Homework 4
Solution

1. Regular languages are closed under union, intersection and difference. And since symmetric difference of S1 and S2 is equivalent to \((S1 \cup S2) - (S1 \cap S2)\), regular language is closed under symmetric difference.

2.
All of the following can be answered using pumping lemma.

(a) Let \(s = a^p b^{p+1}\)
Then \(x = a^\alpha, y = a^\beta, \) and \(z = a^\gamma b^{p+1}\)
Note, \(\alpha + \beta + \gamma = p\).
When \(i = 2\), then \(n_a(s) = \alpha + 2\beta + \gamma = p + \beta \geq p + 1 = nb(s)\), the new string is not in the language.
Therefore, \(L\) is not a regular language.

(b) Let \(s = a^p b^p c^p\)
Then \(x = a^\alpha, y = a^\beta, \) and \(z = a^\gamma b^p c^p\)
Note, \(\alpha + \beta + \gamma = p, n = p\) and \(l = p\).
Let \(x = \epsilon\) and \(|z| = 2p\), then \(\beta = p\) and \(k = \beta\).
When \(i = p\), then \(k = p\beta = 2p = n + l\), which is not in \(L\).
Therefore, \(L\) is not a regular language.

(c) Let \(s = a^{p+1} b^p c^1\).
Then \(x = a^\alpha, y = a^\beta, \) and \(z = a^\gamma b^p c^1\).
Note, \(k = \alpha + \beta + \gamma = p + 1 = n + l\).
When \(i = 0\), then \(k = \alpha + \gamma < p + 1 = n + l\), which is not in \(L\).
Therefore, \(L\) is not a regular language.

(c) Let \(s = a^p b^p\)
Then \(x = a^\alpha, y = a^\beta, \) and \(z = a^\gamma b^p\)
Note, \(n = \alpha + \beta + \gamma = p\).
When \(i = 2\), \(n = \alpha + 2\beta + \gamma = p + \beta \geq p = l\), which is not in \(L\).
Therefore, \(L\) is not a regular language.

(d) Let \(s = an\), where \(n\) is a product of prime number.
Then \(x = a^\alpha, y = a^\beta, \) and \(z = a^\gamma\)
Note, \(\alpha + \beta + \gamma = n > p\), for some value of \(p\).
When \( i = \alpha + \gamma \), \( |xy^iz| = \alpha + \gamma + (\alpha + \gamma)\beta = (\alpha + \gamma)(1 + \beta) \).

Since \( \beta > 0 \), \( xy^iz \) is a product of composite number, which is not a product of prime number. Therefore, L is not a regular language.

(e)
Since \( \{a^n b^k | n > k\} \subset \{a^n b^k | n \neq k - 1\} \), prove that \( L = \{a^n b^k | n \neq k - 1\} \) is not regular.

3.
(a) False, counter example.
Let \( L_1 = \{a^n b^k | n < k\} \), and \( L_2 = \{a^n b^k | n \geq k\} \), and both \( L_1 \) and \( L_2 \) are not regular.
Notice \( L_1 \cup L_2 = a^* b^* \), which is regular.

(b) False, counter example.
Let \( L_1 = a^* b^* \), and \( L_2 = b^* a^* \), which are both regular.
Notice, \( L = \{ w | w \in L_1, w^R \in L_2 \} = \{ a^n b^k | n = k \} \), which is not regular.

(c) L is regular.
\( L = \{a,b\}^+ a a \{a \cup b\}^+ \cup \{a,b\}^+ b b \{a \cup b\}^+ \), which is regular.

(d) L is not regular. Since we have to keep track of the length of \( u \) and \( v \), using pumping lemma, it is trivial to show that it is not regular.