Minimal Delay Traffic Grooming for WDM Star Networks*

Hongsik Choi, Nikhil Garg, and Hyeong-Ah Choi
Department of Computer Science
The George Washington University
Washington, DC 20052
Email: {hongsik, nikhil, choi}@seas.gwu.edu

Abstract

All-optical networks face the challenge of reducing slower opto-electronic conversions by managing assignment of traffic streams to wavelengths in an intelligent manner, while at the same time utilizing bandwidth resources to the maximum. This challenge becomes harder in networks closer to the end users that have insufficient data to saturate single wavelengths as well as traffic streams outnumbering the usable wavelengths, resulting in traffic grooming which requires costly traffic analysis at access nodes. We study the problem of traffic grooming that reduces the need to analyze traffic, for a class of network architecture most used by Metropolitan Area Networks; the star network. The problem being NP-complete, we provide an efficient 2-optimum greedy heuristic for the same, that can be used to intelligently groom traffic at the LANs to reduce latency at the access nodes. Simulation results show that our greedy heuristic achieves a near-optimal solution.

Index Terms: Traffic grooming, WDM star networks, NP-complete, greedy heuristic.

1 Introduction

WDM networks can be viewed as consisting of a Wide Area Network (WAN) that is supported by a number of Metropolitan Area Networks (MAN) that in turn are supported by Local Area Networks (LANs). The local area network nodes are

*This work was supported in part by the DARPA under grant N66001-00-18949 (co-funded by NSA) and by the NSF under grant ANI-9973098.
connected to the end users and currently require less data throughput capacity as individual users do not engage in high-speed connections that fully utilize the fiber bandwidths. WANs are increasingly becoming all-optical and are capable of handling terabits of data. And with the emergence of all-optical LANs using the WDM technology [1], the last-mile-delay in communication is further reduced. However, there are several hurdles yet to be crossed before there can be a truly all-optical network.

An all-optical network is capable of providing large data rates at the cost of omitting the fine granularity of packet switching. This is acceptable for WANs that mostly perform the task of routing traffic meant for edge nodes inside some access network, and hence do not require individual packet analysis at any stage. When we consider an access (MAN) network; the inability of the local traffic to maximally utilize the full potential of the large bandwidth of an optical fiber makes it desirable to groom several traffic streams into a high capacity wavelength channel. The need for traffic grooming [3], [4],[5],[8] is realized even more when a large user population can use only a limited pool of wavelengths. On the other hand, this particular aspect of traffic management makes it difficult to have a fully-optical end-to-end lightpath. The access nodes then become the bottleneck point in the networks. Since they have to perform slow opto-electrical conversions to separate the different traffic streams embedded in a single wavelength, but meant for different destinations across the WDM network and possibly other nodes inside the access network. Reduction of this delay would need an elimination or minimization of the requirement of electronic conversions. With the current level of technology it is not possible to analyze data in the optical domain, so total elimination when providing traffic analysis is not possible. In such a case, clearly, reducing the number of wavelengths that require packet level of switching is the other option. We will consider in this paper optimizing packet level switching to reduce the problem into grooming the traffic in a way that utilizes least number of wavelengths possible to carry such mixed traffic. Hence, the problem will be reduced to maximizing the number of transparent wavelengths that are dedicated to a single traffic stream.

Though several research efforts have addressed the problem of traffic grooming for ring networks[6],[7],[8], and optimal traffic grooming in WDM networks [9], none of the formulations have focused on metropolitan networks exclusively or exploited the network architecture to minimize the latency occurred due to traffic grooming. In this paper, we consider the traffic grooming problem for a single-hop network, typically a star network, an architecture suitable for a metropolitan and local area network. In such an architecture, the access node forms the nucleus of the network, with all the edge nodes around it. The access node is responsible for analyzing the traffic that has to go out to the WAN, or redirected to other nodes in the access network. And the only opto-electronic conversion is done at the hub (access) node. The traffic has to be thus groomed in order to minimize the mixed wavelengths

2
requiring conversions at the hub node. We will term this problem as the Minimal Cost Traffic Grooming Problem (MCTG). The problem turns out to be NP-hard, a complexity we will show in the paper, so we devise a simple greedy heuristic that performs within twice the optimal solution.

In Section 2, we will formally define the MCTG problem and reduce it to a graph-theoretical problem that is easier to analyze. We then provide proof of its complexity, and discuss a polynomial time heuristic for it in Section 4 and define a bound on its performance. The numerical results for the heuristic in comparison to the optimal solutions would be discussed in Section 4.3, and finally the conclusions in Section 5.

