A New Algorithm for Reducing the Number of Add-Drop Multiplexers in WDM Optical Ring Networks

GUANGZHI LI and RAHUL SIMHA
Department of Computer Science
College of William and Mary
Williamsburg, VA 23185

ABSTRACT

This paper considers a network design problem introduced in[11,12] in which resource allocation is performed within a Wavelength Division Multiplexing (WDM) optical ring network to reduce overall equipment cost. The problem can be summarized as follows. The Add-Drop Multiplexers (ADM's) that are used to terminate lightpaths in an optical (SONET/SDH) ring are expensive and therefore a naive implementation that places ADM's for each wavelength at every node would waste expensive resources. However, by carefully assigning connections to wavelengths and routing the resulting lightpaths, one may avoid terminating all wavelengths at every node, and thereby reduce the number of ADM's needed.

While Gerstel et al[11] consider wavelength assignment with respect to this problem, they do not consider the additional complication of routing. In this paper, we consider joint routing and wavelength assignment, and show that the number of ADM's required is significantly less than the number resulting from applying the heuristics in their paper.[11] Our algorithm analyzes connection requirements and, using a decomposition based on Euler paths, performs routing and wavelength assignment.

Keywords: Optical networks, Wavelength division multiplexing, Routing and wavelength assignment, ADM's

1. INTRODUCTION

In spite of high capacities and low error rates, WDM networks are not yet competitive with equivalent electronic networks. One reason is the significant cost of the individual optical hardware components. In this paper, we consider a network design problem introduced by Gerstel et al[11,12] in which resource allocation is performed within a Wavelength Division Multiplexing (WDM) optical ring network to reduce overall equipment cost. More specifically, wavelengths are carefully assigned to minimize the number of Add-Drop Multiplexers (ADM's) that are used to terminate lightpaths in an optical SONET/SDH rings. The problem is important both because ADM's are expensive and SONET/SDH has received much attention recently.

We now describe the problem briefly (a detailed description is given in Section 3). We consider a unidirectional optical ring with a set of connections specified by endpoints for each connection. (We will only examine the assignment of "working" paths and assume that one of several techniques used for "protection" paths is available.) These connections are assumed to be sufficiently long-lived to make off-line wavelength assignment practical.[11] The problem then is to assign a route and a wavelength (along the selected route) to each connection. Since a node may not be a terminating point for some connections, not all wavelengths need to be received at a node. For a node to receive a wavelength, an ADM is used and since ADM's are expensive, it is desirable[11] to intelligently assign wavelengths so that as few wavelengths as possible need to be dropped at every node, thereby minimizing the overall number of ADM's required.

The following ADM property is central to reducing the number of ADM's: two lightpaths that use the same wavelength and terminate at a common node can share the ADM for that wavelength. Then, by increasing this sharing of ADM's, the total number of ADM's can be reduced. Gerstel et al[11] assume routes are fixed and present two heuristics for wavelength assignment. We consider the routing component of the problem as well and show that further ADM reduction results from combining wavelength assignment and routing. Simulation results indicate improvements of about 25% using our algorithm.

Guangzhi Li: E-mail: gli@cs.wm.edu
Rahul Simha: E-mail: simha@cs.wm.edu

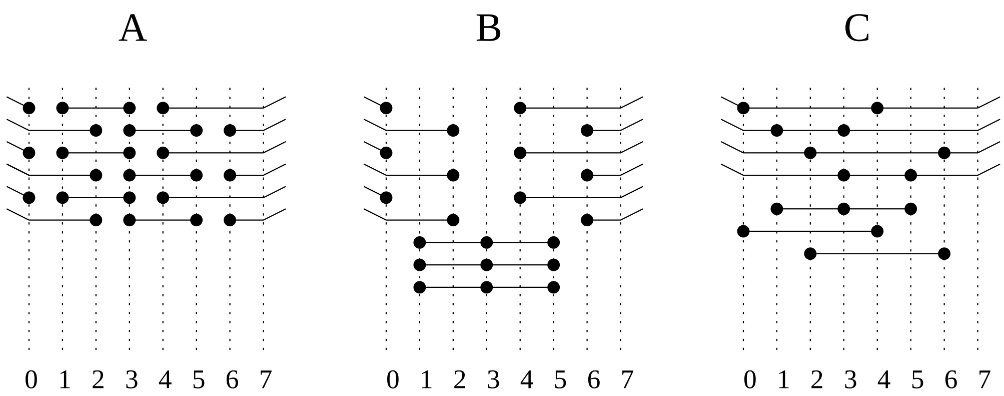

Figure 1. Example to show that routing can reduce the number of ADM's dramatically

