{2,\ldots,\min(s-1, l-1)\}$ such that $(r_{n-1}, r_n, r_{n+1}) = (r'_v, r'_v, r'_{v+1})$.

Call $n_{xt}(R, R')$ the number of common nodes between routes $R$ and $R'$ where cross-talk can occur. Suppose $k$ lightpaths use exactly route $R'$; that is, those $k$ lightpaths are not using only a part of route $R'$ nor are they using a route that includes $R'$. Then, the number of cross-talk components injected by $R'$ on $R$ is $k n_{xt}(R, R')$. Calling $U_{R,R'}(k')$ the probability that some route $R'$ injects $k'$ cross-talk components on route $R$, we therefore obtain

$$U_{R,R'}(k') = n_{xt}(R, R') = U_{R'}(k). \quad (5)$$

Let $XT_R(k)$ be the probability that route $R$ is subject to exactly $k$ cross-talk components. The total number of cross-talk components $k$ seen by route $R$ is the sum of all cross-talk components injected at each node of $R$ by all routes that intersect $R$. Using assumption (A3), the probabilities for establishing lightpaths on different routes are independent such that $U_{R,R'}$ describe independent random variables. Therefore, $\star$ denoting the
convolution operator, and with \( F^R_R = \{R_1, \ldots, R_p\} \), the distribution of \( XT_R \) can be computed as follows:

\[
XT_R = U'_{R,R_1} \ast \cdots \ast U'_{R,R_p}.
\] (6)

C. Blocking Probability Due to QoT

In this subsection, we exhibit the relation between the physical layer (impact of cross-talk on QoT) and the network layer (distribution of the number of cross-talk components). Assuming that physical layer impairments are due to ISI, noise, and self-cross-talk only, the Q factor \( Q_R \) for a route \( R \) is given in Eq. (1).

Using the techniques described in [18], we can pre-compute \( \mu_{1,R}, \mu_{0,R}, \sigma_{0,R}, \sigma_{1,R}, \) and \( \sigma_{x,R} \) for all routes in the network. Since the quantities \( \mu_{1,R}, \mu_{0,R}, \sigma_{0,R}, \sigma_{1,R}, \) and \( \sigma_{x,R} \) are known for each route in the network, we can compute the maximum number of cross-talk components \( N^\max_R \), a route \( R \) can accommodate to maintain a \( Q \) factor above a predetermined threshold \( Q_{th} \) using Eqs. (1) and (2):

\[
N^\max_R = \left[ \frac{(\mu_1 - \mu_0)}{Q_{th} - \sigma_{0,R}^2} - \frac{\sigma_{1,R}^2}{\sigma_{x,R}^2} \right]^{1/2}.
\] (7)

Therefore, the probability that a lightpath is blocked because it does not meet the QoT constraint that this lightpath is subject to \( N^\max_R \) cross-talk components or more; that is,

\[
B_R^{(q)} = \sum_{k > N^\max_R} XT_R(k).
\] (8)

The blocking probabilities due to wavelength continuity and due to QoT are related by

\[
B_R = B_R^{(w)} + (1 - B_R^{(w)})B_R^{(q)}.
\] (9)

Indeed, a call can be blocked due to QoT only if a wavelength is available on the route the call is assigned to, that is, when the call is not blocked due to the wavelength continuity constraint.

IV. INTEGRATION WITH A WAVELENGTH BLOCKING ANALYTICAL MODEL REQUIRING CONDITIONAL PROBABILITIES COMPUTATIONS

Our QoT blocking analytical model is general and can be adapted to various wavelength blocking analytical models. In the previous section, we described how to compute QoT blocking in all-optical networks, assuming that wavelength blocking probabilities \( B_R^{(w)} \) are known. The integration of our QoT computation techniques with certain wavelength blocking computation algorithms, such as that of [14], is straightforward, as is seen in Algorithm 1. However, certain wavelength blocking computation algorithms, such as that of [14], are more complex to integrate with our QoT computation technique because a) the wavelength blocking algorithm itself is iterative, rather than sequential, and b) it relies on the computation of wavelength blocking conditioned on the number of free wavelengths on a given link: \( B_{R_{X,m}}^{[q]} \), which in turn depends on QoT conditional blocking \( B_{R_{X,m}}^{[w]} \).