2 Network Model and Problem Formulation

We consider the following WDM network for our problem. Consider $G$ to be a star network with node set $\{v_0, v_1, \ldots, v_n\}$, where $v_0$ denotes the hub node. Each node $v_i$ for $1 \leq i \leq n$ is connected to $v_0$ by two directed links (one in each direction), and each link supports $W$ wavelengths, and further each wavelength can support up to $g$ low-rate circuits. The hub node is equipped with $2nW$ LTEs (one for each channel in each direction) and a DCS. A Lightpath Terminating Equipment (LTE) is an add-drop multiplexer, capable of pulling individual wavelength signal from a multiplexed signal stream, and inserting different signal down the line, and a Digital Cross Connect Switch (DCS) is the actual equipment that provides groomed traffic. So the hub node is capable of analyzing each traffic stream from any groomed channel and switching it back to a different groomed channel.

A traffic demand for the network is denoted by a matrix $T[(t(i,j)), 1 \leq i, j \leq n$, where $t(i,j)$ denotes the traffic (denoting the number of low-rate circuits) from $i$ to $j$, such that $[(\Sigma \{ t(i,j) \mid 1 \leq j \leq n\}) / g] \leq W$, (1 \leq i \leq n), and $[(\Sigma \{ t(i,j) \mid 1 \leq i \leq n\}) / g] \leq W$, (1 \leq j \leq n). The WAN connected to the star network can be assumed to be one of the nodes connected to the hub node. During the rest of the paper, we are concerned with time costs related to network traffic.

**Minimal Cost Traffic Grooming (MCTG) Problem:** Given such a network $G$, we define the MCTG problem to be the problem of minimizing the total number of circuits that need to be groomed at the hub node, i.e., of maximizing the number of wavelengths that carry a single traffic stream (i.e., any such wavelength $w$ on link $(i,v_0)$ is assigned traffic $t(i,j)$ for some fixed $j$). More precisely, the problem is to define an $n \times n$ decision matrix $A[a(i,j)]$, where $a(i,j) = 1$ if and only if the traffic stream $t(i,j)$ is chosen to be groomed such that

1. $a(i,j) \in \{0, 1\}$ for $1 \leq i, j \leq n$, 

(2) \[ \left( \sum_{i \leq j \leq n} t(i,j) a(i,j) \right) / g \leq W - \sum_{i \leq j \leq n} \{ 1 - a(i,j) \}, \]
(1 \leq i \leq n),
(3) \[ \left( \sum_{1 \leq i \leq n} t(i,j) a(i,j) \right) / g \leq W - \sum_{1 \leq i \leq n} \{ 1 - a(i,j) \}, \]
(1 \leq j \leq n),
(4) satisfying (1)-(3), \( \sum_{1 \leq i, j \leq n} t(i,j) a(i,j) \) is minimized.

Constraints (2) and (3) ensure the availability of at least one wavelength per non-groomed traffic stream, on each outgoing link and incoming link of a node respectively. Constraint (4) is equivalent to maximizing the number of wavelengths assigned to the non-groomed traffic.

Note that in the above formulation, for any \( t(i,j) = 0 \), any optimal solution should assign \( a(i,j) = 1 \). The following example illustrates the formulation of the MCTG problem.

Consider an example of a star network shown in Figure 1. The four nodes are attached to the hub node 0. It is assumed that \( W = 2 \) and \( g = 16 \). The traffic demand \( T(t(i,j)) \) is given as: \( t(1, 2) = 7, t(1, 3) = 6, t(1, 4) = 9, t(2, 1) = 2, t(2, 4) = 8, t(3, 1) = 8, t(3, 2) = 5, t(3, 4) = 8, t(4, 3) = 6 \). One feasible solution \( A(a(i, j)) \) is that \( a(1, 4) = 0, a(2, 1) = 0, a(3, 2) = 0, a(4, 3) = 0 \), and any other \( a(i, j) \)'s are all 1. With this arrangement, the four wavelengths(channels) carrying traffic between (1, 4), (2, 1), (3, 2), and (4, 3) do not require any traffic analysis at the hub node. All the traffic on these wavelengths will be switched to their corresponding destination nodes as soon as it is received at the hub node. On the other hand, an optimal solution would be \( a(1, 4) = 0, a(2, 1) = 0, a(3, 1) = 0, a(4, 2) = 0, a(4, 3) = 0, \) the rest of them being equal to 1. This increases the number of transparent wavelengths by one more than the previous non-optimal solution.

Next, we introduce a graph-theoretic formulation of the MCTG problem and show its equivalence to the MCTG problem. For a given graph \( G(V, E) \), \( E(v) \) denotes the set of edges in \( E \) adjacent to \( v \) in the rest of the paper.