The general routing and wavelength assignment problem (without regard to ADM's) in WDM networks has been studied extensively in the literature.[1,3,6,7,14,22,24,26] The references cited include an up-to-date review,[13] deterministic bounds on the number of wavelengths for the wavelength assignment problem in trees [2,9,15,16,18] and rings,[22,10] probabilistic analyses[4,5] and scheduling.[8,21] These research efforts are either directly or indirectly focused on minimizing the number of wavelengths required or on reducing the scheduling makespan for a given connection set. In this regard, Gerstel et al[11] argue that it is more desirable to focus on cost-effectiveness rather than on the number of wavelengths (or makespan) for the following reasons: (1) Since fiber capacity is very high, multiple connections can easily be "groomed" onto a single wavelength and thus, the number of wavelengths will likely not be a bottleneck resource; and (2) in the near future, the hardware for terminating lightpaths (ADM's) will be a dominant cost in the optical layer. They also argue that, in the near-term, non-ring topologies are not likely to be commercialized.

Our work on the ADM minimization problem is different from[11] in that, as mentioned earlier, we consider both routing and wavelength assignment to reduce the cost. We present a heuristic algorithm that finds a near optimal solution and which combines the advantages of the two heuristics presented in Gerstel et al.[11]

To illustrate the potential benefit in considering routing, we will consider an example given by Gerstel et al[11] that shows how intelligent wavelength assignment can reduce the number of ADM's. This example consists of an 8-node (nodes are numbered from 0 to 7) WDM ring network and a set of full-duplex lightpaths $I = \{(1,3),(4,0),(3,5),(6,2),(1,3),(4,0),\ (3,5),(6,2),(1,3),(4,0),(3,5),(6,2)\}$. In Figure 1, the curved lines at the edges of each sketch signify that the lightpath wraps around the ring; the dashed lines represent the nodes and the dots are the ADM's. In the example, assignment A produces the minimum number of wavelengths by using 6 wavelengths and 24 ADM's. On the other hand, assignment B has fewer ADM's by using more (nine) wavelengths and 21 ADM's.[11] However, this result is based on fixed clockwise routing. If we rearrange routes, we can reduce the number of ADM's dramatically: assignment C only uses 7 wavelengths and 15 ADM's. This example illustrates the cost reduction potential of including routing in the problem formulation. At this point, one may wonder whether routing and assignment may be performed separately, using, say, shortest-path routing for the routing component. However, we show that our algorithm performs significantly better than when shortest-path routing is used with other assignment algorithms.

The rest of the paper is organized as follows. In Section 2, we briefly describe the system structure and our model. In Section 3, we formulate the problem as an integer programming problem with a non-linear objective function and linear restrictions. A lower bound for the number of ADM's and some properties are presented in Section 4. Section 5 describes our heuristic algorithm and presents simulation results. Conclusions are given in Section 6.

2. SYSTEM DESCRIPTION

We consider an undirectional N-node WDM ring network. The N nodes are numbered from 0 to $N-1$ along the clockwise direction. We assume that (1) all connections are full-duplex, (2) all links are bidirectional and (3) the two halves of a duplex connection are routed over the same path using the same wavelength. Thus, we represent the ring

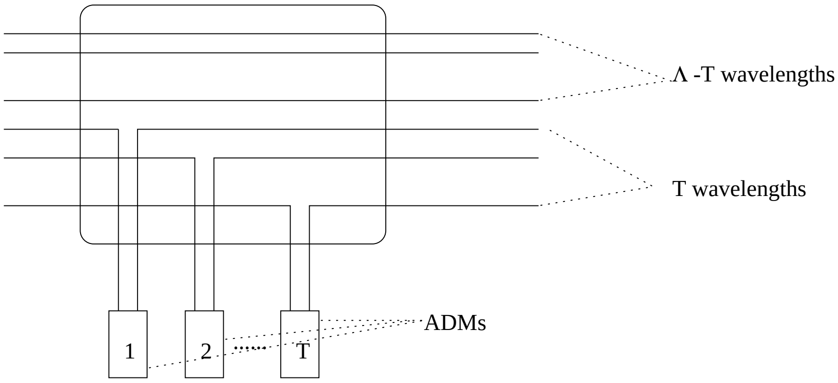

Figure 2. A basic node structure in a undirected WDM ring.

network by a simple undirected circuit. We assume the availability of Λ wavelengths and treat these Λ-wavelength WDM rings as Λ independent SONET rings. (For SONET self-healing, we assume that multiple fibers are used for protected lightpaths). Figure 2 shows a possible node structure in an undirected WDM ring having Λ wavelengths and T ADM's.

