8. Proof: There are four cases to consider.
   (1) \( n = 10m + 1 \). Here \( n^4 \) satisfies the condition sought.
   (2) \( n = 10m + 3 \). Here \( n^2 = 100m^2 + 60m + 9 \) and \( n^4 = 10000m^4 + 12000m^3 + 5400m^2 + 1080m + 81 = 10(1000m^4 + 1200m^3 + 540m^2 + 108m + 8) + 1 \), so the units digit of \( n^4 \) is 1.
   (3) \( n = 10m + 7 \). As in case (2) we need \( n^4 \). For \( n^2 = 100m^2 + 140m + 49 \), and
   \( n^4 = 10000m^4 + 28000m^3 + 29400m^2 + 13720m + 2401 = 10(100m^4 + 2800m^3 + 2940m^2 + 1372m + 240) + 1 \), where the units digit is 1.
   (4) \( n = 10m + 9 \). Fortunately we only need \( n^2 \) here, since \( n^2 = 100m^2 + 180m + 81 = 10(10m^2 + 18m + 8) + 1 \).
   [Note: For any \( n \in \mathbb{Z}^+ \), where \( n \) is odd and \( n \) not divisible by 5, we always find the units
digit in \( n^4 \) to be 1.]

20. Proof: For \( 1 \leq i \leq 5 \), it follows from the division algorithm that \( a_i = 5q_i + r_i \), where
   \( 0 \leq r_i \leq 4 \). So now we shall consider the remainders: \( r_1, r_2, r_3, r_4, r_5 \). For if a selection
   of the remainders adds to a multiple of 5, then the sum of the corresponding elements of
   \( A \) will also sum to a multiple of 5. (Note that for the remainders we need not have five
distinct integers.)
   1) If \( r_i = 0 \) for some \( 1 \leq i \leq 5 \), then \( 5|a_i \) and we are finished. Therefore we shall assume
   from this point on that \( r_i \neq 0 \) for all \( 1 \leq i \leq 5 \).
   2) If \( 1 \leq r_1 = r_2 = r_3 = r_4 = r_5 \leq 4 \), then \( a_1 + a_2 + \ldots + a_5 = 5(q_1 + q_2 + \ldots + q_5) + 5r_1 \),
   and the result follows. Consequently we now narrow our attention to the cases where at
   least two different nonzero remainders occur.
   
   Case 1: (There are at least three 4's). Here the possibilities to consider are (i) \( 4 + 1 \); (ii) \( 4 + 4 + 2 \); and (iii) \( 4 + 4 + 4 + 3 \) — these all lead to the result we are seeking.

   Case 2: (We have one or two 4's). If there is at least one 1, or at least one 2 and one 3,
   then we are done. Otherwise we get one of the following possibilities: (i) \( 4 + 2 + 2 + 2 \)
   or (ii) \( 4 + 3 + 3 \).

   Case 3: (Now there are no 4's and at least one 3.) Then we either have (i) \( 3 + 2 \); (ii) \( 3 + 1 + 1 \); or (iii) \( 3 + 3 + 3 + 1 \).

   Case 4: (We now have only 1's and 2's as summands). The final possibilities are (i) \( 2 + 1 + 1 + 1 \); and (ii) \( 2 + 2 + 1 \).

36. Let \( S = \{1, 2, 3, \ldots, 100\} \) be the sample space for this experiment and let \( A, B, C \) denote
   the following events:
   A: Leslie's selection is divisible by 2: \( \{2, 4, 6, \ldots, 98, 100\} \)
   B: Leslie's selection is divisible by 3: \( \{3, 6, 9, \ldots, 96, 99\} \)

   C: Leslie's selection is divisible by 5: \( \{5, 10, 15, \ldots, 95, 100\} \)
   (a) \( Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B) = \frac{50}{100} + \frac{33}{100} - \frac{6}{100} = \frac{77}{100} = 0.77 \). [Note:
   Here \( A \cap B = \{6, 12, 18, \ldots, 96\} \), the set of integers between 1 and 100 (inclusive) that are
   divisible by 6 — that is, divisible by both 2 and 3.]
   (b) \( Pr(A \cup B \cup C) = Pr(A) + Pr(B) + Pr(C) - Pr(A \cap B) - Pr(A \cap C) - Pr(B \cap C) +
   Pr(A \cap B \cap C) = \frac{50}{100} + \frac{33}{100} + \frac{20}{100} - \frac{16}{100} - \frac{10}{100} - \frac{6}{100} + \frac{3}{100} = \frac{74}{100} = 0.74 \).