A. Overview of the Integration

Now consider the model described in [14]. This model makes the following assumptions, in addition to the assumptions already outlined in Section II:

- Given \( m \) wavelengths are free on link \( j \), the time until a call that uses \( j \) arrives is assumed to be exponentially distributed [19], such that the number of free wavelengths on link \( j \) is modeled as a birth–death process.
- The state of wavelength \( i \) on link \( j \) of a route \( R \) is independent of the state of some other wavelength \( k \neq i \) on the previous link \( j - 1 \) on the same route, given the state of wavelength \( i \) on link \( j - 1 \) or given the state of wavelength \( k \) on link \( j \) (a two-link correlation assumption).

Unlike the algorithm proposed in [13], the wavelength blocking algorithm proposed in [14] is iterative. It is articulated around the computation of the state-dependent arrival rates \( \alpha_i(m) \); assuming \( \alpha_i(m) \) is known, a number of quantities are computed, ultimately leading to the computations of \( B_R^{(w)} \) and \( B_{R_{X,m}}^{(w)} \), and to the update of \( \alpha_i(m) \), which depend on \( B_{R_{X,m}}^{(q)} \). The process is repeated until both \( B_R^{(w)} \) and \( B_{R_{X,m}}^{(w)} \) satisfy a convergence criterion for each route \( R \), each link \( j \), and each number of wavelengths \( m \).

Therefore, we combine the wavelength computation algorithm with QoT blocking computation technique as shown in Algorithm 2 and in Fig. 2. The \( \alpha_i(m) \) are initialized and we obtain \( B_R^{(w)} \) and \( B_{R_{X,m}}^{(w)} \) using [14];
we use these as inputs to our technique to obtain \( B'^{(q)}_R \) and \( B'^{(q)}_{R|X=m} \) and hence \( B_R \) and \( B'_{R|X=m} \); then the \( \alpha_j(m) \) are updated and the loop repeats until convergence of the quantities \( B_R \) and \( B'_{R|X=m} \) for each route \( R \).

**Algorithm 2** Blocking probability computation: main algorithm when conditional blocking probabilities are used by the wavelength blocking computation algorithm.

1: Initialize \( B'^{(w)}_R - B'^{(q)}_R = B_R = 0 \) for every route \( R \).
2: Initialize \( \alpha_j(0) = \lambda_p \cdot \text{(number of routes using link } j) \), \( \alpha_j(0) = 0 \) for every route \( R \), every link \( j \), every wavelength count \( 0 < m < C \).
3: repeat
4: Let \( \bar{B}_R = B_R \).
5: Compute \( B'^{(w)}_R \) for all \( R \): using instance [14].
6: Compute \( B'^{(q)}_{R|X=m} \) for all \( R, j, m \).
7: Compute \( B'^{(q)}_R \) for all \( R \): using Eqs. (3)–(8) as described in Subsections III.B and III.C.
8: Compute \( B'^{(q)}_{R|X=m} \) for all \( R, j, m \): using Eqs. (12)–(16) as described in Subsection IV.B.
9: Compute \( B'^{(q)}_{R|X=m} \) for all \( R, j, m \): using Eq. (11).
10: Compute \( B_Q \) for all \( R \): using Eq. (9).
11: Compute \( \alpha_j(m) \) for all \( R, j, m \): using Eq. (10).
12: until \( (B_R - \bar{B}_R)/B_R < \varepsilon \) for every route \( R \)

### B. State-Dependent Arrival Rates

We now show how to compute the conditional blocking probabilities \( B'^{(q)}_{R|X=m} \), which are needed to compute the blocking probabilities \( B'_{R|X=m} \) in [14].

In [14], state-dependent arrival rates \( \alpha_j(m) \) are defined as

\[
\alpha_j(m) = \sum_{R|X=m} \Lambda_R (1 - B_{R|X=m}),
\]

where \( B_{R|X=m} \) are blocking probabilities conditioned on the number of free wavelengths on a given link. We show here how these conditional blocking probabilities \( B_{R|X=m} \) should be updated to account for (conditional) QoT blocking.