**Maximal Weighted Local Constrained Subgraph (MWLCS) Problem:** Given an integer \( C \) and a bipartite graph \( G = (X, Y; E) \), where each edge \( e \in E \) is associated with an integer \( w(e) \) and each vertex \( v \in X \cup Y \) is associated with an integer \( \alpha(v) \) such that \( \left( \sum_{e \in E(v)} w(e) \right) / C \leq \alpha(v) \) for each \( v \in X \cup Y \), the problem is to find a subset \( E_0 \subseteq E \) of edges such that \( \sum_{e \in E_0} w(e) \) is maximized, and \( \left( \sum_{e \in E(v) \setminus E_0} w(e) \right) / C \leq \alpha(v) - |E(v) \cap E_0| \) for each \( v \in X \cup Y \).

**Theorem 1** The MCTG problem is equivalent to the MWLCS problem when \( \alpha(v) \)'s are all equal for all \( v \in X \cup Y \).
Figure 1: Traffic Grooming in Star Network

Proof: The equivalence can be verified by transforming any instance of the MCTG problem to a corresponding instance of the MWLCS problem. For any instance $G(V, E)$ of MCTG, the transformation to the MWLCS instance $G'(X, Y, E')$ can be done by keeping $X = V - \{v_0\}$, $Y = V - \{v_0\}$, $E'(i, j) = t(i, j)$, where $i \in X, j \in Y; C = g$; and $\alpha(v) = W$ for each $v \in X \cup Y$. By defining $E_0$ to be equal to the inverse of the decision matrix $A$, it can be verified easily that the feasibility constraint of MWLCS reduces to the second and third constraint of the MCTG problem. Similarly, the optimization objective also becomes equivalent. The inverse transformation from MWLCS to MCTG is also the same.

3 Problem Complexity

The problems we described in the last section are not easy to solve and turn out to be NP-complete. We prove here that the decision version of the MWLCS problem
(called **D-WLCS problem**) is NP-complete, consequently proving the NP-hardness of the MWLCS problem. The D-WLCS problem is stated as: given an integer $C$, a bipartite graph $G$ with $w(e)$ and $\alpha(v)$ defined similarly as in the MWLCS problem, and an integer $\gamma$, the problem is to decide whether there exists a subset $E_0 \subseteq E$ such that

1. $[(\sum_{e \in E(v) \setminus E_0})/C] \leq \alpha(v) - |E(v) \cap E_0|$ for each $v \in X \cup Y$

2. $\sum_{e \in E_0} w(e) \geq \gamma$.

The NP-completeness proof is based on the following known NP-complete problem.

**3-Partition Problem**: Given a set $A = \{a_1, \cdots, a_{3m}\}$ of $3m$ integers such that $B/4 < a_i < B/2$ for each $a_i$ ($1 \leq i \leq 3m$) where $B = \sum_{1 \leq i \leq 3m} a_i/m$, the problem is to decide whether $A$ can be partitioned into disjoint $m$ sets $A_1, \cdots, A_m$ such that $\sum_{a_i \in A_j} a_i = B$ for each $1 \leq j \leq m$.

**Theorem 2** The D-WLCS problem is NP-complete.

**Proof**: It is easy to see that the D-WLCS problem can be solved non-deterministically in polynomial time, and we only discuss a polynomial time transformation of the 3-Partition problem to the D-WLCS problem.

Let $A$ be an instance to the 3-Partition problem, then an instance to the D-WLCS problem is constructed as follows, by first assuming $B < g < (m-1)B$. The vertex set $X \cup Y$ is defined as $X = \{x_i \mid 1 \leq i \leq 3m\}$ and $Y = \{y_i \mid 1 \leq i \leq m\} \cup \{z_i \mid 1 \leq i \leq 3m, 1 \leq j \leq m\} \cup \{(x_i, z_i) \mid 1 \leq i \leq 3m\}$. The weight $w(e)$ of each edge $e$ is defined as: (i) $w(x_i, y_j) = a_i$ for each edge of type $(x_i, y_j)$ and (ii) for each edge of type $(x_i, z_i)$, $1 \leq i \leq 3m$, $w(x_i, z_i) = (m-1)B + 2g - (m-1)a_i$. Let $\alpha(v) = (m-1)B/g + 3$ for all $v \in X \cup Y$. Finally, $\gamma$ is defined to be $mB$. One can easily verify that $[(\sum_{e \in E(v)} w(e))/g] \leq \alpha(v)$ for each $v \in X \cup Y$.

