Abstract—Savings in switching costs of an optical cross-connect can be achieved by grouping together a set of consecutive wavelengths and switching them as a single waveband. This technique is known as waveband switching. While previous work has focused on either uniform band sizes or nonuniform band sizes considering a single node, in this paper we focus on the number of wavebands and their sizes for ring topologies. First, we show that such solutions are inadequate when considering the entire network. We then present a novel framework for optimizing the number of wavebands in a ring network for deterministic traffic. The objective of the Band Minimization Problem is to minimize the number of nonuniform wavebands in the network while using the minimum possible number of wavelengths. We show that the problem is NP-hard and present heuristics for it. We then consider a specific type of traffic, namely all-to-all traffic, and present a construction method for achieving the minimum number of wavebands in the ring. Our results show that the number of ports can be reduced by a large amount using waveband switching compared to wavelength switching, for both all-to-all traffic and random traffic. We also numerically evaluate the performance of our waveband design algorithms under dynamic stochastic traffic.

Index Terms—Band minimization, data ordering, ring networks, waveband switching, wavelength division multiplexing (WDM) networks.

I. INTRODUCTION

WAVELENGTH division multiplexing (WDM) is the key technology in today’s optical networks. WDM multiplexes data to be transmitted over the fiber onto multiple wavelengths. With the aim of exploiting the full capacity of the fiber, the number of wavelengths that are multiplexed has increased substantially over the years (e.g., 40–80 wavelengths). A common requirement in an optical network is to be able to switch each wavelength individually. This is known as wavelength switching. Increasing the switching capacity to cope with the higher number of wavelengths is costly for the following reason. Optical cross-connects (OXC)s and reconfigurable optical add/drop multiplexers (ROADMs) are the switching nodes in today’s optical networks. In a wavelength-switched optical node, each wavelength is individually switched, requiring one switch per wavelength (or equivalently, one OXC port per wavelength). An example of a wavelength cross-connect for a ring network is shown in Fig. 1(a); all wavelengths from the network input are demultiplexed, and each wavelength is switched using a switching element shown in Fig. 1(b) [2]. The cost of such wavelength switching nodes can be very high considering the large number of wavelengths possible today [3, Ch. 2].

The number of switches in an OXC can be reduced by using waveband switching. Here, wavelengths are grouped together and switched using a single switch per group [4]. The wavelengths within a waveband are usually consecutive because of the way wavelengths are demultiplexed from a WDM signal, and sometimes because of how certain switches operate [5]. With this technique, the wavelength spectrum can be divided into wavebands, and lightpaths that are in the same waveband can be switched together. Several multigranular optical cross-connects that can switch at both waveband and wavelength granularity were introduced in [6] and [7].

There are two kinds of waveband switching: 1) uniform, and 2) nonuniform waveband switching. In uniform waveband switching, the waveband sizes at a node are all equal, and the same waveband sizes are used at every network node. In the latter kind, the sizes of wavebands can be different within a node and across different nodes [8].

II. PREVIOUS WORK AND MOTIVATION

Two issues pertaining to waveband switching have been studied in the past. The first is the routing and wavelength assignment (RWA) problem for given traffic considering the limited switching capabilities of the nodes. The goal is to choose the route using minimum wavelength resources (i.e., the route on which the wavelength can be grouped and switched together with other wavelengths). RWA algorithms for either dynamic or static traffic are proposed in [9]–[11] with uniform wavebands at the nodes. Similar algorithms for the case of...
different number of uniform wavebands at different nodes are considered in [12].

The second issue involves the waveband design problem for the case of nonuniform wavebands. This design problem mainly tries to find the optimal way of choosing the size of each waveband to minimize the overall switching complexity. It is shown that having nonuniform waveband sizes is advantageous in minimizing switching requirements in [8] and [13], where optimal ways of partitioning the wavelength set into wavebands are shown for various network topologies and traffic patterns. However, in [8] and [13], the authors investigate nonuniform wavebands focusing on a single node. In [13], the optimal wavebands for a switch with a single input and multiple output fibers are obtained. A similar design is presented in [8], which also includes the case of a single switching node with multiple inputs and multiple outputs (i.e., a star network). Additionally, solutions for general network topologies are presented for permutation traffic in [8].

As we show later, single-node solutions are incapable of handling even a simple set of connection requests in a network. In this paper, we consider ring networks, define a waveband optimization problem defined for the entire network, and present a novel formulation of the problem. We solve this problem for a specific deterministic traffic type using two heuristic algorithms. We also introduce an optimal construction method for all-to-all traffic. As it turns out, the waveband sizes are not only nonuniform at a node, but also vary across nodes—a scenario not commonly considered in the literature. We also evaluate the performance of the heuristics for a random traffic type in which there is a random number of connections between each node pair. We then evaluate the performance of the waveband design under stochastic dynamic traffic and show that our design can yield significant performance advantages over other designs.

Other related work has appeared in [14]–[17]. In [14], considering the three-layer multigranular OXC architectures of [11], the authors present an integer linear program (ILP) along with a heuristic algorithm in order to minimize the number of switching ports for the case of nonuniform waveband switching (i.e., fiber, waveband, and wavelength ports in the multigranular OXC). Another ILP model and heuristic are proposed for uniform waveband switching in [15] for mesh networks for the optimal assignment of uniform wavebands (i.e., minimize port count). Tuning bands of ROADM s in ring networks were considered in [16]. However, that paper minimized the worst-case band size of contiguous wavelengths accessed by a transponder and did not consider the optimization of switching costs. The blocking performance of ROADM s is evaluated in [17], where limited add/drop capability is brought by limited tunable transponders.

Our main contribution in this paper is that we solve the problem of waveband optimization in ring networks, which has not been considered from an entire network’s point of view before. We first present a novel framework for waveband optimization in ring networks for deterministic traffic and formulate the Band Minimization Problem in Section III. We first show that the problem is NP-hard. Then, along with a heuristic solution from the literature for a related problem, we develop our own heuristic ROWSWAP and show that it performs very well. Another main contribution is an optimal solution to the Band Minimization Problem for a specific traffic type, namely, all-to-all traffic (Section VI). In Section VII, we present numerical results for waveband optimization under random traffic. The performance of the waveband design under the oft-considered Poisson traffic is evaluated through simulations in Section VIII. Conclusions and some directions for future work are given in Section IX.