Similarly to Eq. (9), the total conditional blocking probabilities depend on the conditional blocking probabilities due to QoT:

\[
B_{R|X=m} = B'^{(w)}_{R|X=m} + (1 - B'^{(w)}_{R|X=m}) B'^{(q)}_{R|X=m}.
\]

We now determine the conditional probabilities \( B'^{(q)}_{R|X=m} \), which are needed to compute the state-dependent arrival rates \( \alpha_j(m) \) through Eqs. (10) and (11). First, we compute the probability \( p_{R|X=m} \) that a given lightpath is established on route \( R \) given \( m \) wavelengths are free on link \( j \), and the probability \( U_{R|X=m}(k) \) that \( R \) is used by exactly \( k \) lightpaths given \( m \) wavelengths are free on link \( j \).

If \( j \) is not a link of \( R \), then \( p_{R|X=m} = p_R \) and \( U_{R|X=m} = U_R \). If \( j \) is a link of \( R \), then at most \( m \) wavelengths are free on \( R \). Consider the case where \( j \) is a link of route \( R \). The probability that a given lightpath is established on route \( R \) given \( m \) wavelengths are free on link \( j \) is now

\[
P_{R|X=m} = \frac{\Lambda_R (1 - B_{R|X=m})}{M_R} = \frac{1 - B_{R|X=m}}{C} \quad (12)
\]
since \( M_R = 1 \) for each route \( R \).

We adapt Eq. (4) accounting for the fact that \( m \), not \( C \), wavelengths at most can be used by a lightpath on \( R \):

\[
U_{R|X=m}(k) = \frac{m!}{k!} (p_{R|X=m})^k (1 - p_{R|X=m})^{m-k} \quad \text{if } k = 0, \ldots, m,
\]

\[
\text{if } k = m + 1, \ldots, C.
\]

The probability that route \( R' \) injects \( k \) cross-talk components of route \( R \), given \( m \) wavelengths are free on \( j \), follows from Eq. (5):

\[
U_{R,R'|X=m}(kn_x(R,R')) = U_{R|X=m}(k).
\]

The distribution of \( XT_R \) given \( m \) wavelengths are free on \( R \) is

\[
\text{XT}_{R|X=m} = U_{R,R_1|X=m} \ast U_{R,R_2|X=m} \ast \ldots \ast U_{R,R_p|X=m}.
\]

and the blocking probability due to QoT conditioned on the state of link \( j \) is

\[
B'^{(q)}_{R|X=m} = \sum_{k > X^\text{max}} \text{XT}_{R|X=m}(k).
\]

The state-dependent arrival rates \( \alpha_j(m) \), which depend on \( B'^{(q)}_{R|X=m} \), can then be updated, using Eqs. (11) and (16), and, for instance, Section III-A of [14].
V. PERFORMANCE

A. Validation by Simulation Results

In this section, we evaluate our analytical model for blocking probability in all-optical networks impaired by cross-talk. Evaluation is performed on three different topologies of increasing complexity: a ring of six nodes, a mesh of eight nodes (Fig. 3), and the NSF-NET topology (Fig. 4). The physical parameters are chosen to emulate regional-sized networks; in particular, as in [4], we scaled down the length of each link of the originally continental-sized NSFNET network by a factor of 10 in order to obtain a fully transparent topology with a diameter of a few hundred kilometers, where the shortest path between any two nodes is short enough such that any node can be reached (without regeneration) from any other node. Unless otherwise stated, we used the parameters given in Table I. Let $B$, $B^w$, and $B^q$ be the mean (taken over the set of the node pairs) blocking probability, blocking probability due to the wavelength continuity constraint, and blocking probability due to QoT, respectively. The analytical results are obtained by stopping the iterative algorithm when the difference for the blocking probabilities between two successive iterations differ by less than 1% for each route. The simulation results are obtained by simulating the routing and wavelength assignment of $10^5$ calls. We use separate counters to determine blocking due to wavelength and due to QoT to compute $B^w$, $B^q$, and $B$ using Eq. (9). Each data point is obtained by repeating this process 10 times in order to compute 95% confidence intervals, which are shown on the plots.

Similar to the analysis, simulations use precomputed tables to compute $Q$ factors; however, in the simulation, we consider that cross-talk is propagated from the node where the leak occurs to the end of the considered lightpath, irrespective of where the leak occurs, as can be seen in Eq. (2). With the physical parameters given in Table I, it can be shown that the cross-talk variance decreases slowly with the propagation distance [18]; therefore, our analytical method tends to underestimate cross-talk variances by assuming longer propagation distances, and hence to underestimate QoT blocking.