Next, we are given a (multi) set of full-duplex connection requirements $I = \{(s_0, t_0), (s_1, t_1), ...\}$ in which several connections may have the same source-destination pair. Two connections (lightpaths) are *adjacent* if they share a common endpoint. If two adjacent lightpaths use the same wavelength, then they are using the same SONET ring and one ADM at the common node can be shared by them. The problem then is to assign a route and a wavelength to each connection, such that we can set up as many connections as possible with as few ADM's as possible. A precise statement of the problem follows next.

3. PROBLEM FORMULATION

We now present a formal statement of the problem by casting it into an integer programming framework. This framework is useful because an exact solution can be obtained for small size problems using standard packages. While these same packages also provide general heuristics for the integer programming problem, however, we believe that our heuristic will work best for medium to large-scale size problems because it carefully exploits the problem structure.

Consider the set of source-destination pairs and let $m_i, i = 0, ..., K-1$ denote the number of connections whose endpoints coincide with pair i (we assume that there are K different connections), and let $m = (m_0, ..., m_{K-1})$, the vector of such counts. For each pair i we have and only have two possible routes, clockwise and counter-clockwise, which are denoted as l_i and r_i respectively. Thus, there are $P \stackrel{\triangle}{=} 2K$ possible paths on which connections can be routed.

Let $A = (a_{ij})$ be the $P \times K$ path-pair incidence matrix,

$$a_{ij} = \begin{cases} 1 & \text{if path } i \text{ connects pair } j, \\ 0 & \text{otherwise} \end{cases}$$

Let $B = (b_{ij})$ be the $P \times N$ path-edge incidence matrix,

$$b_{ij} = \begin{cases} 1 & \text{if path } i \text{ pass through link } j, \\ 0 & \text{otherwise} \end{cases}$$

Let $C = (c_{ij})$ be the $P \times P$ path-path incidence matrix, where c_{ij} is the number of common termination nodes with $c_{ii} = 0$. The other entries are taken from the set $\{0, 1, 2\}$.

A routing and wavelength assignment can be represented by a $P \times \Lambda$ path-wavelength assignment matrix, denoted by $D = (d_{ij})$ where

$$d_{ij} = \begin{cases} 1 & \text{if path } i \text{ is assigned wavelength } j, \\ 0 & \text{otherwise} \end{cases}$$

Next, we describe a general cost function that considers both the cost of an ADM as well as the revenue from assigning (as opposed to rejecting) a connection. The latter cost arises from the fixed number of wavelengths, which means some connections may not be assigned. If each connection is valued as x, then the total value from the assigned connections is $x \cdot \mathbf{1}_{1\times P} \cdot D \cdot \mathbf{1}_{\Lambda\times 1}$. However, each connection needs two ADM's. The total number of shared ADM's is $\sum_{j=0}^{\Lambda-1}\sum_{i=0}^{P-1} d_{ij} \cdot C_{i\bullet} \cdot D_{\bullet j}$, where $C_{i\bullet}$ is the i-th row of C, $D_{\bullet j}$ is the j-th column of D. If the cost of an ADM is y, we can compute the net value as $(x-2y)\cdot \mathbf{1}_{1\times P} \cdot D \cdot \mathbf{1}_{\Lambda\times 1} + y \cdot \sum_{j=0}^{\Lambda-1}\sum_{i=0}^{P-1} d_{ij} \cdot C_{i\bullet} \cdot D_{\bullet j}$, which is a non-linear integer function of parameters $d_{ij}, i = 0, ..., P-1, j = 0, ..., \Lambda - 1$. The feasible domain is restricted by the connection requirements $\mathbf{1}_{1\times\Lambda} \cdot D^\tau \cdot A \leq m$ and wavelength constraint $D^\tau \cdot B \leq \mathbf{1}_{\Lambda\times N}$ (each wavelength on each link can only support one lightpath). Here τ is the matrix transpose operator.

Thus, the problem can be formulated as:

$$\max\ (x-2y)\cdot \mathbf{1}_{1\times P} \cdot D \cdot \mathbf{1}_{\Lambda\times 1} + y \cdot \sum_{j=0}^{\Lambda-1}\sum_{i=0}^{P-1} d_{ij} \cdot C_{i\bullet} \cdot D_{\bullet j}$$

subject to

$$\begin{aligned} \mathbf{1}_{1\times\Lambda} \cdot D^\tau \cdot A &\leq m \\ D^\tau \cdot B &\leq \mathbf{1}_{\Lambda\times N} \\ d_{ij} &= 0, 1, \quad d_{ij} \in D. \end{aligned}$$

where $\mathbf{1}$ is matrix (or vector) in which each entry is unity.

