1. (4 pt each) In each of the following languages, answer true or false. (No partial credit will be given.)

(a) \(HA_{DFA} = \{ \langle B, w \rangle \mid B \text{ is a DFA that halts on input string } w \text{ and reports accept or reject } \} \) is decidable.

(b) \(HA_{NFA} = \{ \langle B, w \rangle \mid B \text{ is a NFA that halts on input string } w \text{ and reports accept or reject } \} \) is decidable.

(c) \(A_{REX} = \{ \langle R, w \rangle \mid R \text{ is a regular expression that generates input string } w \} \) is decidable.

(d) \(A_{CFG} = \{ \langle G, w \rangle \mid G \text{ is a CFG that generates input string } w \} \) is decidable.

(e) \(A_{TM} = \{ \langle M, w \rangle \mid M \text{ is a TM that accepts input string } w \} \) is decidable.

(f) \(HA_{TM} = \{ \langle M, w \rangle \mid M \text{ is a TM that halts on input string } w \text{ and reports accept or reject } \} \) is decidable.

(g) \(NE_{DFA} = \{ \langle A \rangle \mid A \text{ is a DFA and } L(A) \neq \emptyset \} \) is decidable.

(h) \(NE_{CFG} = \{ \langle G \rangle \mid G \text{ is a CFG and } L(G) \neq \emptyset \} \) is decidable.

(i) \(NE_{TM} = \{ \langle M \rangle \mid M \text{ is a TM and } L(M) \neq \emptyset \} \) is decidable.

(j) \(NEQ_{DFA} = \{ \langle A, B \rangle \mid A \text{ and } B \text{ are DFAs and } L(A) \neq L(B) \} \) is decidable.

(k) \(NEQ_{CFG} = \{ \langle G, H \rangle \mid G \text{ and } H \text{ are CFGs and } L(G) \neq L(H) \} \) is decidable.

(l) \(NEQ_{TM} = \{ \langle M_1, M_2 \rangle \mid M_1 \text{ and } M_2 \text{ are TMs and } L(M_1) \neq L(M_2) \} \) is decidable.

3. (5 pts each)

(a) Show that the set of all Turing machines is countable.

(b) Show that the set of all languages over \(\Sigma^*\) is uncountable.

(c) Show that there exists a language that is not Turing-recognizable.

(d) Give an example of a language that is not Turing-recognizable.

(e) Give an example of a language that is not Turing-decidable, but Turing-recognizable.

(f) Justify your answer in (e).
3. (10 pts each)

(a) Give formal descriptions of a Turing machine that decides the following language $L$.

$L = \{a^ib^jc^k \mid i, j, k \geq 0, \ i = j \text{ and } j > k \}$.

(b) Discuss how a Turing machine $M$ should be designed for the following.

(i) Each cell of the input tape has 0, 1, or a blank symbol $b$.

(ii) $M$ can starts on any cell (not necessarily the first cell) containing a symbol 0, 1, or $b$.

(iii) $M$ halts if and only if the tape has a 0 somewhere on it.

Note that your $M$ should always halt if a 0 is included. Otherwise, it will not halt and keep checking.