III. WAVEBANDING FOR DETERMINISTIC TRAFFIC

We consider ring topologies—both unidirectional (with a single fiber per link, oriented in one direction) and bidirectional (with two fibers per link, oriented in opposite directions). There is much work in optical networking that focuses on ring topologies [18], [19]. Ring topologies are useful to study because widely deployed synchronous optical network (SONET) rings have evolved into WDM rings. Furthermore, solutions for these simple topologies can provide insight into solving the much more complex case of mesh topologies. We define a deterministic traffic to be a specified set of lightpaths (LPs, or connections) that the network is required to support without blocking (i.e., the LPs can be provisioned) in the worst case. The actual LP traffic in the network may be random and/or dynamic; as long as the set of active LPs at any time is a subset of the deterministic traffic, the network can support the traffic.

Let us suppose we are given such a deterministic traffic. In this section, we present a formulation for the problem of minimizing the number of wavebands in the network, discuss the complexity of the problem, and present solution approaches. We first show by example that single-node wavebanding solutions (as in [13]) do not work when the network is considered as a whole.

Switching of nonuniform wavebands that are different across nodes can be realized as follows. At a node, the incoming WDM signal is demultiplexed into the required bands using filters with different cutoff/pass bands (an architecture to do this is provided in [5], for example). The bands can then be individually switched by a switching element as shown in Fig. 1(b).

A. Inadequacy of Single-Node Solutions

We briefly explain the single-node solution of [13] for reader convenience. Suppose a node has one input fiber and, say, output fibers, and each fiber has wavelengths. In order for the switch to support any breakdown of wavelengths from the input fiber to the output fibers, the authors developed an optimal algorithm that finds the nonuniform waveband sizes giving the minimum number of bands. The algorithm can be explained as follows: Initializing , calculate the size of the band at each step as , and update as . The algorithm stops when . As an example, when , the eight wavelengths are grouped into four bands of size and . The reader can easily verify that wavelengths can be switched to the first output and wavelengths switched to the other output, for any value.

If an LP request arrives that is not part of the deterministic LP set, then obviously it is blocked. However, if the bands are reconfigurable, then a new set of wavebands for a new traffic including this new call could be computed and the bands redone in order to accommodate the new call.
of \(i \{0, 1, \ldots, 8\}\) by combining the bands appropriately. For example, if five wavelengths are desired at the first output port, then the band of size 4 and one of the bands of size 1 can be assigned. Note that uniform banding would have required eight bands of size 1 (which is the same as individual wavelength switching), and so nonuniform banding saves four switches in this case. A similar banding algorithm is given for a single switch (equivalent to a star network with a single source node) in the case of \(P\)-port traffic in [8]. \(P\)-port traffic model includes any possible traffic set that has at most \(P\) connections to/from a node. The banding calculation becomes identical to [13] when \(P = W\).

We now show that the same band sizes (calculated by the algorithm in [13] for a single node) cannot be used at every node to support all traffic for which there are a sufficient number of wavelengths. Consider all-to-all traffic (one LP between every pair of nodes) in a five-node bidirectional ring network. It is known that three wavelengths per fiber are necessary and sufficient to support this traffic (see [20] for example). Furthermore, LPs must be routed along the shorter direction, and all wavelengths are utilized on all links in this solution. Therefore, whenever a wavelength is dropped at a node to terminate a lightpath, the same wavelength must be added by the same node to originate another lightpath. Each wavelength or waveband at a node can be added/dropped (A/D) or bypassed (B). We need only consider one direction of the ring because the switches in the other direction are set correspondingly. The two bands calculated by the algorithm of [13] with \(W = 3, P = 2\) are of sizes 2 and 1. Since each node requires two A/D wavelengths to connect to the four other nodes, the waveband of size 2 should be A/D at each node. Out of the three wavelengths, the A/D band can be wavelengths \{1, 2\} or wavelengths \{2, 3\}. Thus, the three wavelengths are arranged in two bands in one of the following two configurations at each node. A node with the left configuration in Fig. 2 is called Type 1, and with the right configuration is called Type 2. (Note that the two bands cannot both be A/D or both be B because some LPs bypass and some LPs terminate at each node.)

Note that wavelength 2 is always in an A/D band as seen in Fig. 2. Now, since only one of the above two bandings is possible at any node, if wavelength 1 is in an A/D band, then wavelength 3 is in a B band, and vice versa. Of the five nodes in the network, let \(k_1, k_2 = 0, 1, \ldots, 5\), nodes be Type 1, and the other \(k_2 = 5 - k_1\) be Type 2. It is easy to see that \(k_1 = 0, 1\) would not allow any LP to be provisioned on wavelength 1, while \(k_1 = 2\) will let one LP be provisioned, but leaves wavelength 1 unutilized on the other links, and is therefore not sufficient to support the given traffic. Now, when \(k_1 = 3, 4, 5, k_2 = 2, 1, 0\), respectively, and wavelength 3 would have the same problem. Therefore, it is not possible to support all-to-all traffic using just the two bands. An example illustrating this impossibility is given in Fig. 3(a). It can be seen that two bands are sufficient if there are four nodes, but the addition of a fifth node requires one of the nodes to have three bands. A valid band assignment for all-to-all traffic is shown in Fig. 3(b).

It can be shown that two bands are not sufficient for the case of a unidirectional ring in a similar manner. Here, just a three-node example suffices. Three wavelengths are required for a three node unidirectional ring as in Fig. 4. For all-to-all traffic, each node has an LP to the other two nodes, requiring an A/D band of size 2. As in the bidirectional case, the second wavelength will always be in an A/D band. However, there should be
only two nodes with A/D bands on each wavelength since one wavelength is used to connect each pair of nodes. A valid band assignment with one node having three bands is shown in Fig. 4.

B. Novel Framework for Band Optimization

Having shown that a single node cannot be considered in isolation and the network must be considered as a whole, we now proceed to formulate the band minimization problem in the following manner. Suppose the \( N \) nodes of a ring are numbered as \( 1, 2, \ldots, N \), and the \( W \) wavelengths numbered as \( 1, 2, \ldots, W \). Let \( \beta_n \) denote the number of wavebands at node \( n \) and \( b_{n,i} \) denote the size of the \( i \)-th band at that node \( (i = 1, \ldots, \beta_n) \); thus, \( \sum_{i=1}^{\beta_n} b_{n,i} = W \forall n \).