By solving this integer non-linear programming with variables d_{ij} using standard algorithms, we can get the exact optimal solution for small size problems. However, when N, Λ and P are large, even generic heuristics or approximation algorithms found in commercial packages will be computationally expensive. Also, a tailored heuristic that exploits problem structure is likely to be more effective – the approach taken in this paper. Before presenting such a heuristic, we first derive a lower bound.

4. A LOWER BOUND

We follow the line of reasoning in[11] to derive a lower bound. As discussed earlier, two lightpaths can share an ADM if and only if they are adjacent and use the same wavelength. For each node k, let β_k be the number of lightpaths with node k as one of their terminal nodes. Then node k needs at least $\lceil \beta_k/2 \rceil$ ADM's. This is because at most two lightpaths can share one ADM. Hence, if α is the number of ADM's required, then

$$\alpha \geq \alpha_{lb} = \sum_{k=1}^{N} \lceil \beta_k/2 \rceil.$$

This lower bound can be written as:

$$\alpha_{lb} = |I| + 1/2 \sum_{k=1}^{N} (\beta_k \bmod 2).$$

This lower bound is less than or equal to the lower bound in[11] as we would expect since routing can further reduce the number of ADM's.

Analogous to a result by Gerstel et al,[11] this lower bound is not tight – there exist cases such that the lower bound cannot be reached. One such case is the very same example used by Gerstel et al[11] to illustrate this point. Consider a ring network with $N = 8$ nodes numbered from 0 to 7 in the clockwise direction and let $I = \{(0,5), (5,2), (2,7), (7,4), (4,1), (1,6), (6,3), (3,0)\}$. Note that each lightpath traverses either 3 links or 5 links, so each wavelength can support at most 2 lightpaths. However, any two paths in one wavelengths need at least 3 ADM's. So the total number of ADM's used is at least $8/2 * 3 = 12$. Note that each node is an endpoint of exactly 2 lightpaths, which means $\beta_k = 2, k = 0, ..., 7$. According to the lower bound, $\alpha_{lb} = |I| = 8$. This example shows that the lower bound cannot always be achieved.

5. A HEURISTIC ALGORITHM

5.1. Theoretical background

Our heuristic is based on an Euler-Trail decomposition of the connection set. The intuition is that the trails suggest adjacent connections that can share an ADM. A few theoretical results about Eulerian trails will be useful before presenting our heuristic. We start with some definitions. Given the number of nodes N and a set of connections $I = \{(s_0, t_0), (s_1, t_1), ..., (s_{k-1}, t_{k-1})\}$, we construct an undirected multi-graph $G = (V, I)$, where V represents the N nodes and each edge in G represents a connection in I. In this graph some of the vertices have even degree, and some of them will have odd degree. However, we have following observation:

LEMMA 5.1. *There are an even number of vertices that have odd degree.*

Proof. Each edge is counted twice in the degree count and thus the total degree count is even. Hence, the number of vertices that have odd degrees must be even. ∎

We define a *trail* in G from vertex u to vertex v as an alternating sequence of vertices and edges, for example, $v_1, e_1, v_2, e_2, ..., e_{k-1}, v_k$, where $v_1 = u, v_k = v$, all the edges in the sequence are distinct, and successive vertices v_i and v_{i+1} are endpoints of the intermediate edge e_i. If $u \neq v$, the trail is *open*, otherwise it is *closed*. We call the trail a *nondecomposable trail* if the degree of each trail vertex is less than or equal to 2. The terms *nondecomposable open trail* and *nondecomposable closed trail* follow naturally.

LEMMA 5.2. *(Euler Trail Decomposition[17]) Let $G(V, E)$ be a connected graph with $2k$ vertices of odd degree. Then, the edges of G can be partitioned into k edge-disjoint open trails $E_1, ..., E_k$.*

For each open trail, we have following lemma,

LEMMA 5.3. *Any open trail can be partitioned into several nondecomposable closed trails and one nondecomposable open trail.*

Combining these results, we have following theorem,

THEOREM 5.4. *The multi-graph $G = (V, I)$ can be partitioned into several nondecomposable closed trails and several nondecomposable open trails.*

Now, Gerstel et al[11] have suggested a lightpath splitting approach to assignment: a lightpath is split so that a packet that would have taken the original lightpath is now transmitted over two lightpaths. Because two new lightpath endpoints are created, two additional ADM's may be needed. On the other hand, when a lightpath is split, the ADM's of the two parts may now be shared with other existing lightpaths, thereby increasing assignment flexibility. The Cut-First heuristic in the paper by Gerstel et al[11] is based on this idea. However, lightpath splitting does not necessarily reduce the number of ADM's, and at times can even create more ADM's.[11] This is the reason why sometimes Assign-First, another algorithm proposed in[11] often performs better than Cut-First. (Both algorithms are described later in this paper.) For fixed routing on nondecomposable trails, however, Cut-First performs no worse than Assign-First.

