1. The **symmetric difference** of two sets $S_1$ and $S_2$ is defined as

$$S_1 \oplus S_2 = \{x \mid s \in S_1 \text{ or } x \in S_2, \text{ but } x \text{ is not in both } S_1 \text{ and } S_2\}.$$ 

Show that the family of regular languages is closed under symmetric difference.

2. Show that each of the following languages is not regular.

   (a) $L = \{w \in \Sigma^* \mid n_a(w) < n_b(w)\}$.  
      Note that $n_a(w)$ (or $n_b(w)$) denotes the number of $a$’s (or $b$’s) in $w$.

   (b) $L = \{a^n b^l c^k \mid k \neq n + l\}$

   (c) $L = \{a^n b^l c^k \mid k \geq n + l\}$

   (c) $L = \{a^n b^l \mid n \leq l\}$

   (d) $L = \{a^n \mid n \text{ is the product of two prime numbers.}\}$

   (e) $L = \{a^n b^k \mid n > k\} \cup \{a^n b^k \mid n \neq k - 1\}$

3. Prove or disprove the following.

   (a) If $L_1$ and $L_2$ are non-regular languages, then $L = L_1 \cup L_2$ is also non-regular.

   (b) If $L_1$ and $L_2$ are regular languages, then $L = \{w \mid w \in L_1, w^R \in L_2\}$ is a regular language.

   (c) $L = \{uww^Rv \mid u, v, w \in \{a, b\}^+\}$

   (b) $L = \{uww^Rv \mid u, v, w \in \{a, b\}^+, |u| \geq |v|\}$