Figure 2 shows an example to construct a bipartite graph for a 3-partition problem instance.

We now show that there exists a desired partition of $A$ into $A_1, \cdots, A_m$ if and only if there exists a solution $E_0 \subseteq E$ to the D-WLCS problem such that $\sum_{e \in E_0} w(e) = mB$.

Let $A_1, \cdots, A_m$ be a solution to the 3-Partition problem. Without loss of generality, assume that $a_1, a_2, a_3 \in A_1$, $a_4, a_5, a_6 \in A_2$, \cdots, $a_{3m-2}, a_{3m-1}, a_{3m} \in A_m$. We can then construct an edge set $E_0$ such that $E_0 = \{(x_{3i-2}, y_i), (x_{3i-1}, y_i), (x_{3i}, y_i) \mid 1 \leq i \leq m\}$. Clearly, $\sum_{e \in E_0} w(e) = mB$, and $E_0$ is a feasible solution.

Suppose the D-WLCS problem has a feasible solution, say $E_0$. From the construction of $G$, we note that in any feasible solution, the number of edges incident at
Figure 2: Transformation of 3-Partition problem into MWLCS problem

each $x_i, 1 \leq i \leq 3m$, is at most one. To obtain $\sum_{e \in E_0} w(e) \geq mB$, the only possible arrangement is to have exactly one edge of weight $a_i$ in $E_0$ for each $1 \leq i \leq 3m$. Furthermore, $\sum_{e \in E(y_i) \cap E_0} w(e) = B$ for each $1 \leq i \leq m$. This provides a solution to the 3-Partition problem, which completes the proof of the theorem.

The result in Theorem 2 implies that the MWLCS problem is NP-hard. This together with the equivalence result in Theorem 1 establish the following main theorem in this section.

**Theorem 3** The MCTG problem is NP-hard.

4 A Greedy Heuristic

The MWLCS problem (and consequently, the MCTG problem) was shown to be NP-hard in the previous section. In this section, we present a greedy heuristic algorithm for the MCTG problem, and analyze a performance bound for the algorithm.
4.1 Algorithm

Since the MCTG problem has already been shown to be equivalent to the MWLCS problem, we present a heuristic algorithm to the MWLCS problem. Our greedy heuristic algoredges in the given bipartite graph \( G(X, Y; E) \). Edges are initially sorted such that \( w(e_1) \geq \cdots \geq w(e_m) \), where \( m \) is the number of edges in \( G \). Initially, we assume that \( \alpha(v) = W \) (where \( W \) is the number of wavelengths supported by each link in the formulation of the MCTG problem) for each \( v \in X \cup Y \).

<table>
<thead>
<tr>
<th>Algorithm 4.1</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>MWLCS – HEURISTIC</strong></td>
</tr>
</tbody>
</table>

**Input:** A bipartite graph \( G(X, Y; E) \), \( w(e) \) for each \( e \), \( \alpha(v) = W \) for all \( v \), and \( C \)

**Output:** \( E_0 \subseteq E \)

Set \( E_0 = \emptyset \).
Let \( w(e_1) \geq \cdots \geq w(e_m) \).
for \( i = 1 \) to \( m \) do
  if \( E_0 \cup \{e_i\} \) is a feasible solution, add \( e_i \) to \( E_0 \).
endfor
end Algorithm.

The time complexity of the algorithm is \( O(n^2 \log n) \), which comes from \( O(n^2 \log n) \) sorting time, and \( O(n^2) \) time to check feasibility condition for each edge.

4.2 2-Optimality Analysis of MCTG

In this section, we show that the MWLCS heuristic algorithm guarantees an \( \frac{1}{2} \)-optimal solution to the MWLCS problem, which in turn guarantees a 2-optimal solution to the MCTG problem. In the rest of the paper, \( E_0^{greedy} \) and \( E_0^{optimal} \) are used to define solution sets to the MWLCS problem from greedy and optimal algorithms respectively.

**Lemma 1** \( 2 \ast |E_0^{greedy}| \geq |E_0^{optimal}| \)
Proof: Let $E = \{e_1, e_2, \ldots, e_n\}$ be the edge set for the MWLCS problem, where $w(e_i) \geq w(e_j), 1 \leq i < j \leq n$. Assume that any solution for the problem is represented by a 0/1 vector of size $n$.

Let $X = (x_1, x_2, \ldots, x_n)$ be an optimal solution as opposed to $Y = (y_1, y_2, \ldots, y_n)$ which is a greedy solution. Let $i$ denote the first index for which $x_i \neq y_i$. We build solution $X$ from solution $Y$ by removing edges from solution $Y$ and possibly adding more edges to it.