LEMMA 5.5. *For nondecomposable trails using fixed-routing, Cut-First performs no worse than Assign-First.*

Proof. Let L denote the lightpath load. According to,[11] the Cut-First algorithm achieves $|I| + L - 1$ ADM's for closed trails and $|I| + L$ ADM's for open trails. However, without lightpath splitting and with $L > 1$, we need at least $|I| + 2L - 1$ ADM's for closed trails and at least $|I| + 2L - 2$ ADM's for open trails. Thus, Cut-First can perform better than Assign-First. ∎

Any nondecomposable trail can be routed along either the clockwise or counter-clockwise direction. We route the nondecomposable trails along the direction with lower load. With this background, we are now in a position to present our so-named ReRouting heuristic.

5.2. ReRouting: a new assignment heuristic

ReRouting algorithm:

1. Construct the path-edge graph $G = (R, I)$.
2. Partition G into several nondecomposable trails.
3. For each nondecomposable trail, determine the routing direction with smaller load.
4. For each nondecomposable trail with direction determined, apply the Cut-First heuristic (see below).
5. All lightpaths are now assigned. The last step is to check for possible merging of lightpaths, since two adjacent lightpaths of the same wavelength can be merged.

For reader convenience, we list both heuristics presented by Gerstel et al,[11] Cut-First and Assign-First. (A formula used in their paper is slightly modified here).

Cut-First heuristic:

1. Choose a node, i, such that $i = \min_j\{2x_j + \min\{\sigma_j, \tau_j\}, L - \max\{\sigma_j, \tau_j\}\}$. where σ_i, τ_i, x_i are the total number of lightpath origins, terminations, and crossover at node i respectively.
2. Split each lightpath crossing node i into two parts, one part on each side of node i, and treat each part as an independent lightpath.
3. Along the ring (clockwise or counter-clockwise), node by node, assign wavelengths to lightpaths by the rule: wavelength w is preferred if there is a lightpath that ends at this node which is assigned wavelength w. For all lightpaths that originate at this node, assign the lowest numbered preferred wavelength, if available, otherwise assign it to the lowest numbered wavelength available.
4. Finally, check for possible merges of the lightpaths.

Assign-First heuristic:

1. Choose a node, i, such that $i = \min_j\{2x_j + \min\{\sigma_j, \tau_j\}\}$ where σ_i, τ_i, x_i are the same as in Cut-First.
2. Assign all lightpaths that originate at or crossover node i with different wavelengths.
3. Along the ring clockwise or counter-clockwise, node by node, assign wavelengths to lightpaths, as in the Cut-First heuristic.
4. Finally, check for possible merges of the lightpaths.

Note that in Assign-First, we have slightly modified the formula in selecting the initial node, for the following reason. Gerstel et al[11] estimate an upper bound of Assign-First as

$$\alpha \leq \alpha_{lb} + \min_i\{x_i + \min\{\sigma_i, \tau_i\}\}$$

and based on this upper bound, select the initial node. Now, consider the following example: a ring network with $N = 5$ nodes from 1 to 5 in the clockwise direction, and $I = \{(1,3), (3,5), (5,2), (2,4), (4,1)\}$. Note that $\alpha_{lb} = 5, x_i = \sigma_i = \tau_i = 1, i = 1, ..., 5$. However, because this instance forms a nondecomposable closed trail with $L = 2$, at least $|I| + 2L - 1 = 8$ ADM's are required, which is more than the upper bound. Actually, each crossover lightpath may cause 2 additional ADM's, because its origin may not match an available termination and its termination may not match an available origin because of wavelength overlap. Therefore,