In a ring network, a wavelength is in either an A/D band or a B band. Let \( \gamma_{w,n} \) be a binary variable representing the band to which wavelength \( w \) belongs; if it is in a B band, \( \gamma_{w,n} = 0 \), and \( \gamma_{w,n} = 1 \) otherwise. Let \( \Gamma \) be the \( \times \) \( N \) binary matrix consisting of all the \( \gamma \) values. Given a deterministic traffic, an RWA algorithm may be used to provision the LPs; \( \Gamma \) is determined by the output of the RWA algorithm. Let \( r_w \) denote the row vector of \( \Gamma \) corresponding to wavelength \( w \), and let \( c_n \) denote the column vector of \( \Gamma \) corresponding to node \( n \).

While there is much literature on RWA algorithms, the goal is often to minimize the number of wavelengths required to support a given traffic. The output of the RWA algorithm has not been analyzed further from a banding point of view. For the results of a specific RWA algorithm that \( \Gamma \) represents, observe that runs of 1’s or runs of 0’s in a specific column, say \( c_n \), represent opportunities for wavebanding at node \( n \). Recall that a 0 represents B and 1 represents A/D, so a string of consecutive 0’s (respectively, 1’s) in a column means that all of those corresponding consecutive wavelengths are bypassed (respectively, added/dropped) at node \( n \), and hence can be switched as a single band. From this point of view, the number of bands at node \( n \) can be obtained by simply counting the strings of consecutive 1’s and 0’s in column \( c_n \). Let us denote the total number of bands in the ring network as \( B \), i.e., \( B = \sum_{n=1}^{N} \beta_n \). Here, we illustrate the calculation of bands using the example of Fig. 3(b). For the given RWA, \( \Gamma \) is obtained as

\[
\Gamma = \begin{pmatrix}
1 & 1 & 1 & 1 & 0 \\
0 & 1 & 0 & 1 & 1 \\
1 & 0 & 1 & 0 & 1
\end{pmatrix}.
\]  

The number of bands at each node with this configuration can be found as \( \beta_1 - 3, \beta_2 - 2, \beta_3 - 3, \beta_4 - 2, \beta_5 - 2 \), and the total number of bands is \( B = 12 \). If banding were not done, then each of the three wavelengths would be switched independently at each node, leading to a total of 15 ports, so we get a saving of three ports, across the network. Can we do better? Suppose that the wavelength assignment is the same but the order of wavelengths is different. The following matrix shows \( \Gamma \)’ with the first and second rows (wavelengths) switched:

\[
\Gamma' = \begin{pmatrix}
0 & 1 & 0 & 1 & 1 \\
1 & 1 & 1 & 1 & 0 \\
1 & 0 & 1 & 0 & 1
\end{pmatrix}.
\]  

The number of bands now is \( \beta_1 = 2, \beta_2 = 2, \beta_3 = 2, \beta_4 = 2, \beta_5 = 3 \), giving \( B = 11 \). Thus, the number of bands can be optimized by reordering the wavelengths assigned to LPs by an RWA algorithm.

IV. PROBLEM DEFINITION AND COMPLEXITY

The Band Minimization Problem (BMP) can now be posed as the problem of reordering the rows of a given matrix \( \Gamma \) to obtain a new matrix \( \Gamma' \) so that the total number of consecutive runs of 1’s and 0’s in each column, summed over all columns, is minimized.\(^2\) It turns out that this problem is almost identical to a well-known problem in the low-power chip design community known as the Data Ordering Problem (DOP). The problem was first defined in [21] as follows: Given a binary matrix, a _transformation in a column vector is defined as the number of changes from 1 to 0, or vice versa. From this definition, the number of transitions in \( e_n \) is one less than the number of bands \( (\beta_n) \) at node \( n \).

Hence, the total number of transitions \( \tau = H - N \). The number of transitions between two row vectors of wavelengths \( i \) and \( j \) is the Hamming distance between the binary vectors \( r_i \) and \( r_j \) (defined as \( d(r_i, r_j) \)). The DOP is stated as follows: Find a permutation \( \sigma \) of the row vectors \( r_1, r_2, \ldots, r_W \) of \( \Gamma \) such that the total number of transitions \( \tau = \sum_{i=0}^{W-1} d(r_{\sigma(i)}, r_{\sigma((i+1)\mod W)}) \) is minimized. It is shown in [21] that DOP is NP-hard.

We note that the matrix \( \Gamma \) may sometimes contain _don’t cares_ rather than just 1’s and 0’s. Such a situation will occur, for example, when not all wavelengths are occupied on all links for a given deterministic traffic, and so there are some band switches that do not have to be set to either B or A/D. Under this situation, a more complex problem called _Band Minimization Problem with Don’t Cares_ arises. There are some results for the related Data Ordering Problem with Don’t Cares in the literature [22].

A. Complexity of BMP

**Proposition 1:** BMP is NP-hard.

**Proof:** We prove this by showing that any instance of the DOP can be reduced to BMP in polynomial time. Take any two-dimensional matrix that is an input to DOP. Call this matrix \( \Gamma_{\text{DOP}} \). For a row \( r_i \) of \( \Gamma_{\text{DOP}} \), we apply the following transformation. We find the first two 1’s on this row, say at columns \( n \) and \( m \), and create an LP on the ring network between nodes \( n \) and \( m \) on wavelength \( i \) [e.g., between 1’s at the second and fourth positions of \( r_i \) of \( \Gamma_{\text{DOP}} \) in Fig. 5(a)]. We continue adding LPs on the same wavelength in between those nodes corresponding to the locations of consecutive 1’s. We finally add an LP for the last and first 1 locations for the cyclic connection (e.g., between the fourth and second positions of \( r_1 \)). We repeat this process for every \( r_i \) of \( \Gamma_{\text{DOP}} \).\(^3\) In this way, any instance of DOP can be used to construct an instance of BMP. Since DOP is NP-hard [21], BMP is also NP-hard.

B. Characterization of the Problem

We now show how the Data Ordering Problem is related to some other well-known problems. These relationships may

\(^2\)Other banding problems—for example, the minimization of the maximum number of bands over all nodes—are possible, but are not considered in this paper. It is also possible to define a joint routing, wavelength assignment, and wavebanding problem. However, this does not necessarily result in the minimum number of wavelengths required to support a traffic [8].