- Case 1: $x_i = 1, y_i = 0$ Clearly, this is a contradiction to the greedy algorithm where $e_i$ is the largest weight edge that can be added to the solution without violating the feasibility constraint (as indicated by $x_i = 1$), has not been added to the greedy solution.

- Case 2: $x_i = 0, y_i = 1$ If edge $e_i$ is removed from the greedy solution, at most 2 edges, each incident at vertices on either end of $e_i$ and belonging to $X$, can be added to the solution, without violating the feasibility constraint. Since for each extra edge added to the solution, the right hand side value of the feasibility constraint decreases by 1 for some vertex, while the left hand side value decreases by less than 1. Extra edges can be added only when they decrease the left hand value by exactly 1, in which case they will always be added, even by greedy algorithm.

Thus we can transform solution $Y$ to solution $X$ by adding to $Y$ at most twice the number of edges that have been removed from $Y$. Since the total number of edges that can be removed from $Y$ cannot exceed the number of edges in $Y$, and any edge that already exists in $X$ also exists in $Y$, $2 \ast |E_0^{greedy}| \geq |E_0^{optimal}|$. $\blacksquare$

For any edge set $E' \subseteq E$, let $w(E') = \sum \{w(e) \mid e \in E'\}$. We then establish the following result.

**Theorem 4** $2 \ast w(E_0^{greedy}) \geq w(E_0^{optimal})$

Proof: From Lemma 1, while transforming a greedy solution $Y$ to an optimal solution $X$, each edge that was removed was the largest weight edge that did not exist in $X$ but existed in $Y$. For each edge $e_i$ that is removed, at most two edges were added, and any new edges considered have to have weight at most $w(e_i)$, or else either they already exist in both $X$ and $Y$, or the assumption that $e_i$ is the largest weight edge not in $X$ but in $Y$, is a contradiction. If the two edges added are $e_i, e_m$, then

$$w(e_i) \geq w(e_i)$$  \hspace{1cm} (1)

$$w(e_i) \geq w(e_m)$$  \hspace{1cm} (2)

$$2 \ast w(e_i) \geq w(e_i) + w(e_m)$$  \hspace{1cm} (3)
Hence from Lemma 1 and the above result, we can show \(2w(E^\text{greedy}) \geq w(E^\text{optimal})\).

We will further strengthen our claim in Theorem 4 from numerical results discussed in Section 4.3, and show that the actual performance of the greedy heuristic is much better than the theoretical bound of at least half of optimal.

### 4.3 Experimental Results

We implemented an Integer Programming formulation for the MWLCS problem to obtain optimal solutions for comparison with greedy results. Our integer program formulation was solved using ILOG’s CPLEX. To cover a wide range of solutions, the test network traffics were randomly generated for networks of different sizes. The average was taken over 500 test cases for each network size. For all the test cases, \(g = 16\). The traffic between each pair of nodes in the network has been randomly chosen to occupy at most half the bandwidth available for each wavelength.

![Graph showing Greedy and Optimal Traffic Grooming Performance]

**Figure 3:** Average Deviation of Greedy Heuristic from Optimal Solution

The performance has been measured in terms of percentage deviation of the greedy solution \((S_G)\) from the optimal solution \((S_O)\), given by \(100 \times (1 - \frac{S_G}{S_O})\). As is
Figure 4: Maximum Deviation of Greedy Heuristic from Optimal Solution

evident from Figure 3, the average performance degradation for the greedy heuristic remains well within 10% of the optimal solution, and improves as more wavelengths are available in the network. Figure 4 provides the maximum deviation observed between the greedy and optimal solutions among all the test traffics for a particular size of the network. It follows the trend of the average deviation from the optimal solution for the previous figure, and remains close to the optimal solution and well within the bound of 2 shown in the previous section. The maximum deviation however increases if the badwidth resources are tight with respect to the network size. For a network with 5 nodes and 2 wavelengths, the maximum deviation was observed to go as high as 36%.

5 Conclusion

All-optical networks are capable of satisfying the needs of the current and future high-speed network traffic. Although the network architecture contains bottleneck points that require slower opto-electronic conversions and hence do considerable
damage to the latency of the system. We studied the reduction of such damage by efficiently grooming traffic that requires less analysis by the access nodes that form the focal point of the star network architecture most suitable for all-optical networks at the MAN or LAN level. We then presented in the paper the hardness of the problem and proposed a simpler but highly effective greedy method as a solution to it. The numerical results show that the greedy heuristic performs closer to the optimum solution than the theoretical bound, making it suitable to adapt for traffic grooming in a single-hop architecture metropolitan area network.

References