$$\alpha \leq \alpha_{lb} + \min_i\{2x_i + \min\{\sigma_i, \tau_i\}\}.$$

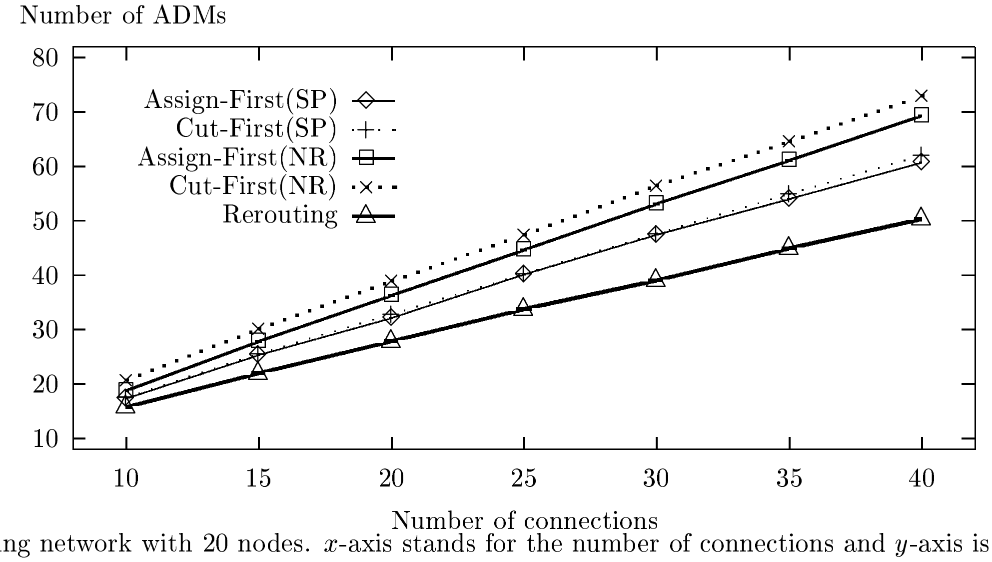

Figure 3. A ring network with 20 nodes. x-axis stands for the number of connections and y-axis is the number of required ADM's according to different heuristics.

5.3. Heuristic comparison

To evaluate the performance of our ReRouting algorithm, we implemented the three heuristics: Cut-First, Assign-First, and ReRouting, where Cut-First and Assign-First are applied on both random routing and shortest path routing. We need a tie-breaking criterion when two or more wavelengths can be assigned to a lightpath in step 3 of each algorithm. In our implementation of these algorithms, without loss of generality, we always assign the shortest lightpath first, and assign it to the lowest numbered eligible wavelength.

At first, we fix the number of nodes in the ring network as 20 and let the number of connections increase from 10 to 40 with in steps of 5. For each sample point and each heuristic algorithm, we took an average of 20 simulation runs. Figures 3 and 4 show the number of ADM's and wavelengths required by the different heuristics respectively. In this case, both the number of ADM's and wavelengths increase with the number of connections.

Observe that Assign-First uses fewer ADM's but more wavelengths in comparison with Cut-First. This is reasonable because Cut-First increases the number of connections and uses the lower bound of wavelengths by splitting some connections into two. However, if we use shortest path routing, these differences will decrease. And shortest-path routing policy always outperforms random routing policy for both Assign-First and Cut-First. Among these five heuristics, ReRouting saves almost 30% (in ADM's) compared with random routing; it also saves almost 15% ADM's compared with shortest-path routing policies, thereby showing that prior shortest path routing alone is not as effective as our ReRouting algorithm. However, Figure 4 shows that although the number of wavelengths required by our ReRouting algorithm is much less than the one required by random routing based on Assign-First and Cut-First, it is slightly more than that required by shortest-path routing.

We also studied the effect of increasing network size by fixing the number of connections as 20 and letting the number of nodes in our ring networks increase from 10 to 40 in steps of 5. Again, each sample point is simulated with 20 runs. The results are presented in Figure 5 and Figure 6. Naturally, the number of ADM's increases with network size, although slower than in Figure 3. This is because increasing the network size decreases the chance of two connections sharing one terminal node and the total number of ADM's is bounded by two times of the number of connections except of Cut-First. Again, ReRouting saves almost 30% of ADM's compared with Cut-First (with Random-Routing policy). In other cases, there is almost a 15% difference. From Figure 6, we find that ReRouting does not have much of an advantage in the number of wavelengths if the network has many nodes and few connections. However, if it has few nodes and more connections, the number of wavelengths required by ReRouting is just slightly higher than the shortest-path routing policy.

We define the "network load" as the ratio of the number of connections to the number of nodes in the ring. Then we fix the network load and study the effect of increasing network size by letting the number of nodes in our ring increase from 10 to 40 in steps of 5. The network load is taken to be 2. As before, each sample point is simulated