We first present results for the ring topology with 6 nodes with 32 wavelengths per link ($C=32$) in Fig. 5. For this set of parameters, the blocking probability due to wavelength continuity is several orders of magnitude lower than that due to QoT, and therefore $B^q \approx B$. Our technique estimates accurately the blocking probability in a wide operation range (blocking probabilities varying over 4 orders of magnitude).

In Fig. 6, we show the blocking probability due to QoT for the mesh of 8 nodes with 16 wavelengths ($C$ = 16) and 12 wavelengths ($C$ = 12) in Fig. 7. For these topologies, the blocking probability due to wavelength continuity is several orders of magnitude lower than that due to QoT, and therefore $B^q \approx B$. Our technique estimates accurately the blocking probability in a wide operation range (blocking probabilities varying over 4 orders of magnitude).

Notice here that, since $B^w \ll B^q$ for all routes, in practice our technique does not rely at all on any underlying wavelength blocking computation technique for the set of parameters utilized here. The two techniques described in Section IV yield identical results.

### TABLE I

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Span length</td>
<td>70 km</td>
</tr>
<tr>
<td>Signal peak power</td>
<td>2 mW</td>
</tr>
<tr>
<td>Bit rate</td>
<td>10 Gbps</td>
</tr>
<tr>
<td>Pulse shape</td>
<td>Super-Gaussian nonreturn to zero</td>
</tr>
<tr>
<td>Port cross-talk</td>
<td>$-30$ dB</td>
</tr>
<tr>
<td>WDM grid spacing</td>
<td>25 GHz</td>
</tr>
<tr>
<td>Optical filters bandwidth</td>
<td>25 GHz</td>
</tr>
<tr>
<td>Fiber loss</td>
<td>0.22 dB/km</td>
</tr>
<tr>
<td>Nonlinear coefficient</td>
<td>2.2 (W km)$^{-1}$</td>
</tr>
<tr>
<td>Chromatic dispersion</td>
<td>17 ps/nm/km</td>
</tr>
<tr>
<td>Dispersion compensation</td>
<td>100% postcompensation</td>
</tr>
<tr>
<td>Noise figure</td>
<td>6 dB</td>
</tr>
<tr>
<td>Receiver electrical bandwidth</td>
<td>7 GHz</td>
</tr>
<tr>
<td>Minimum $Q$ factor</td>
<td>6</td>
</tr>
</tbody>
</table>
Pointurier et al.

![Graph](image)

**Fig. 5.** Blocking probability for the ring of 6 nodes, 32 wavelengths, −30 dB cross-talk; 95% confidence intervals are given for the simulation curve.

\( N = 16 \) for 2 levels of cross-talk: −25 dB and −30 dB. Again, in this case, the blocking probability due to wavelength continuity is very small compared with that due to QoT, and hence we do not report it. Therefore, the choice of underlying wavelength continuity computation algorithm does not affect the QoT computations here, since wavelength blocking is negligible compared with QoT blocking.

We define the gain in load of a network as follows: given a target blocking probability \( B_i \), a reference cross-talk level \( \eta_r \), and a cross-talk level \( \eta \), the gain in load for cross-talk level \( \eta \) is the ratio between the offered network load such that the total blocking probability in the network for cross-talk level \( \eta \) is \( B_i \), and the offered network load such that the call blocking probability in the network for cross-talk level \( \eta_r \) is \( B_i \). It can be seen in Table II, which was obtained using our analytical method, and where \( B_i \) was fixed to 0.001 and \( \eta_r \) to −25 dB, that the cross-talk level has a dramatic influence on the admissible load in the network. For instance, if the cross-talk level drops by only 5 dB from −25 dB to −30 dB, then the network can be loaded 16 times more while still achieving a 0.001 average total blocking probability.

In Fig. 7, we report the blocking probability for the scaled NSFNET topology with 16 wavelengths \( (C = 16) \). Again \( B^w \) is negligible compared with \( B^q \) and we report \( B^q \) only. Our model is very accurate over more than 3 orders of magnitude in terms of blocking probability.