\(^3\)The assumption of having at least two 1’s in a row of \( \Gamma_{\text{DOP}} \) does not affect the complexity of DOP.
be exploited to obtain improved solutions to the DOP. Specifically, we show how we can transform this problem into a Minimum Weight Hamiltonian Path Problem (MWHPP). We can construct a graph $G = (V, E)$ consisting of $W$ nodes, each corresponding to one row of $\Gamma$. Every node of this graph is connected to every other node with an edge having the weight as the Hamming distance between the corresponding rows. Hence, the edge between nodes $v_i$ and $v_j$ has the weight $d(r_i, r_j)$ for every $i$ and $j$. Then, the Hamiltonian path that visits every node in this graph with the minimum total weight gives the solution to the DOP. In other words, the Minimum Weight Hamiltonian Path gives the permutation of the rows of $\Gamma$ for the minimum number of transitions.

Different from the general version of the MWHPP, the graph in this case is a complete graph with the edge weights being integers. Several methods for solving the DOP are summarized in [21] and [23].

We now give an example for a bidirectional ring with $N = 7$. The $\Gamma$ matrix obtained by the RWA algorithm and the graph generated from $\Gamma$ are shown in Fig. 6(a) and (b), respectively. The numbers on the edges of the graph in Fig. 6(b) denote the weights. The bold path shows the Minimum Weight Hamiltonian Path calculated on this graph with a total weight of 15. Therefore, the minimum number of bands for the bidirectional ring of seven nodes is $15 + 7 = 22$.

V. HEURISTIC SOLUTIONS

We now present two heuristic solutions to the BMP called ROWSWAP and GREEDY. ROWSWAP is our own heuristic, whereas GREEDY is a heuristic from the literature to solve the DOP.

A. ROWSWAP Algorithm

This is implemented with a recursive function that takes the matrix $\Gamma$ as input. Given the row vectors of $\Gamma$, it takes a pivot vector and moves it to another row only if this reordering reduces $\text{min}_{\text{total bands}}$, which is the current minimum value obtained by the algorithm until that point. The output matrix of this reordering is denoted by $\Gamma'$. If a reordering occurs, $\text{min}_{\text{total bands}}$ is updated, and the same procedure as above is applied to the new reordered matrix by recursively calling the same function with $\Gamma'$ as the input matrix. All the row vectors $r_i$ ($1 \leq i \leq W$) are used as a pivot vector starting with the first row $r_1$. When the algorithm stops, the value of $\text{min}_{\text{total bands}}$ gives the minimum total number of bands achieved by this algorithm. We show the outline of this heuristic in Algorithm 1. In Algorithm 1, $\text{calcBands}$ is a function that computes the number of bands in the input matrix. We also use the same heuristic for the case when $\Gamma$ includes don’t cares. The function $\text{calcBands}$ counts the wavebands of $\Gamma$ similarly by assigning don’t care to the value of either 0 or 1 that is in an adjacent row and same column.

Algorithm 1: Pseudocode for the ROWSWAP heuristic

```
input: $\Gamma$
output: $\Gamma'$ with reordered rows of $\Gamma$
1 Initialization: $\text{min}_{\text{total bands}} = \text{calcBands}(\Gamma)$;
2 ROWSWAP($\Gamma$);

Function ROWSWAP($Matrix'$):

for $p \leftarrow 1$ to $W - 1$ do
   for $i \leftarrow p + 1$ to $W$ do
      Form $Matrix'$ by moving $r_p$ to insert after the row $r_i$
      if $\text{calcBands}(Matrix') < \text{min}_{\text{total bands}}$
         then
            $\text{min}_{\text{total bands}} = \text{calcBands}(Matrix')$
            $\Gamma' = Matrix'$
            ROWSWAP($Matrix'$)
   end
end
```

B. GREEDY Algorithm [21]

We can first form the transition matrix $\Delta$ of size $W \times W$ whose elements $\delta_{i,j}$ are the Hamming distances between the rows of $\Gamma$, $r_i$ and $r_j$. GREEDY initially takes the minimum element in this matrix, which is the pair of rows having the minimum Hamming distance between each other. This pair constitutes the initial sequence of rows. In the next step, it finds two rows having the minimum distances to the rows in the initial sequence. It adds these rows to the two ends of the sequence. At each step, it similarly finds the rows with the minimum distance to the rows at the ends of the sequence and adds these rows to the ends of the sequence. The algorithm stops when every row is
Algorithms 2: Pseudocode for the GREEDY heuristic

```
input: \( I \) : \( S \) is the set of rows in \( I \)
output: \( \Gamma \) : \( S' \) be the set of rows in \( \Gamma' = \{ r'_1; \ldots; r'_{|S'|} \} \)

1. Create Transition matrix \( \Delta : \delta_{i,j} = d(r_i, r_j) \) \( \forall i, j \)
2. Find the row pair with minimum Hamming distance:
   \( (i, j) = \arg \min_{i < j} \delta_{i,j} \)
3. Create initial \( \Gamma' : \Gamma' = \{ r_i; r_j \} \)
4. For \( p = 1 \) to \([W/2]\) do
   5. \( i = \arg \min \frac{d(r'_i, r_i)}{\forall r_i \in S \setminus \{ r'_i \}} \)
   6. Update \( \Gamma' : \Gamma' = \{ r'_i; \Gamma'; r_j \} \)
end
```

GREEDY can be modified easily to be used with don't cares. The Hamming distance calculation is done considering the don't cares such that if any of the elements is a don't care, then the distance between them is considered to be 0. At each iteration, the value of the don't care of the chosen row is assigned to the corresponding element of the just added row at the same column. For example, if \( \{x, x, 1, 1\} \) with \( x \)'s denoting don't cares is added next to \( \{1, 1, 0, 0\} \), then it would be added as \( \{1, 1, 1, 1\} \).