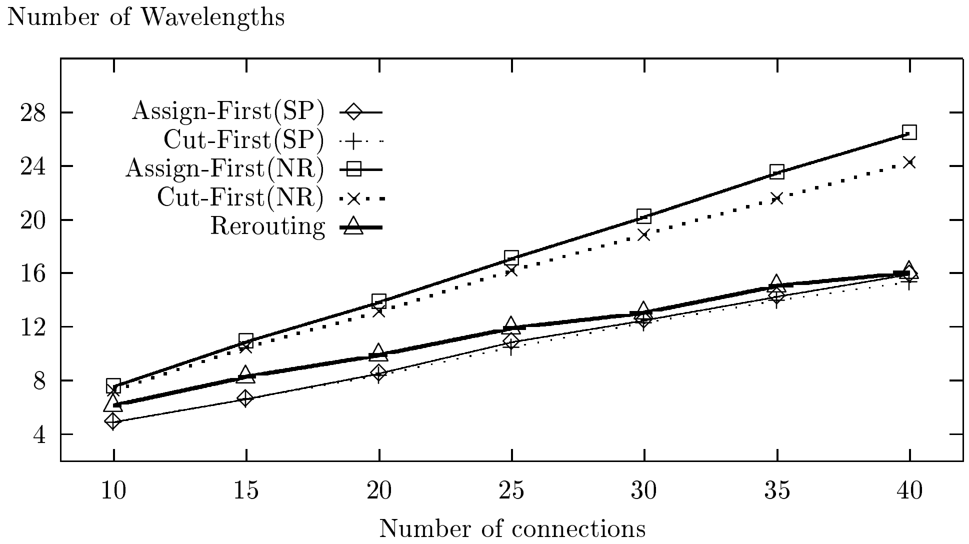

Figure 4. A ring network with 20 nodes. x-axis stands for the number of connections and y-axis is the number of required wavelengths according to different heuristics.

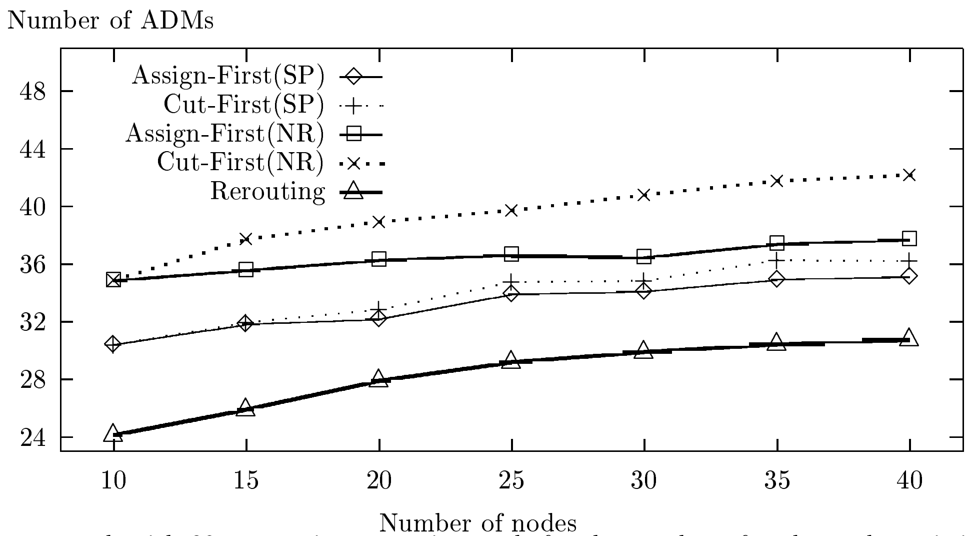

Figure 5. A ring network with 20 connections. x-axis stands for the number of nodes and y-axis is the number of required ADM's according to different heuristics.

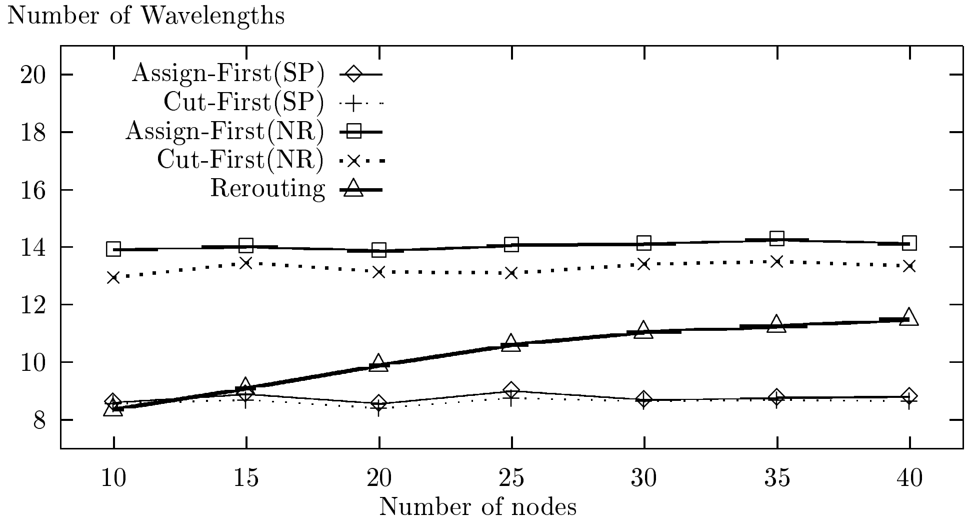

Figure 6. A ring network with 20 connections. x-axis stands for the number of nodes and y-axis is the number of required wavelengths according to different heuristics.