Figures 8 and 9 depict the scenario where the number of wavelengths is 8 \( (C = 8) \) instead of 16. Here, blocking due to wavelength continuity is not negligible. Figure 8 assumes that the underlying wavelength blocking model is that of [13], whereas Fig. 9 assumes that the underlying wavelength blocking model is that of [14]. In each figure, we report the total blocking probability, as well as blocking probabilities due to wavelength continuity and due to QoT. The wavelength blocking probabilities returned by the two underlying wavelength blocking computing algorithms are very close to one another, with results obtained with the wavelength blocking model from [14] slightly more accurate than those obtained with the model from [13]. As a consequence, QoT and overall blocking rates obtained analytically in Fig. 9 obtained with the model from [14] are also slightly more accurate than those in Fig. 8 obtained with the model from [13], especially for the higher load values. However, this accuracy advantage comes at a computational complexity cost, as explained in Subsection V.B. In either case, for loads lower than 30 Erlangs, QoT blocking dominates wavelength blocking, and the converse is true for loads above 30 Erlangs. Our analytical results predict this behavior accurately.

**B. Computational Complexity**

We determine here the computational complexity of our technique for one iteration of the algorithm. Because our algorithm relies on another algorithm to provide wavelength blocking probabilities, we perform a complexity analysis for each of the wavelength blocking algorithms outlined above. In practice, the algorithm runs through just a few iterations before the blocking probabilities converge and the algorithm terminates. Denote by \( N \) the number of nodes in the network, \( L \) the number of links, \( A \) the maximum number of nodes on a route, \( D \) the maximum number of routes intersecting any given route \( (D = \max_R |R_{ij}|) \), \( C \) the number of wavelengths. The number of routes is \( N(N-1) = O(N^2) \).

Consider first the combination of our QoT blocking computations with the wavelength blocking computations of [13]. In this case, QoT computations dominate the time complexity; more specifically, the evaluation of the multiple convolutions in Eq. (6) is \( O(N^2)(AC)^D \log(AC)) \). This is also the complexity of
the overall blocking probability computation algorithm.

Consider now the combination of our model with the wavelength blocking algorithm presented in [14].

Computation of $B^{(w)}$: it can be shown that the time complexity needed to compute all $B^{(w)}$ as in [14] is $O(N^2CA(N^2+C^2L))$.

Computation of $B^{(q)}$: the statistics required to compute the Q factors are precomputed. The complexity of the computations that lead to $B^{(q)}$ is dominated by the computations of Eq. (15). Each $U'_{R,R'}^{X=q,m}$ can be represented as a vector of at most $AC$ elements. To compute each $XT_{R|R'=m}$, we need to convolve at most $D$ of these vectors, which can be done in time $O((AC)^D D \log(AC))$ using fast Fourier transforms. The time complexity of Eq. (15) and thus of our extensions is $O(N^2LC(AC)^D D \log(AC))$. The time complexity of the full algorithm is hence $O(N^2C(A(N^2+C^2L)+L(AC)^D D \log(AC)))$. Notice here that, because of the conditional computations, the QoT computations are a factor $LC$ more complex if the underlying wavelength blocking model is as in [14] rather than as in [13]. Also, when used in conjunction with [14], the time complexity of our technique is not dominated by QoT blocking computations: both wavelength and QoT blocking have an impact on the overall computational complexity.

On comparable hardware, computing blocking probabilities through simulations typically took several hours, whereas analysis took only a few minutes at most—a gain of 1–3 orders of magnitude in running time.

VI. CONCLUSIONS AND FUTURE WORK

We have presented an iterative technique to compute the blocking probability in all-optical networks impaired by ISI, noise, and channel cross-talk. Starting from published methods that compute blocking probability due to wavelength continuity only, we were able to compute blocking probability due to QoT. Our technique was evaluated on various topologies for realistic physical layer parameters and shown to closely match simulation results. Our technique can be used to predict the impact of physical impairments on the network behavior in terms of lightpath rejection rate and hence to dimension medium- or large-scale all-optical networks.