**VI. APPLICATION TO ALL-TO-ALL TRAFFIC**

We now apply this problem formulation and the heuristic algorithms to a specific type of deterministic traffic, namely all-to-all traffic. This is a special case of deterministic traffic, and it is possible to provide an optimal wavebanding algorithm for this traffic. Recall that in all-to-all traffic, there is exactly one LP between every ordered pair of nodes. This traffic model, though simple, is quite powerful because if the network is designed for this traffic, then any traffic pattern (even with LPs arriving and departing dynamically) can be supported as long as it is a subset of all-to-all traffic, i.e., not more than one LP is required to be provisioned from one node to another node. We also derive a lower bound on the number of bands. We mostly focus on bidirectional rings and turn to unidirectional rings at the end of this section.

For the bidirectional ring, we adopt the optimal RWA algorithm presented in [20], which uses the minimum number of wavelengths. For simplicity, we assume that the number of nodes in the ring is odd in order to remove the ambiguity in routing (so that there is exactly one shortest path between any two nodes) and is denoted by \( N \). However, similar results can be obtained for even \( N \). The optimal algorithm in [20] uses \( (N^2 - 1)/8 \) wavelengths for odd \( N \). For reader convenience, we describe this algorithm in Section VI-A because our bound depends on the algorithm. Every wavelength is fully utilized as a result of this algorithm (i.e., all of the wavelengths are used on all links).

A. Optimal RWA Algorithm

We first explain the algorithm of [20]. The algorithm works by assigning wavelengths to LPs connecting nodes that are “placed” on the ring (starting from a ring with no nodes “placed”), until all nodes are placed and therefore all LPs are established. Let \( n_i \) denote the number of nodes already placed at the beginning of step \( i \). For odd \( N \), this algorithm starts with assigning the wavelength 1 to connect the first three nodes as shown in Fig. 7(a) (i.e., \( n_0 = 3 \)). At each step \( i \), two more nodes (denoted by \( \alpha_{i,1} \) and \( \alpha_{i,2} \)) are added to the ring from the previous step at those places of a cut that divides the ring into two sides, one side having exactly one more node than the other. A wavelength is used to connect each node on the right side to \( \alpha_{i,1} \) and \( \alpha_{i,2} \); the same wavelength also connects \( \alpha_{i,1} \) and \( \alpha_{i,2} \) to a node on the left side. One more wavelength connects the remaining node on the left side to \( \alpha_{i,1} \), \( \alpha_{i,2} \), and \( \alpha_{i,1} \), \( \alpha_{i,2} \) directly to each other [e.g., wavelength 3 in Fig. 7(b)]. The lightpaths formed at the end of step 1 are shown in Fig. 7(b).

This procedure of adding two nodes at each step goes on until the required number of nodes \( N \) are added to the ring. This algorithm results in the minimum number of wavelengths, which is \( W = (N^2 - 1)/8 \).

The above RWA algorithm determines the matrix \( \Gamma \). Let us define \( \pi_i \) as a row (wavelength) of \( \Gamma \) with \( i \)'s, and \( \Pi_i \): the number of such rows in \( \Gamma \). We now show that the \( \Gamma \) from the algorithm of [20] gives the following result.

**Proposition 2:** The output matrix \( \Gamma' \) of the optimal RWA algorithm in [20] has \((N - 1)/2\) rows with three \( 1 \)'s and \((N^2 - 4N + 3)/8\) rows with four \( 1 \)'s, (i.e., \( \Pi_3 = (N - 1)/2 \), \( \Pi_4 = (N^2 - 4N + 3)/8 \), and \( \Pi_i = 0 \) \( \forall i \neq 3, 4 \)).

**Proof:** In the algorithm, initially (i.e., \( i = 0 \)) there are three nodes, and at each step two more nodes are added. Therefore, there are a total of \((N - 3)/2\) steps. Initially, there are three lightpaths on the first wavelength, hence the first row of \( \pi_1 \) is \( \pi_2 \). At each step, \( i > 0 \), there is one new wavelength with three lightpaths [the wavelength used to connect the two new nodes to each other; see wavelength 2 in Fig. 7(b)]. The number of wavelengths with three lightpaths (i.e., the number of rows with three \( 1 \)'s) is \( \Pi_3 = \sum_{i=0}^{(N-3)/2} 1 = (N - 1)/2 \). At each step \( i \), a single wavelength is used to connect the new nodes to a pair of old nodes on different sides of the cut, thus creating a wavelength with four lightpaths (i.e., \( \pi_4 \)). Note that at the beginning of step \( i \), there are \( 2i + 1 \) nodes. The additional number
of wavelengths added at step $i$ is $((2i + 1) - 1)/2 = i$ since one new wavelength for a pair of old nodes is used. Therefore, 
$$
P_4 = \sum_{i=1}^{(N-3)/2} i = \frac{N^2 - 4N + 3}{8}. $$
To verify the result, we calculate $\Pi_3 + \Pi_4$ to get $(N^2 - 1)/8$, which is the number of wavelengths used by this algorithm.

We use this proposition in the derivation of the lower bound in Section VI-B.

### B. Lower Bound for Bidirectional Rings

Let $t_{i,j}$ be the minimum number of transitions between a row with $i$ 1’s and a row with $j$ 1’s. The minimum number of transitions depends on a special property of the $\Gamma$ matrix. Since it is all-to-all traffic and each pair of nodes is assigned only one wavelength, no two rows can have a sequence starting with a 1 followed by some 0’s and ending with a 1, with the 1’s exactly in the same positions. For example, $\Gamma$ cannot have the two rows in (3) at the same time because they both contain the sequence 1 0 1 at the same positions

$$
\begin{pmatrix}
  0 & 1 & 0 & 1 & 1 \\
  1 & 1 & 0 & 1 & 0
\end{pmatrix},
$$

(3)

Using this property, we can find at most how many 1’s can be at the same positions in any two rows, and hence find the minimum number of bands. We can easily see that between two $\pi_4$’s, there are at most two 1’s at the same positions. The remaining two 1’s have to be at different positions, hence there are at least four transitions between $\pi_4$’s (i.e., $t_{4,4} = 4$). If we consider two $\pi_3$’s, it is trivial to find that the positions of two 1’s can remain unchanged resulting in $t_{3,3} = 2$. However, this would require one of the paths routed along the longer path, which is a violation of the shortest path requirement of the optimal RWA algorithm. With shortest-path routing, $t_{3,5}$ can actually be seen to be 4.\textsuperscript{5}

Knowing that if we have a $\pi_4$, we can again hold the positions of at most two 1’s between a $\pi_4$ and $\pi_3$. However, in this case the minimum number of transitions is three (i.e., $t_{3,4} = 3$).