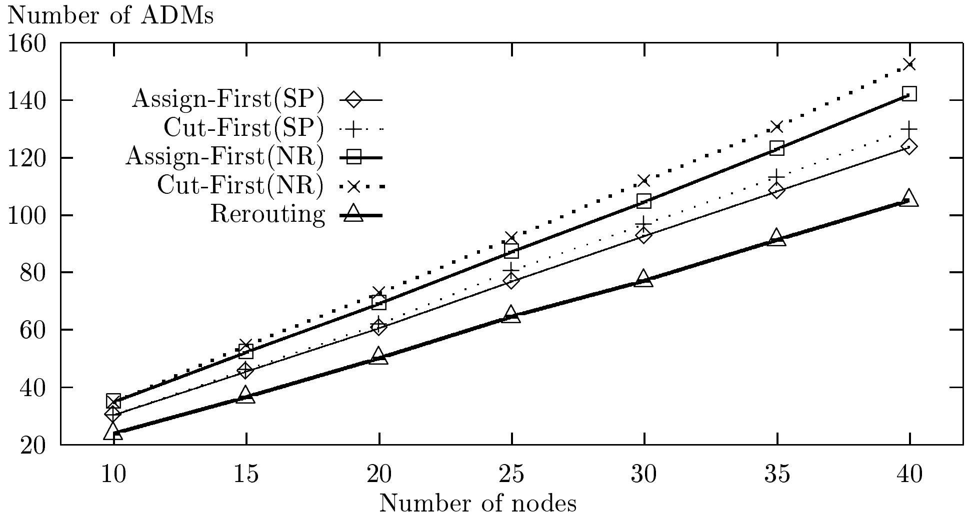

Figure 7. A ring network with load 2. x-axis stands for the number of nodes and y-axis is the number of required ADM's according to different heuristics.

with 20 runs. Figures 7 and 8 show the results. From Figure 7, we observe that the number of ADM's increases with the number of nodes in the network and that, among the algorithms, ReRouting grows the slowest. At the same time, the number of wavelengths required by our new algorithm is just slightly more than the algorithm that uses the fewest wavelengths.

6. CONCLUSION

We have studied a previously articulated[11] cost-effectiveness issue in optical WDM ring networks: the problem of reducing the number of Add-Drop Multiplexers (ADM's) by careful wavelength assignment. Whereas the previous approach did not consider routing, we have shown that significant reduction can be obtained by combining routing and wavelength assignment. Our heuristic is based on an Eulerian Trail decomposition applied to a transformation of the problem instance. Simulation results show that our algorithm can significantly reduce the number of ADM's.

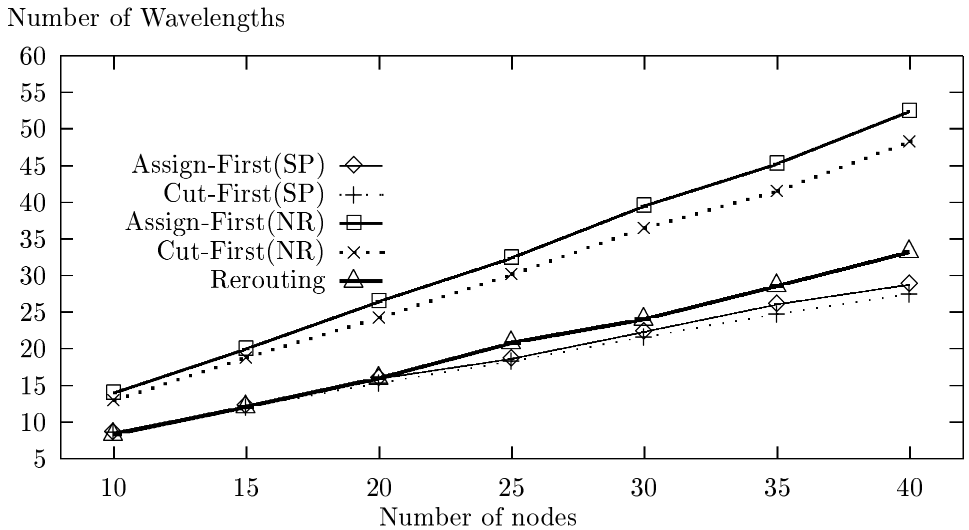

Figure 8. A ring network with load 2. x-axis stands for the number of nodes and y-axis is the number of required wavelengths according to different heuristics.

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