This paper is the first work to our knowledge to analytically model blocking behavior of all-optical net-
works subject to quality of transmission impairments; however, the technique relies on several restrictive assumptions, in particular, fixed routing and first fit routing. The authors believe that, although changing the routing or wavelength assignment schemes does impact the blocking rate, and in particular the wavelength blocking rate, the technique developed in this paper is sufficient for quick dimensioning of all-optical networks. In addition, our technique is independent of the algorithm used to compute wavelength blocking and can be extended to account for different routing schemes (e.g., fixed alternate routing is investigated in [14] for wavelength blocking modeling) and more cross-talk models; however, differentiating between adjacent and nonadjacent channel cross-talk requires the removal of a key assumption, namely, the equivalence of wavelengths in the system, and is left for future work. Although our model is applicable to the “island of transparency” architecture for very large, continental all-optical networks, it does not account for other architectures with sparse regeneration. In addition, the wavelength equivalence assumption does not allow the inclusion of interchannel effects (XPM, FWM), which could be limiting impairments in certain next-generation all-optical networks. Finally, polarization mode dispersion (PMD), which will be a major impairment in future, very-high-speed optical networks, is a statistical impairment of a different nature than ISI, ASE noise, and cross-talk and hence needs to be studied separately.

ACKNOWLEDGMENTS

Thanks to Onur Turkcu of The George Washington University for providing some of the simulation results included in Section V. This work is an extended version of a paper presented at IEEE INFOCOM 2007 [20]. This work was supported in part by the U.S. National Science Foundation (NSF) under grants CNS-0520060 and CNS-0519911.

REFERENCES


Yvan Pointurier (S’02-M’06) received a Diplôme d’Ingénieur from École Centrale de Lille (France), an M.S. degree from the Department of Electrical and Computer Science at the University of Virginia (USA) in 2002, and a Ph.D. from the Charles L. Brown Department of Electrical and Computer Engineering in 2006, also at the University of Virginia. After a two-year Postdoctoral Fellowship in the Department of Electrical and Computer Engineering at McGill University in Montréal, Canada, where he is currently a Senior Researcher. His research interests span design, optimization, and monitoring of networks in general, and optical networks in particular. Dr. Pointurier is a corecipient of the Best Paper Award at the IEEE ICC 2006 Symposium on Optical Systems and Networks.
Maïté Brandt-Pearce received her B.S. in electrical engineering, with a second major in applied mathematics, from Rice University in 1985. She completed an M.E.E. in 1989 and a Ph.D. in electrical engineering in 1993, both also from Rice University. She worked with Lockheed in support of NASA Johnson Space Center from 1985 until 1989. In 1993, Dr. Brandt-Pearce joined the Charles L. Brown Department of Electrical and Computer Engineering at the University of Virginia, where she is currently a Full Professor. In 2005 she spent her sabbatical at the Eurecom Institute in Sophia Antipolis, France. Dr. Brandt-Pearce’s research interests lie in the mathematical and numerical description and optimization of communication systems with multiple simultaneous components from different sources. This interest has found applications in a variety of research projects including spread-spectrum multiple-access schemes, multiuser demodulation and detection, the study of nonlinear effects on fiber-optic multiuser-multichannel communications, optical networks subject to physical layer degradations, free-space optical multiuser communications, and radar signal processing and tracking of multiple targets. Dr. Brandt-Pearce is the recipient of an NSF CAREER Award, an NSF RIA, and an ORAU Junior Faculty Enhancement Award. She is a corecipient of Best Paper Awards at the ICC 2006 Symposium on Optical Systems and Networks. She is a member of Tau Beta Pi, Eta Kappa Nu, and a senior member of the IEEE. She was an Associate Editor for the IEEE Transactions on Communications from 1999 to 2006. She has served on the technical program committee for numerous conferences and was the 2009 General Chair for the Asilomar Conference on Signals, Systems, and Computers.

Suresh Subramaniam (S’95-M’97-SM’07) received his Ph.D. degree in electrical engineering from the University of Washington, Seattle, in 1997. He is a Professor in the Department of Electrical and Computer Engineering at the George Washington University, Washington, DC. His research interests are in the architectural, algorithmic, and performance aspects of communication networks, with particular emphasis on optical and wireless ad hoc networks. Dr. Subramaniam is a coeditor of the books Optical WDM Networks—Principles and Practice and Emerging Optical Network Technologies: Architectures, Protocols, and Performance. He has been on the program committees of several conferences including INFOCOM, ICC, GLOBECOM, OFC, and Broadnets, and served as TPC Co-Chair for the optical networks symposia at GLOBECOM 2006 and ICC 2007. He currently serves on the editorial boards of the IEEE/ACM Transactions on Networking, Optical Switching and Networking, and KICS Journal of Communications and Networks. He is a corecipient of Best Paper Awards at the ICC 2006 Symposium on Optical Systems and Networks and at the 1997 SPIE Conference on All-Optical Communication Systems.