**Theorem 1 (Lower Bound on the Number of Bands in a Bidirectional Ring):** A lower bound on the number of bands in a bidirectional ring with $N$ nodes ($N$ odd) is $L = (N^2 - 7)/2$.

**Proof:** Suppose we are given $\Pi_3$ rows with three 1’s and $\Pi_4$ rows with four 1’s ($\Pi_3, \Pi_4 > 1$). We want to construct a matrix consisting of these rows with the minimum number of transitions. We start with the initial matrix $\Gamma_1$ consisting of only $\pi_4$’s. We note that the initial total number of transitions $\tau_1$ in $\Gamma_1$ is $\Pi_4(1) + t_{4,4} = 4\Pi_4 - 4$. We want to insert a row $\pi_3$ between the rows of the initial matrix. There are two possible cases of this insertion. The first case is inserting $\pi_3$ at the top or bottom of the matrix. The second case is inserting $\pi_3$ in between any pair of $\pi_4$’s.\textsuperscript{6}

In the first case, the new transition count becomes $\tau_0 - \tau_1 + t_{3,4} + 4\Pi_4 - 4$. In the second case, with this insertion it removes a transition between a pair of $\pi_4$’s and adds two transitions between $\pi_4$ and $\pi_3$. Hence, the new transition count is $\tau_2 = \tau_1 - t_{4,4} + 2t_{3,4} = 4\Pi_4 - 2$. Note that $\tau_2$ obtained in the second case is smaller than in the first case. Suppose we continue with the construction of the matrix as follows.

Suppose that at step $i$ of the construction, we are inserting the $i$th $\pi_3$ into the matrix $\Gamma_i$ with transition count $\tau_i$, and $\Gamma_i$ has $\pi_4$’s at the top and bottom. The following cases arise. Insertion:

1) to either top or bottom of $\Gamma_i$: $\tau_{i+1} = \tau_i + t_{3,4} = \tau_i + 3$;
2) between a pair of $\pi_4$’s: $\tau_{i+1} = \tau_i - t_{4,4} + 2t_{3,4} = \tau_i + 2$;
3) between a pair of $\pi_3$’s: $\tau_{i+1} = \tau_i - t_{3,4} + 2t_{3,3} = \tau_i + 4$;
4) between $\pi_3$ and $\pi_4$: $\tau_{i+1} = \tau_i - t_{3,4} + t_{3,3} + t_{3,4} = \tau_i + 4$.

Note that the second case is the best case in which the transition count increases by 2. Therefore, at every step, in order to have minimum transition count in the matrix, we can insert $\pi_3$ at any position in between a pair of $\pi_4$’s. By induction, we say that as long as we have every $\pi_3$ between a pair of $\pi_4$’s, the ordering within the matrix will not matter and will always give the minimum number of transitions. Since the increment of each step on the transition count is 2, the minimum total number of transitions is $\tau = \tau_1 + 2\Pi_3 = 4\Pi_4 + 21 \Pi_4 - 4$.

Given that $\Pi_3 = (N-1)/2$ and $\Pi_4 = W - \Pi_3$, where $W = (N^2 - 1)/8$, we then obtain the lower bound as $L = \tau + N = ((N^2 - 1)/2) - 3 - (N^2 - 7)/2$.

Now we show that this lower bound is achievable with a special construction method, and hence this method gives the optimal number of wavelengths.

### C. Optimal Construction for Bidirectional Rings

We develop an optimal construction method by a special ordering of the wavelengths (rows) added to the $\Gamma$ at each iteration of the optimal RWA algorithm explained in Section VI-A. Our goal is to obtain a construction that will have every single one of the $\pi_3$’s in between a pair of $\pi_4$’s. Moreover, in that structure, we want every transition between rows to be at the minimum possible transition count $t_{4,4}$ (e.g., every adjacent pair of $(\pi_3, \pi_4)$ and $(\pi_4, \pi_4)$ has a Hamming distance equal to $t_{3,4}$ and $t_{4,4}$, respectively).

Let $\pi_3(i)$ and $\pi_4(i)$ denote a $\pi_3$ and $\pi_4$ added at the $i$th iteration, respectively. We remind the reader that at the $i$th step of the RWA algorithm, $i$ new $\pi_4$’s and one $\pi_3$ are added.

Assume a cut at step $i$ is made right next to where cut at step $i-1$ is (e.g., $\alpha_{i-1}, \alpha_{i}$ is on the clockwise side of $\alpha_{i-1}, \alpha_{i}$, respectively). We now make the following observations for the $i$th iteration.

1) **Observation:** There is a $\pi_4(i)$ that only has a Hamming distance of $t_{4,4}$ to any $\pi_4(k)(k \neq i)$.

**Proof:** While forming $\pi_4(i)$’s, we can have the cut pair $\alpha_{i-1}, \alpha_{i}$ connect to the same nodes that $\alpha_{i-1}, \alpha_{i}$, $\alpha_{i-1}$, $\alpha_{i}$.

\textsuperscript{6}For $\Pi_4 = 1$ (i.e., $N = 5$), this exact construction does not work. However, proceeding in a similar way, a lower bound of 10 can be obtained for $N = 5$. Thus, Theorem 1’s lower bound for $N = 5$ ($L = 9$) is still valid.
connected to in $\pi_4(i - 1)$’s. Therefore, with this assignment, any $\pi_4(i)$ and $\pi_4(i - 1)$ differ at only four positions, which are the different cut points as shown in Fig. 8.

2) **Observation**: The number of transitions between the $\pi_3(i)$ and any $\pi_3(i)$ is $t_{3,i} = 3$.

**Proof**: Since $\pi_3(i)$ and any $\pi_3(i)$ share the same cut points, they only differ at those positions of the node on either side of the cut. A $\pi_3(i)$ connects to one node in either side, whereas $\pi_3(i)$ only connects to one node on one side [see Fig. 7(b)]. Therefore, the number of transitions in between is three.

3) **Observation**: The number of transitions between any pairs of $\pi_4(i)$ of the same iteration $i$ is $t_{4,4} = 4$.

**Proof**: Similar.

First we explain the construction for $N \geq 9$, and later we will explain the special cases of $N = 5$ and $7$. Let us first show the matrix obtained after the first three iterations. (Let $\Gamma(i)$ denote the matrix after the $i$th iteration and $\pi_4(i)$ denote the $i$th added at that iteration. Elements in boldface are the cut-points.)

$$\Gamma(3) = \begin{pmatrix} \pi_3(0) \\ \pi_3(1) \\ \pi_3(2) \\ \pi_3(3) \\ \pi_3(4) \\ \pi_3(5) \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

1) Step 1: We order the rows of the matrix as follows after the third iteration to get:

$$\Gamma'(3) = \left[ \pi_4(1); \pi_3(0); \pi_4(2); \pi_3(2); \pi_4(2); \pi_3(1); \pi_4(3); \pi_3(3); \pi_4(3) \right] .$$

The reader can easily verify that the number of transitions in this matrix after this step is $8t_{3,4} + t_{4,4} = 28$.

2) Step $i (i \geq 4)$: Among the rows added in this iteration, find the $\pi_4(i)$ that has the minimum distance to the $\pi_4(i - 1)$ at the bottom of $\Gamma'(i - 1)$ and add it as the row right below (using observation 1). We call this newly added row $\pi_4^*(i)$. First add $\pi_4^*(i)$ and then the other $\pi_4(i)$’s (in any order); insert the $\pi_3(i)$ between any pair of $\pi_4(i)$’s. We thus form the following sequence $\eta(i) = [\pi_4^*(i); \pi_4(i); \ldots; \pi_3(i); \ldots; \pi_4(i)]$. For example, $\pi_4^*(i)$ to be added right under the row $\pi_4^*(3)$ in (4) in iteration 4 would be $\pi_4^*(4) = [0 \ 1 \ 0 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 0 \ 0 \ 1 \ 1 \ 0 \ 0 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 1 \ 1 \ 0 \ 0 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 0 \ 1 \ 1 \ 0 \ 0 \ 0 \].$

According to observations 2 and 3, $\eta(i)$ has the minimum total number of transitions calculated as $(i - 2)t_{4,4} + 2t_{4,4}$. We append these rows to the modified matrix $\Gamma_{\text{modified}}(i - 1)$ with additional columns corresponding to the cut-points (e.g., 0’s are inserted to those locations of the new cut-points in $\Gamma(i - 1)$) to get $\Gamma(i)$ as follows:

$$\Gamma(i) = \left[ \Gamma_{\text{modified}}(i - 1) \right],$$

(5)

Note that as we append these two matrices, the number of transitions is incremented by $t_{4,4}$. Therefore, the additional transitions in this step is $(i - 1)t_{4,4} + 2t_{4,4} = 4i - 2$.

Clearly, the number of transitions at the initial ordering and at each additional step are the minimum values. Therefore, the final matrix will have the optimal number of bands. Alternatively, we can get the total number of bands by summing the transitions from the above steps as $28 + \sum_{i=4}^{N-3} 4i + 2 - (N^2 - 2N - 7)/2$. Adding this result to $N$, we get $(N^2 - 7)/2$. We thus have the following theorem.

**Theorem 2 (Optimal Number of Bands in a Bidirectional Ring)**: The optimal number of bands in a bidirectional ring with $N$ nodes ($N$ odd) is

$$B_{opt} = \frac{N^2 - 7}{2} \quad \text{for } N > 9.$$
TABLE I

<table>
<thead>
<tr>
<th>N</th>
<th>W</th>
<th>WXC</th>
<th>Lower Bound</th>
<th>Non-Uniform WXC</th>
<th>Reduction in #switches</th>
<th>Reduction after Γ'</th>
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</table>

Fig. 9. (a) Binary reflected Gray code of size 4. (b) Π with minimum bands for a four-node unidirectional ring.

transitions is $5 \times 2 = 10$. In general, the minimum number of transitions obtained by this construction is $\tau = 2(W - 1)$, where $W = \binom{4}{2}$. Therefore, the total number of bands (switching elements) for the entire network is $B = 2(W - 1) + N = N^2 - 2$.

E. Numerical Results

We compare the total number of nonuniform wavebands (i.e., switching elements) calculated by the optimal, GREEDY, and ROWSWAP to the total number of switching elements in a wavelength switching architecture. We include results obtained by the heuristics along with the optimal values in order to be able to assess their performances. In a wavelength-switched network, there are $W$ switching elements (one for each wavelength) per node. Therefore, the total number of bands in the network is $NW$.

First, for a unidirectional ring, the number of wavelengths for all-to-all traffic is $W = \binom{N}{2}$, and the number of switches for wavelength switching is $NW = N^2(N - 1)/2$. Compare this to the number of switches with band switching, which is given by $B = N^2 - 2$ (from Section VI-D). Therefore, the reduction in the number of switches (over the entire network) is $N^2((N - 3)/2) + 2$. For example, a 10-node ring requires $W = 45$ wavelengths and 450 wavelength switches, but only 98 band switches, leading to a 78% reduction in the number of switches. The results get better as the network size increases; for example, a 20-node ring would require 3800 switches with wavelength switching, but only 398 switches with band switching, a 90% reduction in the number of ports.

We next present results for bidirectional rings in Table I. Compared to wavelength switching (WXC), we see that the reduction in bands is significant, starting at 27% for $N = 5$ and increasing with increasing $N$. We see that the two heuristics give very close results to the optimal. Additionally, the optimal values are very close to the lower bound. For $N \geq 11$, GREEDY achieves the lower bound and hence gives optimal results. In the same range, ROWSWAP calculates the number of bands as two more than the lower bound. However, for $N = 7$, ROWSWAP achieves the optimal value, whereas GREEDY’s result is one more than the optimal. The last column shows the further reduction achieved by GREEDY from the bands in the initial $\Gamma$ (i.e., the unoptimized $\Gamma$ resulting from the RWA algorithm).

VII. APPLICATION TO RANDOM TRAFFIC

We now apply the provided solutions to a randomly generated traffic matrix. In this traffic model, there is a random number of LPs generated uniformly randomly from the set $\{0, 1, \ldots, k\}$ between every pair of nodes in the ring. We apply the first-fit wavelength assignment with alternate routing as the RWA algorithm, in which we try to assign each LP to the first available wavelength (e.g., lowest numbered) first on the shortest path direction, and if no such wavelength exists, in the opposite direction. If no wavelength is found on either direction, a new wavelength is created and assigned to the LP on the shortest path. Moreover, contrary to the optimal RWA algorithm used for all-to-all traffic, the wavebands used for switching on both links (e.g., east/west) at a node are not symmetrical (i.e., a wavelength can be added/dropped on the first link, but it may not be used on the second link of a node). Therefore, we create a new matrix $\Gamma_r$ of size $W \times 2N$ that has a column for each link of a node. Note that the RWA strategy used here results in the occurrence of don’t cares as elements of $\Gamma_r$, due to underutilization of some of the wavelengths. We show this strategy on a four-node ring network with one LP between node pairs (1, 2), (2, 3), (1, 4); two LPs between (3, 4) and (2, 4) in Fig. 10. We then apply the heuristics to $\Gamma_r$ and calculate the average percentage reduction in the number of switching elements over 100 trials.

Because of the uncertainty due to don’t cares, heuristics do not improve the waveband results in some cases. For such cases, we just use the number of bands in the original $\Gamma_r$. For GREEDY, making a decision one step at a time based on the Hamming distances to the next pair of rows instead of a holistic approach makes it less efficient with the added degree of complexity with don’t cares. In Table II, the column Overall shows the overall
Lightpath establishment requires a wavelength that is available end-to-end with available waveband switches on each node along the route.

We apply the nonuniform bands calculated for deterministic all-to-all traffic to the case of dynamic traffic. The actual number of wavelengths \( \{W_n\} \) in the system can be different from the minimum number of wavelengths required for all-to-all traffic. We assume \( W_n \geq W \). For \( W_n > W \), the bands calculated can be proportionally expanded depending on the ratio \( W_n/W \) as explained here. Define \( x = W_n\text{mod}W \) and \( y = \lfloor W_n/W \rfloor \). Given the matrix \( \Gamma_R \) for all-to-all traffic, \( x \) rows of \( \Gamma_R \) are duplicated \( y \) times, and \( W - x \) rows \( y - 1 \) times. \( \pi_4 \)’s have the priority to be among the selected \( x \) rows. We call the resulting matrix as \( \Gamma_{R'} \).

We also compare two cases of nonuniform banding. In the first case, a waveband can only be used to do add/drop or bypass as designated by \( \Gamma_R \). We call this case fixed nonuniform wavebands. The second case, in which every band can be configured to do add/drop or bypass on demand (but the band itself is fixed, i.e., at any given time the whole band can be either A/D or B) is just called nonuniform wavebands. We note that in the first case, a lightpath for a source–destination pair can only be established at the wavelength(s) that is designated by \( \Gamma_R \) for that particular pair.

We adopt a first-fit wavelength assignment strategy among the available wavelengths for uniform wavebands and fixed nonuniform banding. For nonuniform wavebands, we modify the first-fit strategy as follows. Let \( w_{k,i,j} \) be the \( k \)-th wavelength for the pair of nodes \( i \) and \( j \) given that there are \( \Omega_{k,i,j} \) such wavelengths designated in \( \Gamma_{R'} \). Whenever there is a lightpath request for the node pair \( \{i,j\} \), we first try to establish the lightpath on the \( w_{k,i,j} \) starting from \( k = 1 \) to \( k = \Omega_{k,i,j} \). If no such wavelength is available, a first-fit strategy is applied on the remaining wavelengths.

We compare the performance of nonuniform banding with uniform banding in a 19-node bidirectional ring network in this section. Results for unidirectional rings are not shown since they are similar. We ran simulations using \( 10^6 \) call arrivals. The load per route and per wavelength is denoted by \( \rho \).

In Fig. 11, we plot blocking probability versus load for a ring network with \( N = 19 \) for \( W_n = 45 \). The number of nonuniform wavebands calculated for \( N = 19 \) (see Table I) is 177. We plot the curves for 171 and 190 number of uniform wavebands, corresponding to 9 and 10 wavebands per node with sizes of 10 and 9, respectively. We see that nonuniform wavebands perform significantly better than uniform wavebands (for both 171 and 190 wavebands) for higher loads. For lower loads (e.g., \( \rho = 0.001 \)), nonuniform wavebands perform the same as uniform with 190 bands. Larger \( W_n \) allows nonuniform wavebands to perform better. We note that except for lower loads, fixed nonuniform wavebands perform the same as nonuniform wavebands. At lower loads, nonuniform wavebands perform significantly better.

In Fig. 12(a), we plot blocking probability versus number of uniform wavebands per node for \( W_n = 90 \). The straight lines correspond to the blocking values obtained with nonuniform wavebands with 9.3 bands per node on average (\( b_{\text{max}} = 13, B = 177 \)) with different load values. We see that for \( \rho = 0.003 \),
In Fig. 12(b), we plot blocking probability versus number of wavelengths \( W_a \) for two different load values. For \( \rho = 0.001 \), for lower \( W_a \), uniform wavebands perform better than nonuniform wavebands. This is due to the fact that with lower \( W_a \) that are closer to \( W \), the probability of a duplicate lightpath (multiple lightpaths on the same route) being blocked is very high since with the nonuniform switch configurations there is a limited set of available wavelengths for any route. However, we see that with higher \( W_a \), nonuniform banding starts to perform better for \( W_a > 80 \). For \( \rho = 0.003 \), nonuniform banding starts to perform better than uniform for \( W > 45 \). In general, the performance of nonuniform banding gets better with higher \( W_a \), whereas the performance of uniform banding gets worse.

### IX. Conclusion and Future Work

Wavebanding saves switching port costs in optical cross-connects by grouping together wavelengths and switching them as a band. Optimal ways of waveband switching for ring networks are presented in this paper. We introduced a novel framework to minimize the number of nonuniform wavebands to support a deterministic traffic in ring networks and presented algorithms that attempted to optimize the number of bands. We then obtained optimal solutions for bidirectional and unidirectional rings under all-to-all traffic. We also evaluated the heuristics’ performance under a random traffic model. The results showed a significant reduction in the number of switches over the entire network. We also obtained the performance with dynamic stochastic traffic and showed that nonuniform wavebanding is advantageous over uniform wavebanding in most cases in terms of blocking probability. Future work includes extension of this framework to mesh network architectures and designing dynamic RWA algorithms considering nonuniform waveband designs. Another possible extension is solving the Band Minimization Problem without the minimum wavelength requirement.

### References


